S. N. Atluri, A. K. Amos

# Large Space Structures: Dynamics and Control

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With 166 Figures

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# Preface

This monograph is intended to provide a snapshot of the status and opportunities for advancement in the technologies of dynamics and control of large flexible spacecraft structures. It is a reflection of the serious dialog and assessments going on all over the world, across a wide variety of scientific and technical disciplines, as we contemplate the next major milestone in mankind's romance with space: the transition from exploration and experimentation to commercial and defense exploitation.

This exploitation is already in full swing in the space communications area. Both military and civilian objectives are being pursued with increasingly more sophisticated systems such as large antenna reflectors with active shape control. Both the NATO and Warsaw pact alliances are pursuing permanent space stations in orbit: large structural systems whose development calls for in-situ fabrication and/or assembly and whose operation will demand innovations in controls technology.

The last ten years have witnessed a fairly brisk research activity in the dynamics and control of large space structures in order to establish a technology base for the development of advanced spacecraft systems envisioned for the future. They have spanned a wide spectrum of activity from fundamental methods development to systems concept studies and laboratory experimentation and demonstrations. Some flight experiments have also been conducted for various purposes such as the characterization of the space enviroment, durability of materials and devices in that environment, assembly and repair operations, and the dynamic behavior of flexible structures. It is this last area that has prompted this monogram. The emphasis is clearly on the basic analytical and experimental methods development aspects of the technology.

The principal aims of this monopgrah are to bring together the view points of the structural dynamicists and the control theoreticians and, through this interdisciplinary dialogue, to facilitate further coordinated efforts in resolving outstanding technical problems in the nonlinear dynamics and control of highly flexible space structures.

In Chapter 1, Noor and Mikulas deal with several issues pertinent to the development of equivalent-continuum models for beam-like and plate-like lattices. These issues include: (i) a definition of the equivalent continuum, (ii) characterization of the continuum model and (ii) different approaches for the generation of stiffness, inertia, and thermal properties of the equivalent continuum. In Chapter 2, Atluri and Iura discuss computational methods to treat nonlinearities of the structural, the inertial, and of the damping (due to flexible hysteritic joints) type that arise in the context of dynamics and control of LSS. Both semi-discrete type and space-time type methods to analyse the transient nonlinear response are discussed. Algorithms for implementing control on nonlinear semi-discrete type coupled systems of ordinary differential equations, are discussed. Reduced-order structural modeling techniques for both the equivalent-continuum models of LSS, as well as for truss and frame type lattice structures are discussed. Simple finite element methods for beam type LSS undergoing large rotational deformations, and field-boundary-element methods for shell-type LSS undergoing large deformations are discussed. Exact and explicit expressions for tangent stiffness operators of truss and frame type LSS are given. Finally, Atluri and Iura discuss the mechanical coupling between structural members and piezo-electric-film type control actuators, and give expressions for the actuator forces as functions of the excitation voltages and the magnitudes of generalized internal forces in the member

In Chapter 3, Hu, Skelton, and Yang discuss the question of the contribution of a specific mode of linear vibration to the norm of the nodal response vector of a structure, in a situation wherein the promary consideration is the accuracy of dynamic response of a structure at specific locations. This question is of importance in deciding which modes are to be retained in creating a reduced order structural model for the purpose of designing the controllers. In Chapter 4, Modi and Ibrahim discuss a general formulation, applicable to a class of space platforms with flexible extendable members, for analyzing transient dynamic situations involving complex interactions between deployment, attitude dynamics, and flexural rigidity. In Chapter 5, Park surveys partitioned solution procedures for large-scale simulation of dynamics and control of space structures which involve structural elements capable of large overall as well as flexible motions.

In Chapter 6, Zak discusses the case of pulse excitations of structures. In such cases, if a finite dimensional discrete model of the structure is employed, a loss of contribution of the high-frequency modes to the dynamic response may result. Zak shows that the thus unmodelled part of the response can be represented by a system of thin pulses and discusses the associated fundamental dynamical properties of the system. In Chapter 7, Bainum reviews the topics of mathematical models for orbital dynamics of large flexible structures, numerical techniques for synthesizing shape and attitude control laws, and the modelling of environmental disturbance torques due to the interaction of solar pressure on vibrating and thermally deformated structures.

In Chapter 8, Srinivasan discusses the phenomenon of friction between contacting surfaces, and addresses related issues such as quantifying the nature and magnitude of friction forces, quantifying the nature and magnitude of vibratory motion at contacting interface, and predicting the extent of damping that may be present. These topics are of interest in the passive control of space structures. In Chapter 9, Meirovitch discusses the active control concept of the independent-modal-space-method, and the concept of direct feddback control. He also discusses the related issues of deciding suitable control gains, the presence of damping, etc. In Chapter 10, von Flotow discusses the limiting case, labelled the acoustic limit, wherein the control bandwidth includes a very large number of natural modes. He argues that, in this limit, the modal analysis approach to control design is of limited value, and discusses alternate approaches involving wave-propagation formalisms applicable to flexible lattice-type structures.

In Chapter 11, Lynch and Banda present the control design techniques of Linear Quadratic Gaussian with Loop Transfer Recovery, for large space structures, wherein the high-frequency modeling uncertainties necessitate a robust control design. In Chapter 12, Bernstein and Hyland review the machinery of Optimal Projection for Uncertain Systems, for active control of flexible structures, and demonstrate its practical value. In Chapter 13, Kosut presents an approach to the problem of designing a robust control using on-line measurements, emlpoying the methods of parameter identification to obtain a nominal estimate of the plant-transfer function. Non-parametric spectral methods are then used to obtain a freqeuency domain expression for modal uncertainty. If the modal uncertainty exceeds a specified frequency bound, data filters used in the system-identification are modified, and the procedure is repeated. He also presents an analysis which establishes conditions under which the procedure converges to a satisfacotry robust design.

In Chapter 14, Junkins and Rew address the question whether a feed-back control law, designed based on a linear finite dimensional discrete mathematical model of a flexible structure, will stabilize and near-optimally control the real system. They emphasize the development of robust eigenstructure assignment methods and summarize optimization methods in which both the controller and selected strucural parameters are redesigned to improve the robustness. In the final Chapter 15, Khot presents two approaches for the optimum design of a strucutre and its control system with the objective of modifying the structural stiffness in order to achieve both a minimum weight structure, and a desired spectrum of closed-loop eigenvalues and structural frequencies.

The editors believe that these fifteen chapters collectively form a sound foundation for the subject of dynamics and control of flexible structures, wherein rapid scientific advances are to be expected in the next decade or so. It is towards this objective that the editors hope that this monograph would serve as a catalyst.

It is a great pleasure to thank all the authors for their kind cooperation through a timely preparation of their manuscripts. The editors also thank the Harris Corporation, Government Aerospace Systems Division, of Melbourne, Florida for their kind permission to use the illustration that appears on the cover of this manograph. A note of thanks to Ms. Deanna Winkler is also recorded here, for her assistance in the various editoral tasks.

Atlanta and Washington, D. C., July 1987 — Satya N. Atluri, Anthony K. Amos

# Contents

# Continuum Modeling of Large Lattice Structures: Status and Projections

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SUMMARY

The status and some recent developments of continuum modeling for large repetitive lattice structures are summarized. Discussion focuses on a number of aspects including definition of an effective substitute continuum; characterization of the continuum model; and the different approaches for the generation of the properties of the continuum, namely, the constitutive matrix, the matrix of mass densities, and the matrix of thermal coefficients. Also, a simple approach is presented which can be used to generate analytic expressions and/or numerical values of the continuum properties.

Application of the proposed approach to some beamlike and double-layered platelike lattices, currently considered as candidates of large space structures, is described. Future directions of research on continuum modeling are identified. These include needed extensions and applications of continuum modeling as well as computational strategies and modeling techniques.

SYMBOLS

| | |
|---|---|
| $A^{(k)}$ | cross-sectional area of member k of the repeating cell |
| [C] | matrix of stiffness coefficients of the simplified continuum |
| $C_{11}, C_{12}, C_{13}, \ldots C_{88}$ | stiffness coefficients of the simplified continuum (see Tables 1 and 2 and Figs. 4 and 5) |
| $\{C_T\}$ | thermal load vector of the simplified continuum |
| $d_1, d_2, \ldots d_6$ | generalized displacements (see Figs. 4 and 5) |
| $E^{(k)}$ | elastic modulus of the material of member k of the repeating cell |
| $F_1, F_2, \ldots F_6$ | generalized internal forces in the continuum beam model (see Fig. 4) |
| $G_{11}, G_{12}, G_{13}$ | partitions of the matrix $[G]_c$, see Eqs. 37 |
| $[G]_c$ | geometric stiffness matrix of the continuum |
| [g] | geometric stiffness matrix of the simplified continuum |
| $[g^{(k)}]$ | geometric stiffness matrix of member k of the repeating cell |
| [K] | stiffness matrix of the repeating cell |
| $[K]_c$ | stiffness matrix of continuum |
| $K_{11}, K_{12}, K_{13}, K_{22}, K_{23}, K_{33}$ | partitions of the matrix $[K]_c$, see Eqs. 14 |

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| | |
|---|---|
| $K$ | kinetic energy density of the continuum |
| $L^{(k)}$ | length of member k of the repeating cell |
| $M_2$, $M_3$, $M_t$ | bending and twisting moments in continuum beam model (see Fig. 4) |
| $M_{11}$, $M_{22}$, $M_{12}$ | bending and twisting stress resultants in continuum plate model (see Fig. 5) |
| $M_{11}$, $M_{12}$, $M_{22}$ | partitions of the matrix $[M]_c$, see Eqs. 15 |
| $[M]_c$ | matrix of density parameters of the continuum |
| $[m]$ | matrix of density parameters of the simplified continuum model |
| $m_{11}$, $m_{12}$, $m_{13}$, ... $m_{66}$ | density parameters of the simplified continuum (see Tables 1 and 2 and Figs. 4 and 5) |
| $[m^{(k)}]$ | consistent mass matrix of member k of the repeating cell |
| $\bar{N}$ | axial force in beamlike lattices (see Fig. 4) |
| $N_{11}$, $N_{22}$, $N_{12}$ | extensional stress resultants in continuum plate model (see Fig. 5) |
| $\{P_T\}$ | thermal load vector of the repeating cell |
| $\{P_T\}_c$ | thermal load vector of the continuum |
| $\{P_{T1}\}$, $\{P_{T2}\}$, $\{P_{T3}\}$ | partitions of the vector $\{P_T\}_c$, see Eqs. 16 |
| $Q_{12}$, $Q_{13}$ | transverse shearing forces in the y and z directions in continuum beam model |
| $Q_1$, $Q_2$ | transverse shear stress resultants in continuum plate model |
| $[R^{(k)}]$ | transformation matrix whose entries are products of direction cosines of member k |
| $S_1$, $S_2$, ... $S_8$ | stress resultants in the continuum plate model (see Fig. 5) |
| $T^o$ | temperature parameter (see Eqs. 26 and 32) |
| $T^{(k)}$ | temperature of member k of the repeating cell |
| $\{T\}_c$ | vector of temperature parameters used in describing the continuum |
| $U$ | thermoelastic strain energy density of the continuum |
| $U_1$, $U_2$ | contributions to the strain energy of the linear and quadratic terms in the temperature parameters |
| $U_o$ | isothermal strain energy |
| $u$, $v$, $w$ | displacement components in the coordinate directions |
| $\bar{u}$, $\bar{v}$, $\bar{w}$ | displacement parameters characterizing warping and cross-sectional distortions |
| $u^o$, $v^o$, $w^o$ | displacement parameters in the coordinate directions |
| $\{u\}^{(k)}$ | vector of nodal displacements of member k of the repeating cell |
| $\{u\}_c$ | vector of displacement parameters used in describing the continuum |
| $\{u_{c1}\}$, $\{u_{c2}\}$ | partitions of the vector $\{u_c\}$, see Eqs. 13 |
| $x$, $y$, $z$ | Cartesian coordinates |
| $\alpha^{(k)}$ | coefficient of thermal expansion of member k of the repeating cell |
| $[\Gamma_u^{(k)}]$, $[\Gamma_\epsilon^{(k)}]$, $[\Gamma_T^{(k)}]$ $[\bar{\Gamma}_\Delta^{(k)}]$ | transformation matrices (see Eqs. 6, 7, 8 and 34) |

| | |
|---|---|
| $\gamma^o_{xy}$, $\gamma^o_{xz}$, $\gamma^o_{yz}$ | shearing strain parameters in the coordinate planes |
| $\gamma_{12}$, $\gamma_{13}$, $\gamma_{23}$ | shearing strains in the coordinate planes |
| $\{\Delta\}_c$ | vector of displacement parameters and their spatial derivatives for the continuum |
| $\varepsilon^{(k)}$ | axial strain of member k of the repeating cell |
| $\varepsilon_{11}$, $\varepsilon_{22}$, $\varepsilon_{33}$ | axial strains in the coordinate directions |
| $\varepsilon^o_x$, $\varepsilon^o_y$, $\varepsilon^o_z$ | extensional strain parameters in the coordinate directions |
| $\{\varepsilon\}_c$ | vector of strain parameters used in describing the continuum |
| $\{\varepsilon\}^{(k)}$ | vector of strain components in the coordinate directions used in the expansion of $\varepsilon^{(k)}$. |
| $\{\varepsilon_{c1}\}$, $\{\varepsilon_{c2}\}$, $\{\varepsilon_{c3}\}$ | partitions of the vector $\{\varepsilon\}_c$, see Eqs. 12 |
| $\bar{\kappa}$ | strain parameter (see Eqs. 25) |
| $\kappa^o_x$, $\kappa^o_y$, $\kappa^o_z$, $\kappa^o_t$, $2\kappa^o_{xy}$ | curvature changes and twist parameters |
| $\rho$ | mass density of the material (see Fig. 6) |
| $\phi_x$, $\phi_y$, $\phi_z$ | rotation components |
| $\psi^o$ | strain parameter (see Eqs. 25) |
| $\Omega$ | characteristic geometric property of the repeating cell of the lattice (length of repeating cell for beamlike lattices and planform area of repeating cell for platelike lattices) |
| $\omega$ | frequency of vibration (see Figs. 8-11) |
| $(\ell, m, n)^{(k)}$ | direction cosines of member k |

$\partial_x \equiv \partial/\partial x$ , $\partial_y \equiv \partial/\partial y$ , $\partial_z \equiv \partial/\partial z$

Superscript t denotes transposition.

## 1. INTRODUCTION

Lattice structures have been used for many years in spanning large areas with few intermediate supports. These structures can combine low cost with light weight and an esthetically pleasing appearance. Also, due to their ease of packaging, transporting, and assembling in space, lattice structures have attracted considerable attention for use in large-area space structures such as the space station, large space mirrors, antennas, multipurpose platforms, and power systems for supporting space operations. A main feature of the large-area lattice structures considered for space applications is that the basic pattern or configuration is repeated many times.

A review of the state-of-the-art in the analysis, design and construction of lattice structures until 1976 is given in [14 and 31]. The currently-used approaches for analyzing large repetitive lattices can be grouped into four classes; namely:

1) direct method
2) discrete field methods
3) periodic structure approaches
4) substitute continuum approaches.

In the first approach (direct method) the structure is analyzed as a system of discrete finite elements, and the methods of solving structural framework problems are applied. It has the obvious drawback of being computationally expensive for large lattices. This is particularly true when a buckling, vibration, or a nonlinear analysis is required.

The second approach (discrete field methods) takes advantage of the regularity of the structure and involves writing the equilibrium and compatibility equations at a typical joint of the lattice and either solving the resulting difference equations directly, or using truncated Taylor series expansions to replace the difference equations by differential equations (see, for example, [15, 16, 50, 51 and 62]). This approach works well for simple lattice configurations, but becomes quite involved for lattices with complex geometry.

The third group of methods are referred to as periodic structure approach, and are based on either: a) the combined use of finite elements and transfer matrix methods, which is efficient only for rotationally periodic (i.e., cyclically symmetric) structures or lattices with simple geometries [33, 64 and 65], or b) the exact representation of the stiffness of an individual member from which the analysis of beamlike lattices with simply supported edges can be performed [4, 5 and 6].

The fourth approach is based on replacing the actual lattice structure by a substitute continuum model which is equivalent to the original structure in some sense, such as the constitutive relations, strain energy and/or kinetic energy (see, for example, [3, 17, 18, 19, 21, 24, 29, 34, 35, 37, 42, 43, 54, 55, 56, 59, 63 and 66]). The use of continuum models to simulate the behavior of planar lattice beams dates back to the previous century [61, p. 483]. It has gained popularity only in recent years and has been applied to a variety of other discrete systems and phenomena including solid and liquid crystals, dislocations and defects, composite materials and biological systems.

The number of publications on continuum modeling of repetitive lattice structures has been steadily increasing. Therefore, there is a need to broaden awareness among practicing engineers and research workers about the recent developments in various aspects of continuum modeling for large lattice structures. The present paper is a modest attempt to fill this void. Specifically, the objectives of this paper are:

1) to assess the effectiveness of the currently used approaches for continuum modeling;

2) to present a simple and rational approach for development of continuum models for large repetitive lattice structures; and

3) to identify the future directions of research which have high potential for realizing the advantages of continuum modeling.

The scope of the present study includes thermoelastic stress analysis, buckling, free vibration, and geometrically nonlinear problems of large lattice structures. Beamlike and platelike repetitive lattices with pin and rigid joints are considered. Continuum modeling of lattices with flexible joints will also be discussed.

## 2. ADVANTAGES OF CONTINUUM MODELING

Before an assessment is made of the different approaches for developing continuum models, the following three advantages of using the continuum modeling approach for analyzing repetitive lattice structures are identified. First, it offers a practical and efficient approach for analyzing large lattice structures. This is particularly true for beamlike and platelike lattices, wherein a dimensionality reduction can result in a substantial reduction in the number of degrees of freedom. Second, it provides a simple means of comparing structural, thermal, and dynamic characteristics of lattices with different configurations and assessing the sensitivity of their responses to variations in material and geometric properties; and third, it provides an effective tool for parameter/system identification and feedback control system design of lattice structures.

## 3. DEFINITION AND KEY ELEMENTS OF A SUBSTITUTE CONTINUUM MODEL

A number of definitions have been given for the substitute continuum model. Herein an *effective continuum model* is defined to be a continuum which has the following characteristics:

1) the same amount of thermoelastic strain and kinetic energies are stored in it as those of the original lattice structure when both are deformed identically;

2) the temperature distribution, loading and boundary conditions of the continuum simulate those of the original lattice structure being modeled;

3) for beamlike and platelike lattices the continuum models are one-dimensional beams and two-dimensional plates, respeetively (see Fig. 1);

4) local deformations are accounted for; and

5) lattices with pin joints are modeled as classical continua, and lattices with rigid (and/or flexible) joints are generally modeled as micropolar continua.

The last two characteristics are perhaps the most important in terms of recent developments and are discussed subsequently.

### 3.1 Local Deformations

The local deformations of two axially loaded planar trusses are shown in Fig. 2. The first truss has double lacing and a single-bay repeating cell. The second truss has single lacing and a double bay repeating cell. The cord members of the first truss remain straight as shown on the top sketch. On the other hand, the actual deformation of the single-laced truss has the zig-zag pattern shown on the top right sketch. On the average, however, the cord members remain straight. Early continuum models averaged these deformations, thereby substantially overestimating the axial stiffness. Recent continuum models, for lattices with more than one bay in their repeating cells, do account for the local deformations [37, 38 and 42].

### 3.2 Ordinary Versus Micropolar Continua

A contrast between the ordinary and micropolar continua is made in Fig. 3. For an

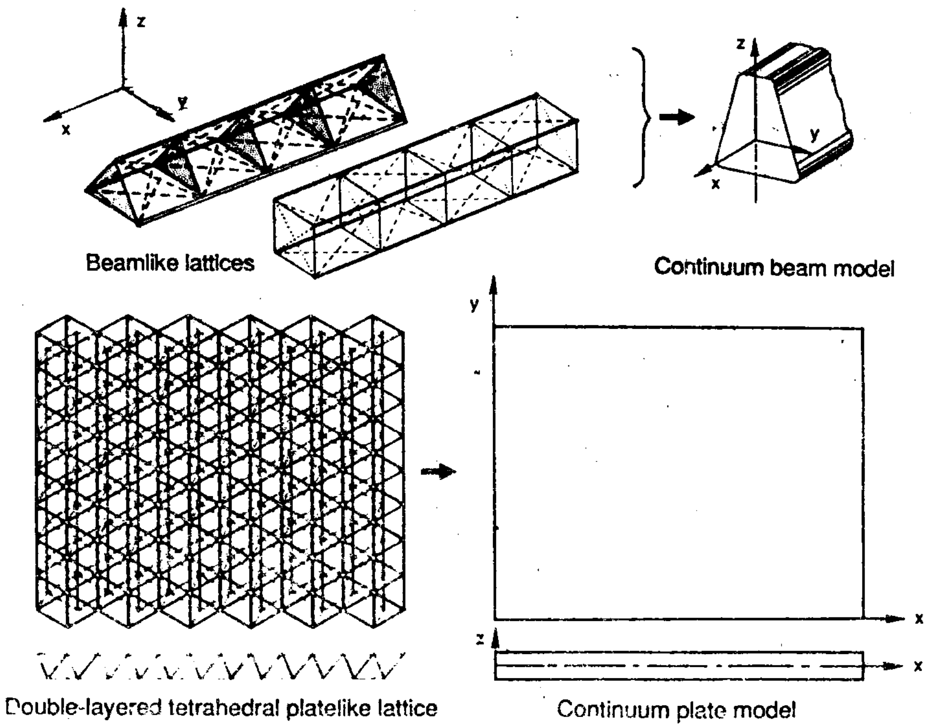



Figure 1 - Continuum models for beamlike and platelike lattice structures.

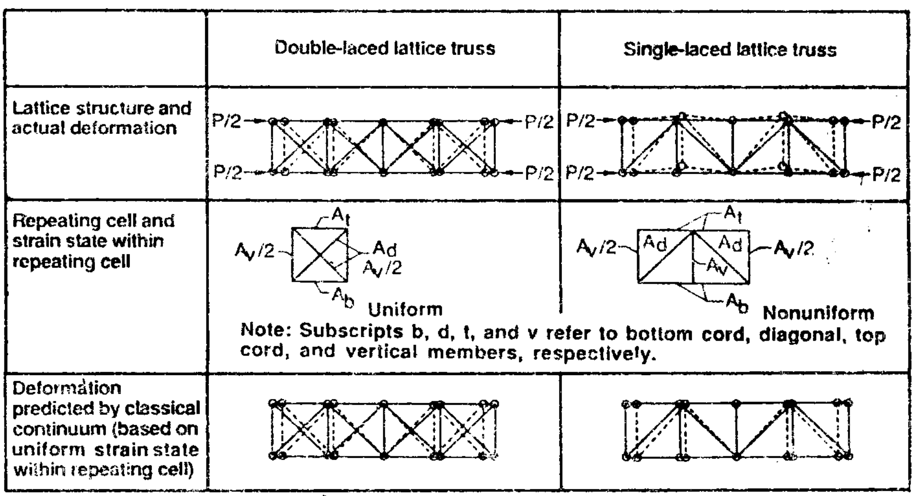



Figure 2 - Local deformations in planar lattice trusses subjected to axial loading.

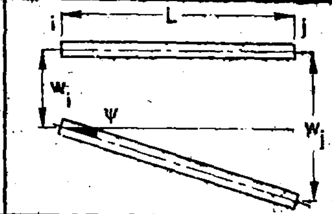
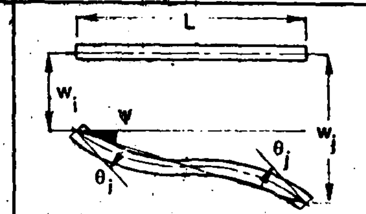
| | Truss member (pin joints) | Beam member (rigid joints) |
|---|---|---|
| Deformation pattern |   |   |
| Rotations | $\psi = \frac{1}{L}(w_j - w_i)$ | $\psi = \frac{1}{L}(w_j - w_i)$<br>$\theta_i, \theta_j$ (joint rotations) |
| Appropriate continuum | Ordinary (displacement field only) | Micropolar (independent displacement and rotation fields) |

Figure 3 - Deformation patterns for pin-jointed and rigid-jointed one-dimensional members

axially loaded pin-jointed truss member the transverse motion is completely characterized by the joint displacements. The member rotation ψ is related to the joint displacements $w_i$ and $w_j$. Therefore, the appropriate continuum to use in modeling pin-jointed trusses is the ordinary continuum for which the displacement field completely characterizes the motion of the structure.

On the other hand, for a rigid-jointed member, the transverse motion is characterized by both the joint displacements $w_i$, $w_j$ as well as the joint rotations $\theta_i$, $\theta_j$ which are independent degrees of freedom. Therefore, the appropriate continuum to use in modeling rigid-jointed flexural members is one whose motion is characterized by both a displacement field and an independent rotation field (referred to as microrotation field). The micropolar continuum is such a continuum.

## 3.3 Characterization of the Substitute Continuum Model

The substitute continuum model is characterized by the thermoelastic constitutive relations and density parameters which are determined in terms of the geometric and material properties of the original lattice structure. The thermoelastic constitutive relations and density parameters of the continuum can then be used to determine: a) the thermoelastic strain and kinetic energies; b) the governing differential equations; and whenever appropriate c) equivalent discrete finite element models.

## 3.4 Comments on Continuum Models

The following three comments regarding continuum models seem to be in order:

1. For some lattices the substitute continuum models may not have much resemblance to the continuum theories commonly used in engineering practice. Also, for complicated lattices the continuum models may be fairly complicated, and therefore, not useful for practical applications.

2. The accuracy of the predictions of the continuum approximation increases with the increase in the number of repeating cells (or modules) constituting the original lattice structure.

3. The response of the substitute continuum model (which simulates that of the original lattice structure) can be generated through: a) exact (or analytic) solution of the governing differential equations, or b) application of a discretization technique such as Rayleigh-Ritz technique or the finite element method.

## 4. DEVELOPMENT OF SUBSTITUTE CONTINUUM MODELS FOR STRESS ANALYSIS AND FREE VIBRATION PROBLEMS

A number of approaches have been proposed for developing continuum models, and for determining the appropriate constitutive relations and density parameters. These approaches include:

a) relating the force or deformation characteristics (or both) of a small segment of the lattice to those of a small segment of the continuum [20, 21, 22, 25, 26, 27, 28, 29 and 53];

b) using the discrete field method to obtain the governing difference equations of the lattice and either solving them directly or converting them to approximate differential equations [16, 50 and 51];

c) applying homogenization techniques based on using multiple-scale asymptotic expansions (see [7, 8, 11 and 32]); and

d) using energy equivalence concepts. The potential and kinetic energies of a typical (repeating) cell of the lattice are equated to those of the continuum, after expanding the nodal displacements of the lattice in a Taylor series.

The latter approach has been applied to a number of beamlike and platelike lattices. Computerized symbolic manipulation was used to generate analytic expressions for the stiffness and density parameters of the continuum (see [37, 38, 39, 41 and 42]). More recently, an equivalent approach was proposed for generating the properties of simplified one- and two-dimensional continuum models of beamlike and platelike lattice trusses with pin joints, which does not require the use of computerized symbolic manipulation (see [46]). Rather, numerical values of the stiffness and mass coefficients can be obtained by using a small Fortran program on an IBM PC (see [49]). A modified version of this approach is described subsequently.

The three key elements of the foregoing approach are:

1) introduction of kinematic and temperature assumptions to reduce the dimensionality of the continuum;

2) expansion of each of the nodal displacements, strain components, and temperature in a Taylor series; and

3) generation of four transformation matrices which relate nodal displacements, axial strains and temperatures of individual members of the repeating cell to the displacements, strain and temperature parameters of the continuum.

The procedure consists of the three major phases which are discussed subsequently for the case of lattices with pin joints.

Phase 1 - Generation of the Thermoelastic Stiffnesses of a Repeating Cell

1) A repeating cell (or module) is isolated from the lattice grid. The axial strain, temperature, and consistent mass matrix of a typical member, k, of the repeating cell are given by $\varepsilon^{(k)}$, $T^{(k)}$ and $[m^{(k)}]$, respectively.

2) The axial strain $\varepsilon^{(k)}$ of member k is expressed in terms of the vector of strain components in the coordinate directions through the following matrix equation:

$$\varepsilon^{(k)} = [R^{(k)}]\{\varepsilon\}^{(k)} \tag{1}$$

where

$$\{\varepsilon\}^{(k)} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \tag{2}$$

$$[R^{(k)}] = [\ell^2 \quad m^2 \quad n^2 \quad \ell m \quad \ell n \quad mn]^{(k)} \tag{3}$$

$\varepsilon_{11}$, $\varepsilon_{22}$, $\varepsilon_{33}$ are the axial strains in the coordinate directions; $\gamma_{12}$, $\gamma_{13}$, $\gamma_{23}$ are the shearing strains; and ($\ell$, m, n) are the direction cosines of the member.

For simplicity, in the present study the strain state is assumed to be uniform within each repeating cell. Variation of the strain state within the repeating cell can be accounted for by expanding $\{\varepsilon\}^{(k)}$ in a Taylor series about the center of the repeating cell. The number of terms in the Taylor series expansion is equal to the number of independent deformation modes of the repeating cell.

3) The stiffness matrix and the thermal load vector of the repeating cell are generated using the following equations:

$$[K] = \sum_{\text{members}} (EAL)^{(k)} [R^{(k)}]^t [R^{(k)}] \tag{4}$$

$$\{P_T\} = \sum_{\text{members}} (\alpha EAL)^{(k)} [R^{(k)}]^t T^{(k)} \tag{5}$$

where E, $\hat{A}$, L, $\alpha$ are the elastic modulus, cross-sectional area, length and coefficient of thermal expansion of member k; and superscript t denotes transposition.

The thermoelastic stiffnesses of the equivalent three-dimensional classical continuum are obtained by dividing the right-hand sides of Eqs. 4 and 5 by the volume of the repeating cell. Note that for members shared by n repeating cells, their cross sectional areas in Eqs. 4 and 5 are divided by n.