Acoustic and Electromagnetic Waves

D. S. JONES

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D. S. Jones

Department of Mathematical Sciences
University of Dundee

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Preface

Much research has been devoted to problems in electromagnetism and acoustics in the past two decades. Often the investigations run along parallel lines, the main differences being due to the vector character of the electromagnetic field and the scalar nature of sound waves. Sometimes these differences are significant because of boundary conditions or polarization effects but often the analysis is very similar especially in those cases when the model in electromagnetics leads to a discussion of scalar fields. It was therefore thought that a book which traced the common strands and brought out any appropriate modifications in going from one topic to the other would be helpful to workers in both areas.

One aim of the book is to enable the reader to come within striking distance of modern research. It would therefore have been desirable to include numerous references to recent papers to assist the reader in becoming acquainted with the many contributors to various subjects over the past twenty years. However, the implementation of such a policy has proved to be impracticable. The last twenty years has constituted a period of intense activity and there would not be space for both a text and a complete list of relevant papers. For instance, in microstrips, a topic hardly touched upon, there have been 10 000 papers and there are now specialized journals appearing frequently which deal with areas forming a small fraction of the book. The sheer pace of publication renders any collation on a reasonable scale unmanageable. Accordingly, all efforts to maintain an up-to-date comprehensive list of references have been abandoned. Nevertheless, the hope is that the treatment of fundamentals in this book will place the student in a firm position to undertake research and a few references have been added for those with specialized interests.

As far as electromagnetism is concerned the starting point of this book is Maxwell's equations. The reader is assumed to have reached this point through the customary courses on static electricity, magnetostatics, and current electricity. The book follows logically on these courses in that its viewpoint is macroscopic. No endeavour is made to relate the macroscopic laws to any theory, such as wave mechanics, of microscopic structure. Similarly, fluid motion is regarded as a macroscopic phenomenon whose governing equations are assumed to be familiar to the reader though a brief derivation of the equations pertinent to the propagation of sound is given.

The first chapter is devoted to the general properties of solutions of the equations controlling acoustics and electromagnetism in matter which has certain macroscopic properties. The chief objective, therefore, is a delineation of the methods of construction whether by separation of variables, potentials, or integral representations. Energy considerations and the boundary conditions to be met in traversing the interface between two media are also examined. Chapter 2 is general in nature as well, being occupied with relativistic effects in mechanics, the transmission of light, and moving media.

After Chapter 2 specific problems are considered in more detail. Thus, in Chapter 3, the radiation is determined from a moving source of constant strength and from a fixed source of varying magnitude. The patterns to be expected when the sources are distributed on slender bodies are also studied. Next, the attributes when fields are confined to the interior of a hollow finite container are discussed in Chapter 4. The propagation in tubes and waveguides, together with the influence of intervening obstacles, is the burden of Chapter 5.

The theme of Chapter 6 is the transmission of waves in an infinite domain in the presence of a plane boundary across which the material properties change discontinuously. There are various possibilities depending on whether the media are homogeneous, isotropic, anisotropic, or inhomogeneous and particular attention is paid to the problem of a stratified atmosphere over a plane earth. The guiding of waves outside cylindrical structures is the subject matter of Chapter 7.

Chapters 8 and 9 attend to the scattering of waves by objects, smooth in the one chapter and possessing edges in the other. Both chapters begin with an examination of some exact solutions and then employ these as a basis to develop approximate techniques for general shapes.

The last chapter studies waves of arbitrary time variations, in contrast to Chapters 4–9 where harmonic waves form the main substance. Here will be found short descriptions of SEM (singularity expansion method), nonlinear waves, and features not normally encountered with harmonic waves.

A variety of mathematical techniques and special functions, with their own peculiar traits, is deployed. In one or two places, explanation accompanies the main text but those used most often have been collected together for convenience in seven appendixes. Five of these deal with special functions; two cover tensors and the asymptotic evaluation of integrals respectively. When equations in the appendixes are referred to in the principal text they are prefixed by the identifying letter of the appendix. Similarly, a cross-reference to an equation in another chapter will carry the chapter number first in the absence of any other indicator.

Each chapter has a set of exercises at its end. Some of these are to test how well the reader has grasped the content. There are also some which extend theory begun in the chapter or give details of notions which were excluded for one reason or another from the bulk of the discourse.

Naturally, the parts dealing with electromagnetism have something in common with the author's earlier book The Theory of Electromagnetism published by Pergamon in 1964, which has been out of print for some years. But many topics are covered in this book which were not included in the earlier one and, even where the subject matter overlaps, there are considerable changes in the treatment, emphasis, and approach.

Finally, the author wishes to express his gratitude to Mrs E. D. Ross whose forbearance, good humour, perseverance, and skill were responsible for the production of a readable typescript from a very ragged manuscript. The author's obligation to his wife Ivy is too great to ever be expunged—without her unwavering backing and inspiration nothing would have been achieved.

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Contents

1. The representation of acoustic and electromagnetic fields

MUCH of the analysis of the wave motion in acoustics and electromagnetism has common features but the relevant equations can be derived only when the reader has a suitable background knowledge. In this book it will be assumed that the reader has available certain mathematical information and is familiar with, for example, vectors, the divergence theorem, and Cauchy's theorem from previous study. More advanced techniques will also be called on but it is hoped that various explanatory appendixes will aid the understanding of these.

For the sections on electromagnetism it will be supposed that the reader is acquainted with the theory of experiments which lead us to believe that the behaviour of macroscopic electromagnetic phenomena is governed by Maxwell's equations (this knowledge is not required in order to read the material on sound waves). In most undergraduate courses this is achieved by discussing electrostatics and magnetostatics based on Coulomb's law of force between charges, by considering the magnetic forces of steady currents using the laws based on Ampere's experiments and by employing the fundamental results about varying fields that were obtained by Faraday. Accordingly, we shall assume that the reader has this basic information and shall develop from that point our investigation of the predictions ensuing from Maxwell's equations, making no attempt to deduce the principal properties of electrostatics, magnetostatics, and steady currents except in so far as they are required for some particular purpose.

This first chapter will be devoted to deriving general formulae without any relation to specific physical problems; the application to special problems will be undertaken in later chapters.

The equations of sound waves

1.1 The basic equations

Acoustics might be described as the theory of the propagation of small disturbances in fluids. The term fluid includes both liquids and gases. The main difference between liquids and solids is that no force is required to alter the shape of a liquid, provided that sufficient time is available, whereas considerable forces must usually be deployed to deform a solid. Rapid deformations are resisted by liquids—a display of *viscosity*. Liquids of all types, from the mobile to the highly viscous, exist but we shall generally be concerned with freely flowing substances such as water. Another property of liquids is their great resistance to change of volume so that it is very difficult to compress them or to persuade them to expand.

Gases, on the other hand, are distinguished from liquids by their ability to change volume comparatively easily. If they are given more space than they ordinarily occupy they fill the space uniformly. Also they can be compressed into a very small volume by the application of suitable forces. A gas exhibits viscosity so that liquids and gases may be roughly categorized as incompressible and compressible fluids respectively.

In the macroscopic theory of fluids, a fluid is regarded as a continuous distribution. The connection between the microscopic point of view, which considers all substances to be composed of molecules, and the macroscopic will not be discussed here though it is often convenient to think of a fluid as a conglomeration of particles. Such a particle should, however, be visualized as a little bit of fluid rather than as a molecule.

When the position of every particle of the fluid is known at every instant a complete picture of the flow is available. The problem of finding the positions consists in discovering expressions for the location (x, y, z) of a particle at time t given that it was at (X, Y, Z) at the time τ earlier. Then the equations

$$x = x(X, Y, Z, t), \qquad y = y(X, Y, Z, t), \qquad z = z(X, Y, Z, t) \tag{1.1}$$

furnish a curve as t varies which is the *path* of the particle that started from (X, Y, Z). The velocity of this particle has three components (u, v, w) at time t given by

$$u = \partial x/\partial t, \qquad v = \partial y/\partial t, \qquad w = \partial z/\partial t \tag{1.2}$$

when x, y, z are regarded as functions of X, Y, Z, and t.

If a particle initially at (X, Y, Z) arrives at (x, y, z) at time t one commencing from $(X+\xi, Y, Z)$ where ξ is small will be at $(x+\xi\,\partial x/\partial X, y+\xi\,\partial y/\partial X, z+\xi\,\partial z/\partial X)$ at the same time. Therefore the fluid which originally occupied a small rectangular parallelepiped with sides ξ, η, ζ parallel to the coordinate axes will fill a volume given by the determinant

$$\begin{vmatrix} \dfrac{\partial x}{\partial X} & \dfrac{\partial y}{\partial X} & \dfrac{\partial z}{\partial X} \\ \dfrac{\partial x}{\partial Y} & \dfrac{\partial y}{\partial Y} & \dfrac{\partial z}{\partial Y} \\ \dfrac{\partial x}{\partial Z} & \dfrac{\partial y}{\partial Z} & \dfrac{\partial z}{\partial Z} \end{vmatrix} \xi\eta\zeta$$