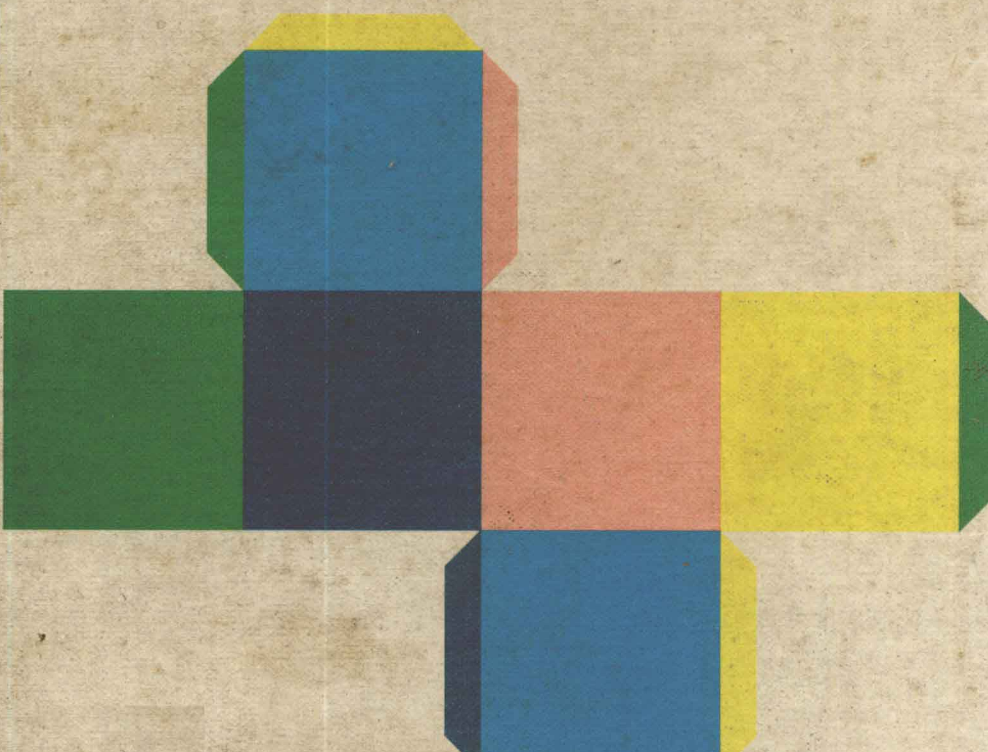
MODERN ELEMENTARY MATHEMATICS



KOVACH

Modern Elementary Mathematics

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Modern

Elementary

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To

C. D. H.

my favorite professor

Preface

This book is intended for those readers who have a need to bolster their foundations in mathematics. Because of its unique spiral approach, students with widely differing backgrounds will find it helpful.

Each of the four main topics has four chapters devoted to it. The first merely introduces the subject in an intuitive way and presents some of the necessary language and symbolism. Succeeding chapters build on this introduction and carry the work to higher levels.

This method of approach and the subject matter presented make this an ideal text for preparing elementary teachers in mathematics. More than this, the prospective secondary teacher will find here all the necessary material with which to bridge any gaps in his preparation. There is enough material for two three-semester-hour courses.

Although the book presupposes two years of high school mathematics—one year of algebra and one year of geometry—only a very modest recall of these subjects is required. All the topics are presented from first principles so that the book should have great value for use in in-service courses both at the elementary and secondary level.

Because of the spiral method of presentation, students, teachers, and parents will find that the book is ideal for self-study. In spite of the word "elementary" in the title many college students will find the book helpful in preparing themselves for rigorous courses in calculus or abstract algebra.

The book aims to raise the mathematical sophistication of the student from a near zero level to a most respectable one. This is accomplished in such a subtle way that the reader is hardly conscious of it. He does not meet the first formal proof until Chapter 9, yet the text carries him to the forefront of mathematical research in certain areas.

Not only are all the important ideas in elementary mathematics explored, but the reader is given many instances of the way mathematicians think and work. For this reason, this is an excellent text for courses in the foundations of mathematics and for survey courses for liberal arts students.

There is an unusually complete chapter on problem solving which gives practical hints for the solution of *word problems*. The characteristics of each type are explained, yet the solutions are not presented as "cookbook" procedures. The text is further enhanced for the reader because of an expanded section providing hints and answers to more than 500 exercises. There is also an index of symbols and a section describing some of the mathematics curriculum studies.

Our experience with the notes from which this book was written was most gratifying. Students who had weak backgrounds and those who had been exposed to too much rigor too soon found the material pleasant to the taste and easy to digest. We hope you assimilate the contents as easily.

We wish to acknowledge how very much we owe to the excellent teachers of mathematics who have struggled with us. They cannot, however, be in any way responsible for any errors or omissions we have made.

The first twelve chapters have been previously published by Holden-Day, Inc., under the title *Introduction to Modern Elementary Mathematics*.

Ladis D. Kovach

February 1968

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Introduction

1. Why *new* mathematics?

You've heard the expression, "Times aren't like they used to be." Perhaps you've even said this yourself and if someone were to ask you to be more specific, you might say, "Well, for one thing, newspapers aren't like they used to be—they're making the print much smaller nowadays"; or, "They're making belts and collars a lot tighter than they used to"; or, perhaps, "Staircases are steeper than they used to be"; or, "The distance to the corner bus stop is farther than it used to be."

And now to these troubles is added another one. They're not making arithmetic the way they used to. Parents are finding that they can no longer help their children with their arithmetic homework. It seems that the reason for this is *the new mathematics.* For some reason—known only to mathematicians?—the whole business of arithmetic has been turned topsy-turvy. The system that has been adequate for hundreds of years is no longer adequate. Pupils must learn new concepts, new words, new methods. And, of course, before the pupils can learn these things the teacher must learn them also. It is no wonder that one over-worked, frustrated teacher said recently, "It's bad enough to be living in an age of anxiety and devastating uncertainty—and now this new math on top of everything!"

It does seem so entirely unnecessary, doesn't it? But we are going to try to give you some idea of *why* the new mathematics came into being and what it's all about.

In the past—meaning when *we* were in elementary school—the emphasis was on speed and accuracy in manipulating numbers. We learned certain methods and became quite proficient in arithmetic skills. We also learned certain rules which we followed blindly. Some of you may remember one or two of these rules. For example, "invert the divisor and multiply," or "to change a decimal to a percent move the decimal point two places to the right." We followed these rules blindly and whatever the teacher said was *the law*. It never occurred to any of us to question *why* a certain method produced the correct answer. The teacher said to do it a certain way and that was good enough for us. And, strangely enough, most of us managed to get good grades in arithmetic. In other words, we learned our arithmetic skills quite well.

The next step was to go from arithmetic to algebra. Now many of us found to our sorrow that our skill in manipulating *numbers* did not help much in algebra. Here we had to manipulate *letters* and *equations*. This was much more difficult; we had to learn more rules and these rules became more mysterious and more confusing. For example, "a minus times a minus is a plus," or "change the sign of the subtrahend and add," or "transpose the unknown to the left side of the equation and the numbers to the right side." The fact that we didn't understand most of these rules didn't really matter. We learned them and we learned how to manipulate letters and equations. That is, *some* of us did. Others fell by the wayside and decided that mathematics was not for them—they were not cut out for it. This feeling was quite general among girls who very often justified their failure by saying that no one in their family was any good in algebra and they weren't about to be the exception.

The hardy ones who survived their first encounter with high school algebra went on to geometry. We found in geometry that our skill in manipulating numbers and our knowledge that "two minuses make a plus" did not seem to help us very much here. Geometry turned out to be an entirely different sort of business, consisting of strange things like theorems, axioms, propositions, and corollaries. Many students never did learn the difference between these. Those of us who were ambitious amd worked hard learned the theorems and memorized the proofs. We could say, for example, that the reason two angles were equal was because of "proposition 14, page 74." Or we wrote as the reason for some statement, "by Corollary 5." Again the mortality rate was quite high and a large percentage of those who survived algebra succumbed to geometry. In fact, the only ones left at the end of 10th grade geometry were a few individuals who were pointed toward college to become mathematics majors or scientists.

It is hardly worth considering these "last of the Mohicans," but we will continue. After geometry came more algebra and then more geometry but there's no use talking any more about these. Let's go on to trigonometry. In this subject we studied triangles and the relationships between the angles and the sides of these triangles. We found again, unfortunately, that neither our skill in manipulating numbers, nor our skill in "transposing," nor our ability to recite theorems did us much good in trigonometry. We had to start from scratch and work very hard in order to pass this course. A few more students fell by the wayside, but not too many, because most of them got lost in mathematics back at the previous two levels.

Those of us who survived high school mathematics and went on to college came face to face with calculus. What happened here is now ancient history. It's a repetition of the same things that we encountered before. Again we lost a large number of students. Some of these decided to go to a technical institute and learn a trade, while others changed their majors.

You can see what kind of a mathematical structure we have built here. It is certainly not a stable one. There is danger at any instant that the whole thing will topple and come crashing to the ground. It is to prevent this toppling, to build a more *stable* structure, that we are changing our methods of teaching mathematics. We are trying to change our method of teaching mathematics so that the broadest part of mathematics education is at the base, so that we can have something to build upon. This broadest part, of course, comes during the elementary school period.

Usually about this time someone will ask the question, "Most pupils do *not* become mathematics majors, so why get all excited about building a good foundation in mathematics?" The main reasons are that the growth of science and technology in recent years has made it necessary for the intelligent citizen to know more about these subjects. Since science is based on mathematics, and can hardly be understood without a knowledge of mathematics, an understanding of mathematics is essential to everyone. Another factor is the widespread use today of electronic computers. A large percentage of our population is either directly connected with computers or has someone in its immediate circle of family or friends who is. Some of these people may be using computers for business applications, some for scientific applications, some may be maintaining or servicing computers, and some may be writing programs for computers. In other words, we have an entirely new industry today and since so many people are involved in this, it becomes necessary for *everyone* to know something about it in order to talk intelligently.

The availability of the electronic computer has made mathematics more useful to people who before had no need for it. Today we find that social scientists, doctors of medicine, biologists, economists, businessmen, and even home economists and physical educators are using the computer to solve specialized problems in their fields. Of course, this requires that they have a greater knowledge of mathematics than was necessary in former years.

The rapid growth of science and technology in recent years has been due, in part, to advances in mathematics and the utilization of the electronic computer. In order to design complex systems[1] it is mandatory that a mathematical model be built first and the various characteristics of the system be analyzed on a computer. It is also essential that the mathematical model be as true to life as possible and this is most difficult to achieve. The many different and unpredictable things that can affect the operation of a system require that the most sophisticated mathematics be used in the analysis.

Then, too, there have been some new types of mathematics that have been developed. *Linear programming*, that is, the branch of mathematics concerned with solving simultaneously a system of inequalities, is useful in a variety of situations. Its application ranges from a determination of the most efficient supply system for the armed forces of a nation to the determination of the types and amounts of food needed by a hospital to provide the necessary nutrients most economically. *Graph theory*, a branch of combinatorial topology, has developed recently and is useful in analyzing complex electrical circuits. Thus, today more mathematics is required for careers in science and

[1] By a "system" is meant the totality of equipment and knowledge needed to accomplish a certain task, such as, making a landing on the moon, making observations from a satellite, or installing an automatic traffic control system.

engineering *and* also in many other fields. Today a higher percentage of our pupils will need to study more advanced mathematics. If they do not get a solid foundation in the elementary school, they may be handicapped later.

To summarize, mathematics was formerly taught in a way that tended to emphasize *skills* and to ignore real *understanding*. Knowledge gained in this way was almost impossible to extend. Yet, better training in mathematics is necessary in both technical and nontechnical fields today. Moreover, the intelligent citizen must have a better understanding of science and technology since rapid developments in these areas are affecting him personally as well as influencing national policies.

2. History of some modern programs

There is some danger in the use of the word "modern" in connection with mathematics programs. It implies that we are in some sort of experimental phase. Actually, the "experimenting" was done in approximately the decade between 1950 and 1960. During this time new methods and new materials were tried out in experimental classrooms all over the United States. Many teachers spent extra hours on Saturdays, after school, early in the morning to become familiar with new concepts so that they could teach them to their classes. On the basis of these experiments, carried out by hundreds of teachers in thousands of classes, new elementary mathematics textbooks were written. These books are available today and are being enthusiastically adopted all over the country. In fact, the decade from 1960 to 1970 might be called the *implementation* decade in contrast to the previous *experimental* decade.

In this section we will describe some of the more famous of the modern programs. We believe that in this way we can convey to you some of the spirit of modern mathematics. We call your attention, however, to an important point. While on the surface it may *seem* that the various programs differ considerably in their approach and treatment, we will show in the next section that there are points of similarity which are quite important. With this in mind, you may discover these similarities yourself as you read.

School Mathematics Study Group (SMSG)

Although not a pioneering group, the SMSG is perhaps the best known because it has had more members, more experimental classes, and has produced more publications than any other group. SMSG has probably inspired the writing of more textbooks than any other organization.

A brief history of SMSG can, perhaps, best be given by quoting from a government publication.[2]

Consideration of the mathematics programs at the elementary school level by the School Mathematics Study Group began with a conference on elementary school mathematics called in 1959 by Professor E. G. Begle, director of the project. In attendance were university professors of mathematics, high school and elementary school teachers, supervisors and education specialists with specific interest in mathematics, psychologists, and representatives from scientific and governmental organizations having an interest in mathematics. From this conference came the recommendation that a critical study of elementary school mathematics curriculum be undertaken. Among the aspects of the total elementary mathematics program suggested for study were: (1) the grade placement of topics; (2) development of concepts and mathematical principles; (3) the possible introduction of new topics particularly from geometry; (4) topics for able learners; (5) training for teachers; (6) the relation of elementary school mathematics to future study of the subject; (7) methods and materials for effective classroom instruction; and (8) the application of findings on concept-formation from psychology and child development to the learning of mathematics.

In March of 1960, a detailed outline of a suggested program for grades 4, 5, and 6 was developed. For 8 weeks during the summer of 1960 a writing team composed of classroom teachers or supervisors, mathematicians, and mathematics educators worked together to prepare materials. At this time units comprising a complete course for grade 4 and sample units for grades 5 and 6 with accompanying teachers' manuals were prepared. The format of the units is the "write-in" text workbook type with explanation and instruction; space is allowed for

[2] "Elementary School Mathematics: New Directions" by Edwina Deans, U. S. Department of Health, Education, and Welfare; U. S. Government Printing Office, 1963, no. OE-29042.