GRAVITATION, QUANTUM FIELDS AND SUPERSTRINGS

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PREFACE

The UGC Instructional Conference on 'Gravitation, Quantum Fields and Superstrings' held at Madras in December 1986 was organised by the Department of Theoretical Physics, University of Madras in collabaration with the Institute of Mathematical Sciences, Madras. The conference was second in a series aimed at bringing about an active interaction between people working on gravity and astrophysics on the one hand and field theory and elementary particles on the other. It was a sequel to the UGC supported instructional conference on 'Gravitation, Gauge Theories and the Early Universe' held at the Indian Institute of Science, Bangalore during May-June 1985.

This work comprises the content of courses of lectures on four broad topics, namely i) Structure formation in the universe, ii) Black holes and singularities, iii) Quantum field theory methods and iv) Superstrings. Each course included workshop sessions to elucidate the topics covered in the main lectures and to discuss problems. It has not been possibe to include a couple of lectrues in this volume as they have been published elsewhere.

We are grateful to the University Grants Commission, Delhi for providing the financial support to organise the conference. We also thank the Tata Institute of Fundamental Research, Bombay and the Institute of Mathematical Sciences, Madras for providing token grants. The conference was held in the Indian Institute of Technology, Madras. The participants and lecturers were also housed there. We are indebted to the I.I.T. authorities for providing these facilities. Our special thanks are due to Dr.B.Vishwanathan (Chemistry department), Dr.S.G.Kamath and Prof. V.B.Johri (Mathematics department) and Prof. V. Balakrishnan (Physics department) of I.I.T. Madras in this connection. We thank the members of the theoretical physics groups in the University of Madras and the Institute of Mathematical Sciences who helped us in the organisation of the conference.

We acknowledge the invaluable help rendered by Sri.J.Segar (Department of Theoretical Physics, University of Madras) in the preparation of the typescripts. We thank Smt. C. Balambal, Sri. R.Kasthurirangan and Sri. Sukumar of Devi Reprographics, Adayar, Madras for typing the manuscripts neatly and accurately.

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Madras
June 1988.

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CONTENTS

OBSERVATIONAL COSMOLOGY

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LECTURE I

Cosmology is the study of the large scale structure of the universe we live in. I shall begin by describing the so called 'standard' cosmology not because I believe it to be the correct model of our universe but as a theoretical framework to organise the relevant concepts. I am aware that such a procedure is questionable on epistemological grounds but have chosen it as a framework general enough to encompass various notions currently considered important.

The basic assumption is that our universe can be approximated to a good degree by a spatially homogeneous and is isotropic model. It should be emphasised that isotropy homogeneity but not vice versa (the cylinder is an example of a homogeneous but nonisotropic space).

The assumption of homogeneity and isotropy by itself has some interesting consequences. This implies that inter 'particle' distances are of the type

$$d(t) = R(t)r \qquad (1.1)$$

Thus

$$v(t) = \dot{r}(t) = \dot{R}(t)\, r = \dot{R}(t)/R(t)\, d(t) = H(t)\, d(t) \qquad (1.2)$$

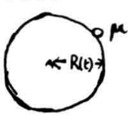
which is the famous hubble's law. If the unverse is expanding $H(t) > 0$. When the velocity $v(t) < c$, the redshift of the spectral lines $\sim v/c = z$ and one gets

$$z \sim H(t)/c\; d(t) \qquad (1.3)$$

Which is the famous redshift-distance relation. It should be appreciated that this is just a reflection of the isotropy and homogeneity and not of any detailed cosmological model.

Before developing a cosmological model based on general relativity theory, the presently accepted classical theory of gravitation, it is instructive to examine a cosmology based on newtonian gravitation and mechanics as it shares some essential features with a more complete treatment.

Let the test particle mass be μ and let $R(t)$ be the radius of the sphere of dust which will act gravitationally on μ. The energy conservation equation reads



$$\frac{1}{2}\mu \dot{R}^2 - \frac{GM\mu}{R} = -E \qquad (1.4)$$

which can be rewritten as

$$\frac{\dot{R}^2}{2} - \frac{GM}{R} = -1/2\,\kappa c^2 \qquad (1.5)$$

where $-1/2\ kc^2$ is the energy per unit mass. If it is assumed that matter is neither created nor destroyed, the mass conservation implies

$$4\pi\rho R^3/3 = M = \text{constant} \qquad (1.6)$$

As argued before, homogeneity and isotropy is sufficient to yield

$$v(t) = H(t)\, r(t) \qquad H(t) = \frac{\dot{R}(t)}{R(t)} \qquad (1.7)$$

When $k = 0$, the equation

$\dot{R}^2 = 2GM/R$ has the simple solution

$$\frac{2}{3} R^{3/2}(t) = \sqrt{2GM}.t \quad \text{i.e.} \quad R(t) = (9GM/2)^{1/3} t^{2/3}$$

and $$\rho(t) \sim t^{-2} \qquad (1.8)$$

close to $R \sim 0$, the kc^2 term is unimportant and the above behaviour is correct if $k = 0$. If k is positive, R cannot grow beyond

$$R_{max} = 2GM/kc^2 \qquad (1.9)$$

By a suitable choice of scale, k can be made to be ± 1. If k is negative, R(t) can increase indefinitely. For large R, the $kc^2/2$ term completely dominates and one has

$$R(t) = \sqrt{k}\ .\ ct \qquad (1.10)$$

LECTURE II

Let us turn our attention to a (cosmological) model of isotropic and homogeneous universe based on general relativity. The line element of such a (Friedman) universe can be written in the form

$$ds^2 = c^2 dt^2 - R^2(t)\,\{\frac{dr^2}{1-kr^2} + r^2\,(d\theta^2 + \sin^2\theta\, d\phi^2)\} \quad (2.1)$$

where $k = \pm 1, 0$. The curvature of a spatial hypersurface t = const is

$$^{(3)}R = \frac{6k}{R^2(t_0)} \quad (2.2)$$

which gives a very simple geometrical interpretation to the parameter k. If we approximate the matter distribution by a fluid the Einstein equations become (oo components)

$$\dot{R}^2 - 8\pi G\rho R^2/3 = -kc^2 \quad (2.?)$$

which takes exactly the same form as in Newtonian Cosmology even though k has a radically different interpretation (so does c!). The spatial components of Einstein's equation yield

$$\frac{2\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{kc^2}{R^2} = -8\pi Gp/c^2 \quad (2.4)$$

Differentiating (1) w.r.t.t,

$$2\dot{R}\ddot{R} - \frac{8\pi}{3}G\,2R\dot{R}\rho - \frac{8\pi}{3}GR^2\dot{\rho} = 0 \quad (2.5)$$

Eliminating $\ddot{R}$

$$2\dot{R}\ddot{R} + \frac{\dot{R}^3}{R} + \frac{kc^2\dot{R}}{R} = -\frac{8\pi G}{c^2}\,p\,\dot{R}R$$

$$\therefore\ 2\dot{R}\ddot{R} + \frac{\dot{R}^3}{R} - \frac{\dot{R}^3}{R} + \frac{8\pi}{3}G\rho R\dot{R} = -\frac{8\pi G}{c^2}\,p\dot{R}R$$

Therefore

$$\therefore 8\pi G\rho R\dot{R} + \frac{8\pi}{3} G R^2\dot{\rho} + \frac{8\pi G}{c^2} p \dot{R}R = 0$$

$$\text{i.e. } (\rho R^3)^{\cdot} + (R^3)^{\cdot}\frac{p}{c^2} = 0 \tag{2.6}$$

When the pressure is negligibly small

$$\rho R^3 = \text{constant} \tag{2.7}$$

Consequently

$$\rho(t) = \rho(t_0)\frac{R^3(t_0)}{R^3(t)} \tag{2.8}$$

The solution of the field equations is

$$\left.\begin{aligned} R &= \alpha(1-\text{Cos }\omega) \\ ct &= \alpha(\omega-\text{Sin }\omega) \end{aligned}\right\} \quad k = +1 \qquad \alpha = \frac{4\pi G}{3c^2}\rho R^3$$

$$R = \left(\frac{9GM}{2}\right)^{1/3} t^{2/3} \qquad k = 0$$

$$\left.\begin{aligned} R &= \alpha(\text{Cosh }\omega-1) \\ ct &= \alpha(\text{Sinh }\omega-\omega) \end{aligned}\right\} \quad k = -1 \tag{2.9}$$

Particle Horizon:

The propagation of light rays along radial direction is governed by

$$dS^2 = 0 = c^2dt^2 - R^2(t)\frac{dr^2}{1-kr^2} \tag{2.10}$$

Hence

$$c\int_{t_1}^{t}\frac{dt}{R(t)} = -\int_{r_1}^{0}\frac{dr}{\sqrt{1-kr^2}} = \int_{0}^{r_1}\frac{dr}{\sqrt{1-kr^2}} \tag{2.11}$$

The function

$$F(r) = \int_0^r \frac{dy}{\sqrt{1-ky^2}} = \sin^{-1} r \qquad k = 1$$

$$= r \qquad 0$$

$$= \sinh^{-1} r \qquad -1$$

If the universe has no singularity in the past i.e. steady state universes, $t_1 \to -\infty$ is possible. If $\int_{t_1}^{t_0} dt/R(t)$ diverges as $t_1 \to -\infty$ or as $t_1 \to 0$ if there is a singularity, it is possible to receive signals sent sufficiently early. If on the other hand

$$\int_0^{t_0} dt/R(t) \quad \text{exists,} \tag{2.13}$$

the observer will be able to receive signals only from particles that are within radius $r_H(t_o)$ or equivalently proper distance

$$d_H(t_0) = CR(t_0) \int_0^{t_0} dt'/R(t') \qquad = R(t_0) \int_0^{r_H(t_0)} dr/\sqrt{1-kr^2} \tag{2.14}$$

This is called the Particle Horizon. Whenever grows faster than $R^{-2-\varepsilon}$ as $R \to 0$ particle Horizons will be present.

If the greatest part of the t integral comes from matter dominated era, then $d_H(t_o)$ can be worked out analytically:

$$d_H(t_0) = \frac{R(t\ )}{R_0 H_0 \sqrt{2q_0 - 1}} \cos^{-1}\left[1 - \frac{(2q_0 - 1)R(t\ )}{q_0 \quad R_0}\right]$$

$$= 2/H_0 \; (R(t\ )/R_0)^{3/2}$$

$$= \frac{R(t\ )}{R_0 H_0 \sqrt{1-2q_0}} \cosh^{-1}\left[1 + \frac{(1-2q_0)R(t\ )}{q_0 \quad R_0}\right] \tag{2.15}$$

where $q = -R\ddot{R}/\dot{R}^2$ is the deceleration parameter.

In the early part of the matter dominated era

$$R(t) \ll R_0$$

Therefore

$$d_H(t) \sim \frac{R(t)}{R_0 H_0 \sqrt{2q_0-1}} \quad \frac{\sqrt{2}\sqrt{2q_0-1}}{\sqrt{q_0}} \sqrt{\frac{R}{R_0}}$$

$$\sim H_0^{-1} (q_0/2)^{-1/2} (R(t)/R_0)^{3/2} \qquad (2.16)$$

when $q_0 > 1/2$ the universe is spatially finite. Looking out in any direction we can see particles out to a fractions of the circumference $L = 2\pi R$

$$\frac{d_H(t)}{L(t)} = \frac{1}{2\pi} \cos^{-1}\left(1 - \frac{(2q_0-1)R}{q_0 \quad R_0}\right) \qquad (2.17)$$

When R reaches its maximum value

$$R_m = \frac{2q_0 R_0}{2q_0-1}$$

$$d_m = 1/2 \quad L_m \qquad (2.18)$$

and we are able to see the antipodes.

Just as there are some comoving particles that we cannot see, there may in general be events that we shall never see. An event that occurs at t_1, r_1, will become visible at $r = 0$ at t given by

$$\int_{t_1}^{t} dt'/R(t') = \int_0^{r_1} dr'/\sqrt{1-kr'^2} \qquad (2.19)$$

If the t' integral diverges as $t \to \infty$ or as $t \to T$ (the time of next contraction) then it will in principle be possible to receive signals from any event if we wait long enough. On the other hand if

$$\int_{t_1}^{t_{max}} dt'/R(t') \quad \text{converges,}$$

it will be possible to receive signals from events for which

$$\int_0^{r_1} dr/\sqrt{1-kr^2} \leq \int_0^{t_{max}} dt'/R(t') \qquad (2.20)$$

This is called the event Horizon. For $k = 1$

$$d_E(t_1) = \frac{R(t_1)}{R_0 H_0 \sqrt{2q_0-1}} \left(2\pi - \cos^{-1}\left\{1 - \frac{(2q_0-1)R(t_1)}{q_0 \quad R_0}\right\}\right) \qquad (2.21)$$

for $q_0 < 1/2$ $\quad R \sim t$ $\quad$ no event Horizon

$q_0 = 1/2$ $\quad R \sim t^{2/3}$ $\quad$ no event Horizon

$q_0 > 1/2$ $\quad \int^{t_{max}} dt/R$ converges $\quad$ event Horizon

Age of the Universe:

The Einstein equation in the matter dominated era can be recast as

$$(\dot{R}/R_0)^2 = H_0^2 \, [1-2q_0 + 2q_0 \, (R_0/R)]$$

$$t = 1/H_0 \int_0^{R/R_0} dx \, [1-2q_0 + 2q_0 \, (R_0/R)] \qquad (2.22)$$

Thus the present age of the universe is