FUNDAMENTALS OF PHYSICS

Third Edition Extended

Volume One

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WILEY

JOHN WILEY & SONS New York · Chichester · Brisbane · Toronto · Singapore

Text and cover design: Karin Gerdes Kincheloe
Production supervised by Lucille Buonocore
Illustration supervised by John Balbalis
Copy editing supervised by Deborah Herbert
Photo Research: Anita Duncan

Library of Congress Cataloging-in-Publication Data:

Halliday, David
Fundamentals of physics.

Includes index.
1. Physics. I. Resnick, Robert
II. Title.
QC21.2.H35 1988c 530 88-17380
ISBN 0-471-61916-7

Printed in the United States of America
10 9 8 7 6 5 4

PREFACE

This third (1988) edition of *Fundamentals of Physics* is a major revision of both the second (1981) edition of that text and of its revised printing (1986). Although we have retained the basic framework of these earlier versions, we have virtually rewritten the entire book. Users of the earlier editions can appreciate the changes better if we list them in some detail.

(a) In the words of one reviewer, "you have succeeded in maintaining the overall level throughout but have substantially lowered the learning threshold." Many new techniques have been used to achieve this. For example, many hints on problem solving are sprinkled throughout the early chapters, each one focusing on a chronic student hang-up. (A complete list of these Hints appears on page xiii). A larger set of worked examples—now called Sample Problems to reflect their consistent focus—is included to provide problem-solving models for all aspects of each chapter; several are put in extended question and answer format to reveal directly the pathways followed by experienced problem solvers. There are more but shorter sections per chapter for easier digestion of the material, and more use of subheads is made within sections for greater clarity and emphasis. In the body of the text, relationships are typically displayed and discussed before they are formally derived, a more inductive procedure that we think will prove effective. Often these formal derivations appear in separate sections or subsections. And, we have greatly expanded the set of confidence-building exercises for homework while increasing the number of problems as well.

(b) Greater clarity has been achieved in many ways. A more student-oriented style is employed than before and a two-column format has been adopted for easier reading. Chapter-head photographs are now included along with captions that make a valid attention-grabbing point about the contents of each chapter. Opening sections discuss the relevance of the topics to be treated in each chapter in order to motivate students from the start. Throughout the chapter, photographs and diagrams are featured in greater numbers, with self-contained captions, to reinforce the text material. In each chapter there are examples that deal with practical and applied situations. Chapters conclude with a detailed Review and Summary section for student reference and study.

(c) The sets of chapter-ending questions, exercises, and problems are by far the largest and most varied of any introductory physics text. We have edited the highly praised sets of the earlier edition to achieve even greater clarity and interest and have added a substantial number of new applied and conceptual ones. A more generous use of figures and photographs serves better to illustrate the questions, exercises, and problems than before.

The thought questions have always been a special feature of our books. They are used as sources of classroom discussion and for clarification of homework concepts. There are nearly 30 per chapter. Their total number, now over 1400 in the entire book, is greater than before and they relate even more to everyday phenomena, serve to arouse curiosity and interest, and stress conceptual aspects of physics.

Exercises typically involve one step or formula or represent a single application and are used for building student confidence. They now constitute about 45 percent of the exercise-problem sets. In preparing the new set of problems, we have been careful not to discard the many tried and true problems that have survived the test of the classroom for many years. Long-time users of our text will not find their favorites missing. Of the substantial number of new problems, many fit the "real world" category of student interest and these and the others serve different pedagogic objectives as well. Amongst the problems are a small number of advanced ones, as well, identified by stars* next to their number. A typical chapter has about 31 exercises and 37 problems, the total number of exercises and problems in the entire book being about 3400.

By labeling exercises "E" and problems "P" and organizing them in order of difficulty for each section of the chapter, we have simplified the selection process for teachers from the voluminous material now made available. The variation of level and the breadth of scope have been enlarged. Hence, teachers can vary the content emphasis and the level of difficulty to suit their tastes and the preparation of the student body while still having a very adequate supply for many years of instruction. Indeed, the book is now somewhat longer principally because of all the self-study and learning features that are now included.

(d) Our treatment of modern physics has been enhanced. There are two entirely new modern physics chapters, one on Relativity and the other on Quarks, Leptons and the Big Bang in Volume Two. And, in rewriting the earlier chapters, we have sought to pave the way more effectively than in previous editions for the systematic study of modern physics presented in the later chapters. We have done this in three ways. (i) In appropriate places we have called attention—by specific example—to the impact of relativistic and quantum ideas on our daily lives. (ii) We have stressed those concepts (conservation principles, symmetry arguments, reference frames, role of aesthetics, similarity of methods, use of models, field concepts, wave concepts, etc.) that are common to both classical and modern physics. (iii) Finally, we have included a number of short optional sections in which selected relativistic and quantum ideas are presented in ways that lay the foundation for the detailed and systematic treatments of relativity, atomic, nuclear, solid state, and particle physics given in later chapters.

(e) To emphasize the relevance of what physicists do, and further motivate the student, we include, within the chapters, numerous applications of physics in engineering, technology, medicine, and familiar everyday phenomena. In addition, we feature 21 separate, self-contained essays, written by distinguished scientists and distributed at appropriate locations in the text, on the application of physics to special topics of student interest such as sports, toys, amusement parks, medicine, lasers, holography, space, superconductivity, concert-hall acoustics, and many more. (See the Table of Contents.)

(f) In the interests of simplification and of greater clarity for students, certain rearrangements of material have been made. For example, motion in one dimension is now treated before vectors. A better balance of the material on rotational motion in mechanics is achieved over two chapters by presenting the simpler concepts in kinematics and dynamics first, and then the more difficult concepts, enabling the instructors to more easily choose the depth desired. Similarly, formerly-scattered material —such as on the Doppler effect or on special relativity —has been drawn together in one place for greater conceptual unity. Material on elasticity, now somewhat longer, fits more naturally into the chapter on equilibrium. There are, of course, many other smaller rearrangements too numerous to mention here.

Like the second edition, this edition is available in a single volume of 42 chapters, ending with relativity, and in an Extended Version of 49 chapters that contains in addition a development of quantum physics and its applications to atoms, solids, nuclei, and particles. The Extended Version is also available as a two volume set: Volume One covers Mechanics and Thermodynamics (Chapters 1–22) and Volume Two covers Electricity and Magnetism, Optics, and Modern Physics (Chapters 23–49). The former is meant for introductory courses that treat modern quantum physics in a subsequent separate course or semester. There are also numerous optional sections throughout the text that are of an advanced, historical, general, or specialized nature.

Indeed, just as a textbook alone is not a course, so a course does not include the entire textbook. We have consciously made available much more material than any one course or instructor is expected to "cover." More can be "uncovered" by doing less. The process of physics and its essential unity can be revealed by judicious selective coverage of many fewer chapters than are contained here and by coverage of only portions of many included chapters. Rather than give numerous examples of such coherent selections, we urge the instructor to be guided by his or her own interests and circumstances and to plan ahead so that some topics in modern physics are always included.

A textbook contains far more contributions to the elucidation of a subject than those made by the authors alone. As before, John Merrill (Brigham Young University) has been of special service for all aspects of this work, as has Edward Derringh (Wentworth Institute of Technology). Albert Bartlett (University of Colorado) has been of particular help with the essays and Benjamin Chi (SUNY Albany) with the figures and photographs. At John Wiley, publishers, we have been fortunate to receive strong coordination and support from Robert McConnin and Catherine Faduska, physics editors, with notable contributions from John Balbalis, Lucille Buonocore, Deborah Herbert, Karin Kincheloe, Safra Nimrod, and other members of the production team. We are grateful to all these persons.

Our external reviewers have been outstanding and we acknowledge here our debt to each member of that team, namely, Joseph Buschi (Manhattan College), Philip A. Casabella (Rensselaer Polytechnic Institute), Randall Caton (Christopher Newport College), Roger Clapp (University of South Florida), William P. Crumment (Montana College of Science and Technology), Robert Endorf (University of Cincinnati), E. Paul Esposito (University of Cincinnati), Andrew L. Gardner (Brigham Young University), John Gieniec (Central Missouri State University), Leonard Kleinman (University of Texas at Austin), Kenneth Krane (Oregon State University), Howard C. McAllister (University of Hawaii at Manoa), Manuel Schwartz (University of Louisville), John Spangler (St. Norbert College), Ross Spencer (Brigham Young University), Harold Stokes, (Brigham Young University), David Toot (Alfred University), Donald Wieber (Contra Costa College), and George U. Williams (University of Utah).

We thank all the essayists for their valuable contributions and cooperative spirit. Kathaleen Guyette has been superb in providing the wide range of secretarial services required.

We hope that the final product proves worthy of the effort and that this Third Edition of *Fundamentals of Physics* will contribute to the enhancement of physics education.

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January, 1988

THE ESSAYISTS

ALBERT A. BARTLETT

Albert A. Bartlett (Essays 4 and 11) is a professor of physics at the University of Colorado, Boulder, where he has been a member of the faculty since 1950. He received his B. A. from Colgate University and his Ph. D. from Harvard in nuclear physics. His interests are centered on teaching physics; he was President of the American Association of Physics Teachers in 1978. He is a founding member of PLAN-Boulder—an environmental organization that is largely responsible for Boulder's Greenbelt and open space land acquisition program.

CHARLES P. BEAN

Charles P. Bean (Essay 14) is Institute Professor of Science at Rensselaer Polytechnic Institute. He received his Ph. D. in physics from the University of Illinois in 1952. For more than 33 years he was a research scientist in the General Electric Research Laboratory and its successor, the General Electric Research and Development Center. While there he made research contributions to the fields of ionic crystals, magnetism, superconductivity, and membrane biophysics. He is a member of the National Academy of Sciences and the American Academy of Arts and Sciences. As an avocation he studies the physics of phenomena in nature.

PETER J. BRANCAZIO

Peter J. Brancazio (Essay 6) is a professor of physics at Brooklyn College, City University of New York. He received his Ph. D. in astrophysics from New York University in 1966. He is the author of two books: *The Nature of Physics* (Macmillan, 1975) and *Sport Science* (Simon & Schuster, 1984). His articles on the physics of baseball, football, and basketball have appeared in *Discover, Physics Today, New Scientist, The Physics Teacher,* and the *American Journal of Physics.* A lifelong athlete and sports fan, he is equally at home on the basketball court and in the classroom.

PATRICIA ELIZABETH CLADIS

Patricia Elizabeth Cladis (Essay 20) was born in Shanghai, China and grew up in Vancouver, British Columbia. She received her Ph. D. in physics from the University of Rochester with a thesis on the dc superconducting transformer. Before joining AT&T Bell Laboratories, she did postdoctoral research at the University of Paris, Orsay, France, where she first learned about liquid crystals and discovered "escape into the third dimension" and point defects in nematics. At Bell Labs she discovered the "reentrant nematic" phase. Currently she uses liquid crystals to study general problems in nonlinear physics. She has published nearly 100 scientific papers and is on the editorial board of the journal *Liquid Crystals.*

ELSA GARMIRE

Elsa Garmire (Essay 19) is professor of electrical engineering and physics, and Director of the Center for Laser Studies at the University of Southern California. Garmire received the A. B. in physics from Harvard University in 1961 and the Ph. D. in physics from M. I. T. in 1965 for research in nonlinear optics under Nobel Prizewinner C. H. Townes. The author of over 120 papers with seven patents, she has been a researcher in quantum electronics and in linear and nonlinear optical devices for 25 years. She is a fellow of the Optical Society of America and of IEEE and has been on the board of both societies. She is associate editor of the journals *Optics Letters* and *Fiber and Integrated Optics,* and was U.S. delegate to the International Commission for Optics.

RUSSELL K. HOBBIE

Russell K. Hobbie (Essays 8 and 21) is a professor of physics at the University of Minnesota. He received his B. S. from M. I. T. and his Ph. D. from Harvard. His research interests include diagnostic radiology, magnetic resonance imaging, impedance cardiography, and computerized medical diagnosis. He is the author of *Intermediate Physics for Medicine and Biology,* published by Wiley, 1988.

TUNG H. JEONG

Tung H. Jeong (Essay 18) received his B. S. from Yale University in 1957, and Ph. D. in nuclear physics from the University of Minnesota in 1963. Presently, he is chairman of the physics department and holder of the Albert Blake Dick endowed chair in Lake Forest College. Besides directing annual summer holography workshops since 1972, he consults and lectures on holography in hundreds of institutions around the world. He is a Fellow of the Optical Society of America and a recipient of the Robert A. Millikan Medal from the American Association of Physics Teachers. His hobbies include skiing, [illegible] playing the violin in the Lake Forest Symphony Orchestra.

KENNETH LAWS

Kenneth Laws (Essay 3) is professor of physics at Dickinson College in Carlisle, Pa., where he has been teaching since 1962. He earned his B. S., M. S., and Ph. D. degrees from Caltech, the University of Pennsylvania, and Bryn Mawr College, respectively. For the last dozen years he has been studying classical ballet at the Central Pennsylvania Youth Ballet, and he has recently been applying the principles of physics to dance movement. This work has lead to numerous lectures and classes around the country, and has culminated in a book, *The Physics of Dance,* published in 1984 (paperback 1986) by Schirmer Books.

PETER LINDENFELD

Peter Lindenfeld (Essay 12) has degrees in electrical engineering and engineering physics from the University of British Columbia and a Ph. D. in physics from Columbia University. Since his graduation he has been at Rutgers University where he is professor of physics. His research and publications are on low temperature physics and superconductivity as well as on activities related to physics teaching. He is a fellow of the American Physical Society and an honorary life member of the New Jersey Section of the American Association of Physics Teachers. He has received awards for his booklet "Radioactive Radiations and their Biological Effects," for his work on a solar calorimeter, and for some of his photographs.

RICHARD L. MORIN

Richard L. Morin (Essays 8 and 21) is an associate professor in the Department of Radiology and Director of the Physics Section in Radiology at the University of Minnesota. He received his undergraduate training in chemistry at Emory University and his Ph. D. in radiological sciences from the University of Oklahoma. His research interests are in the area of computer applications in radiology and nuclear medicine.

SUZANNE R. NAGEL

Suzanne R. Nagel (Essay 17) is head of the Glass Research and Engineering Department at AT&T-Bell Laboratories, Murray Hill, N.J. She received her Ph. D. in ceramic engineering from the University of Illinois in 1972 after her undergraduate studies at Rutgers University. She has authored 30 technical papers in the area of glass science and lightguide technology, and her research has involved the processing and property optimization of optical fibers for communications, as well as close interaction with their manufacture. She is actively involved in promoting careers in science and engineering for women and minorities, and was recently featured in the Chicago Museum of Science and Technology Exhibit "My Daughter the Scientist," which is touring the country.

GERARD K. O'NEILL

Gerard K. O'Neill (Essay 13) holds a bachelor's degree from Swarthmore College, a Ph. D. from Cornell University (1954), and an honorary D. Sc. from Swarthmore. He was a member of the faculty at Princeton University from 1954 to 1985, and was made full professor of physics in 1965. In 1985, he retired early from Princeton, becoming Professor Emeritus of Physics. He is the author of several books and many articles in his field. In March 1985, he was appointed by President Reagan to the National Commission on Space. In 1983 he founded the Geostar Corporation, a communications and navigation satellite company based on patents issued to him. His latest commercial start-up company is O'Neill Communications, Inc.

SALLY K. RIDE

Sally K. Ride (Essay 5) is a NASA Space Shuttle astronaut. She earned a B. S. in physics and a B. A. in English from Stanford University in 1973, and a Ph. D. in physics from Stanford in 1978. After graduate school, she was selected for the Astronaut Corps. She has flown in space twice: on the seventh Space Shuttle mission (STS-7, the second flight of the *Challenger*, launched in June, 1983), and the thirteenth Shuttle mission (STS-41G, launched in October, 1984). In 1986, she was appointed to the Presidential Commission investigating the Space Shuttle *Challenger* accident. Since the completion of the investigation, she has acted as Special Assistant to the Administrator of NASA, helping to develop NASA's long-range plans for human exploration of space. She is currently affiliated with the Center for International Security and Arms Control at Stanford University.

JOHN S. RIGDEN

John S. Rigden (Essay 7) received his Ph. D. from Johns Hopkins University in 1960. After postdoctoral work at Harvard University, he started the physics department at Eastern Nazarene College. After one year at Middlebury College, he moved to St. Louis where he is now professor of physics at the University of Missouri–St. Louis. He is the author of *Physics and the Sound of Music* (Wiley, 1977; Second Edition, 1985). More recently, he has written the definitive biography of the great American physicist, I. I. Rabi: *Rabi: Scientist and Citizen* (Basic Books, 1987). Since 1978 he has been the editor of the *American Journal of Physics*.

JOHN L. ROEDER

John L. Roeder (Essay 2) began investigating the physics of the amusement park with an article in the September 1975 issue of *The Physics Teacher* and now takes his physics classes at The Calhoun School in New York City on field trips to Six Flags Great Adventure, the site of his "original research." In addition to serving as a double Resource Agent — for both the American Association of Physics Teachers and the New York Energy Education Project — John is a cofounder of and the newsletter editor for the Teachers Clearinghouse for Science and Society Education, Inc. He received his A. B. from Washington University and his M. A. and Ph. D. from Princeton University.

WILLIAM A. SHURCLIFF

William A. Shurcliff, Physics Department, Emeritus, Harvard University (Essay 9) received his Ph.D. from Harvard in 1934. He has held positions such as senior scientist and research fellow in numerous government, industrial, and university laboratories, including American Cyanamid Company, Polaroid Corporation, Office of Scientific Research and Development, and the Cambridge Electron Accelerator. He is the author of books on polarized light, solar heated houses, and superinsulated houses.

RAYMOND C. TURNER

Raymond C. Turner (Essay 15) is well known for his work with the physics of toys. He received his B. S. in physics from Carnegie Institute of Technology and his Ph. D. in solid state physics from the University of Pittsburgh in 1966. He is now a professor of physics at Clemson University in South Carolina where he conducts research on electron-spin-resonance studies of polymers. He has presented numerous workshops and lectures at national teachers' meetings on the use of toys in physics education, and he has served on local and national committees of the American Association of Physics Teachers. He has published articles on physics and toys in the *American Journal of Physics* and *The Physics Teacher*.

JEARL WALKER

Jearl Walker (Essays 1, 2, 10 and 16) is professor of physics at Cleveland State University. He received a B. S. in physics from M. I. T. and a Ph. D. in physics from the University of Maryland. Since 1977 he has conducted "The Amateur Scientist" department of *Scientific American*, where he is read in 10 languages in world-wide publication. His book *The Flying Circus of Physics with Answers* is also published in 10 languages.

ESSAY 1
RUSH-HOUR TRAFFIC FLOW

JEARL WALKER
CLEVELAND STATE UNIVERSITY

Traffic lights in a small town usually require no special sequencing. The flow of traffic through a system of lights may be haphazard, but the queues at red lights are seldom long. In contrast, traffic flow in a large city, especially during rush hour, requires careful regulation. Without it, the lines of cars lengthen until many intersections are blocked, throwing the whole region into what is called gridlock. Since only the cars on the perimeter of the congested area can then move, hours may be needed to free the cars trapped in the interior.

Suppose you were to engineer the traffic light system for a one-way street that consists of several lanes along which rush-hour traffic flows. The lights are green for 50 s, yellow for 5 s, and red for 25 s—values that are typically employed for a heavily-traveled route through a city. To promote the traffic flow you may be tempted to increase the duration of the green light or decrease the duration of the red light. However, you must remember that the traffic on the perpendicular streets cannot be held in check for too long or lengthy queues will develop there.

How should you time the onsets of green lights at the intersections? If you arrange for all the lights to turn green simultaneously, then the traffic can move for only 50 s. Upon each onset of the green lights, platoons of cars will move along the route until all the lights on the street turn red. To maximize the distance traveled, the drivers will be tempted to race through the system. Large platoons of cars traveling at, say, 55 mi/h along a crowded city street would resemble a "grand prix," presenting an obvious danger.

A better and safer design is one where the light at each intersection does not turn green until the platoon leaders from the previous intersection reach it. For example, suppose the platoon is initially at intersection 1 and headed toward intersection 2. How long after the light at intersection 1 turns green should the light at intersection 2 turn green?

Let d be the distance between the two intersections and v_p be the speed you wish the platoon to have, probably the speed limit. The required time delay on the green light at intersection 2 is d/v_p. If all the subsequent green lights along the street are also appropriately delayed, drivers traveling at v_p should move smoothly through the whole light system. If the intersections are separated by identical distances, then the delay on the green light from one intersection to the next is always d/v_p. The plan works just as well if the distance between intersections varies. You merely use the appropriate value of d to determine the necessary delay needed for the green light at a given intersection. Racing through the system is then futile; eventually the driver is stopped by a red light, whereupon everyone that had been passed catches up before the light turns green.

The plan is slightly modified by two factors. A platoon leader should see the onset of the green light before reaching an intersection. Otherwise, the leader will slow out of fear of entering the intersection on a red light. To adjust for this factor, you must decrease the delay in turning on the green light by a few seconds. However, if the platoon were stopped by a red light at intersection 1, the platoon leader would require a few extra seconds to respond to the onset of green light there and to accelerate to cruising speed. Thus, the leader does not travel the full distance between intersections at v_p. This factor requires an increase in the delay by a few seconds. In

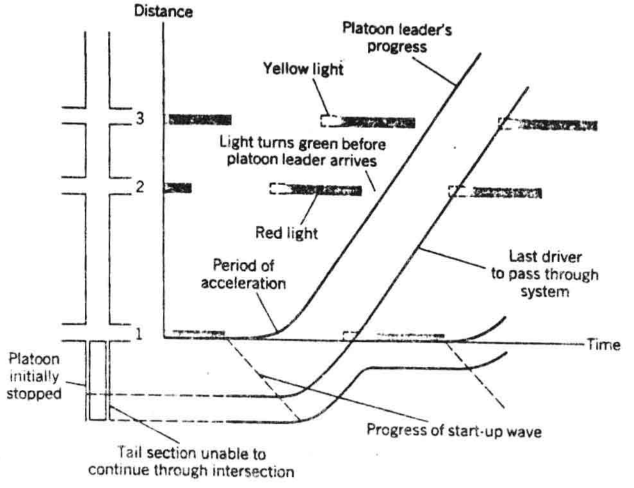

Figure 1 Graphical representation of how a platoon of cars stopped at intersection 1 thereafter proceeds through a system of traffic lights.

some cases, the two factors might offset each other.

All these points are illustrated in Fig. 1. A street system is displayed on the left side. The vertical axis on the graph corresponds to the distance along the street. The platoon, which was initially stopped at intersection 1, travels through the light system. When the platoon leader travels at a constant speed, the slope of the line representing the travel is equal to that speed. The leader's initial period of acceleration is represented by a curved line. Note that the light at intersection 2 is shown turning green a few seconds before the leader reaches it.

With a system of delayed lights, the traffic can still freeze into place. The problem lies in the fact that once a platoon of drivers is stopped and then given another green light, they cannot all accelerate simultaneously. Instead, a "start-up wave" travels from the leader along the length of the platoon. Compared to the leader, the last driver not only has a longer distance to travel to intersection 2 but also must wait for the start-up wave to travel through the platoon. Thus, the last driver in a lengthy platoon is always certain to require much more time to reach the next intersection.

You can measure the speed of a start-up wave by positioning yourself alongside a stationary platoon. Determine the length of the platoon by multiplying the number of cars by an estimate of the average distance between front bumpers. Then measure the time from the onset of the green light until the last car begins to move. The speed of the start-up wave is the length of the platoon divided by the time you measure. A typical value is 5 m/s (= 11 mi/h).

If the tail of a platoon cannot pass through an intersection with the rest of the platoon, then the subsequent platoons may begin to lengthen. Consider a tail section that has been abandoned at intersection 2 as shown in Fig. 2. Call the section *A*. As *A* waits through the red light, the platoon at intersection 1 gets a green light and begins to move forward. Call this platoon *B*. Since the green light at intersection 2 is delayed to match the progress of the leaders of *B*, they arrive at the rear of *A* before the leaders of *A* begin to move. The problem is compounded by the fact that before the last drivers of *A* can move, they must wait for the start-up wave to travel to them from their leaders.

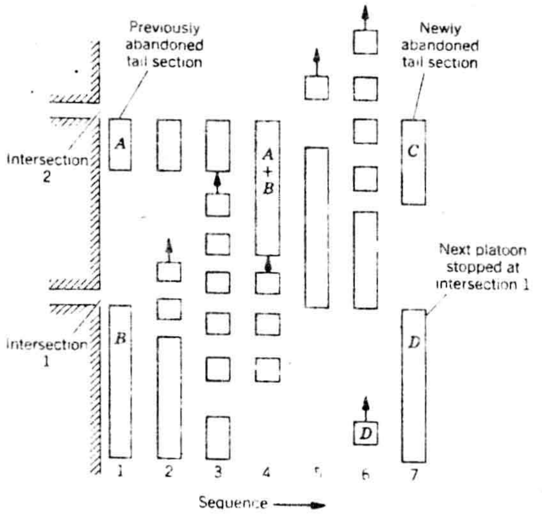

Figure 2 With a delayed light at intersection 2, the platoon length can increase.

The net result is that A and B combine, forming a new platoon at intersection 2. It may be too long to pass entirely through the intersection. Let C be the tail section that is stopped by the next red light at intersection 2. If C is longer than A, the situation has deteriorated. As C waits through the red light, another platoon (call it D) arrives from intersection 1. The combination of C and D may then be longer than the previous combination of A and B, guaranteeing that the next tail section to be abandoned at intersection 2 will be longer than C was. The situation can then quickly become hopeless if with each cycle of the lights the newly abandoned section at intersection 2 is longer than the previous one. Eventually, the abandoned section is so long that it stretches back to intersection 1.

Consider the very next platoon that is waiting at intersection 1. Call the platoon M. When M receives a green light, it moves into intersection 1, blocking it. Since these cars then cannot move until the start-up wave from intersection 2 reaches them, they are still in place when the light at intersection 1 turns red. With intersection 1 blocked, drivers on the perpendicular street cannot move through it during their green light. The queues on the side street lengthen until they block intersections on the streets that run parallel to the route we are considering. The congestion rapidly spreads until the whole region becomes one large parking lot.

A gridlock can develop even if the traffic light system is well designed. I was once trapped in a gridlock when a sudden, heavy snowfall caught the afternoon rush-hour traffic of Cleveland. Since the street I was on was slippery, the platoon leaders proceeded cautiously. The start-up waves also moved slower than normally. Within about 20 min, abandoned sections of platoons stretched back to previously crossed intersections, blocking them. For 2 mi along my route and along five parallel arteries out of the city, traffic came to almost a standstill. I made progress only because cars on the outward end of the route gradually escaped into the suburbs. As they left the pack, a start-up wave moved sedately through the 2-mi platoon, allowing me to creep forward by a few car lengths. The problem worsened as stalled cars blocked lanes. A normal 5 min drive took over 2 h!

A gridlock caused by an unexpected snowfall may be impossible to alleviate. However, a gridlock caused by unexpectedly long platoons may be avoidable if an engineer or a computer intervenes in the normal sequencing of lights. Suppose that through remote sensing you detect the gradual buildup of abandoned sections of platoons at, say, intersection 2. Let A be the section that is abandoned there when you intervene and let B be the platoon that is then stopped at intersection 1. One way to reverse the buildup is to release A early enough that B need not stop as it approaches intersection 2.

If A is already sufficiently long when you intervene, you may need to change the sequencing so that the green lights at intersection 1 and 2 turn on simultaneously. If A is even longer, you will need to turn on the green light at intersection 2 before you turn on the green light at intersection 1.

Which course of action is best can be stated in terms of the ratio $x = L/d$ of the length L of the abandoned section A to the distance d between intersections 1 and 2. If the leaders of B travel to the rear of A at a constant speed v_p, they require a time of $(d - L)/v_p$ to make the trip (see Fig. 3). Meanwhile, a start-up wave travels through A at a speed of v_s, requiring a time of L/v_s to reach the rear of A. If the leaders of B are to reach the rear of A as the cars there begin to move, then the time t between the onset of the green light at intersection 2 and that at intersection 1 should be

$$t = \frac{d-L}{v_p} - \frac{L}{v_s}.$$

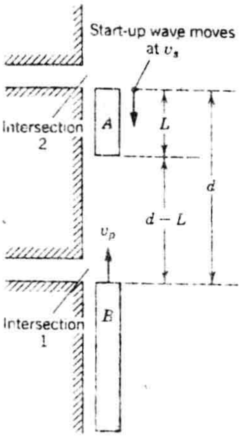

Figure 3 Factors important in determining relative onsets of green lights at successive intersections.

If x is short enough, t is positive, which means that the beginning of the green light at intersection 2 should follow the beginning of the green light at intersection 1 by the time t. If x is long enough, t is negative, which means that the onset of the green light at intersection 2 should precede the onset of the green light at intersection 1 by the time t.

"Switch-over" between the two procedures occurs when t is zero:

$$0 = \frac{d-L}{v_p} - \frac{L}{v_s},$$

which gives x as

$$x = \frac{L}{d} = \frac{v_s}{v_s + v_p}.$$

With typical values of 10 m/s (about 22 mi/h) for the platoon speed and 5 m/s for the speed of the start-up wave, switch-over is required when x is $\frac{1}{3}$. If x is larger than $\frac{1}{3}$, the green light at intersection 2 must turn on before the green light at intersection 1.

You can interpret this result in terms of the speed at which a "wave" of green lights travels along the street. When $x < \frac{1}{3}$, the wave travels in the direction of the traffic and with the same speed. When $x = \frac{1}{3}$, the green lights come on simultaneously. When $x > \frac{1}{3}$, the wave of green lights travels in the opposite direction of the traffic.

This result is only an estimate since the calculation ignores three facts. The leaders of B begin their motion by accelerating from a stop. They must see that the rear of A is moving prior to their arrival or they will slow or stop out of caution. The rear cars in A must, of course, accelerate to the cruising speed prior to the arrival of B. You might enjoy fine tuning the calculation by including these additional factors.

How would you design a light system for optimizing the flow of traffic in two directions on a street? In such a situation is there any way to avoid having cars occasionally stop? Can you simulate traffic platoons moving through a system of street lights on a computer, perhaps making a game out of the simulation? If so, then you could investigate the problems of gridlock without ever having to endure one.

ESSAY 2
FEAR AND TREMBLING AT THE AMUSEMENT PARK

JOHN ROEDER
THE CALHOUN SCHOOL

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The lure of the rides at an amusement park lies in their apparent danger, in illusions of motion and forces—and often in the uncommon experience of rapid rotation. The rides might play such tricks on your common sense that you cherish the experience, thrilling in being hurled toward the ground or whipped around with such abandon that sight of your surroundings blurs. You owe much of the excitement to physics.

Merry-Go-Round

Many of the rides involve rotation because it creates illusions and strange sensations. For example, consider a traditional merry-go-round with imitation horses. When you ride a horse while the ride turns around its center, you feel as though you are being forced radially outward. Is there really such a *centrifugal force* on you? What could generate it? Certainly there is no agent on one side of you pressing you outward. One reason the merry-go-round is fun is that it creates the illusion of a magical centrifugal force. Actually, the only radial force on you is inward as the horse pulls on the lower part of your body, forcing you to continue moving around a circle (Fig. 1). Since the lower part of your body pulls on the rest of you, your body tends to lean outward. You misinterpret the leaning as being due to some unseen agent pushing against you.

Every merry-go-round fan knows that the outer horses are more exciting to ride than the inner horses. The difference is due to the speed the horse gives you. If your speed is small, you require only a small centripetal acceleration to move around a circle. A larger centripetal acceleration is needed when your speed is larger. Since the horses are fastened to the merry-go-round, they each must complete a circle in the same amount of time. The inner horses travel around a small circle with a small speed. When you ride one of them, the acceleration that you undergo is small, as is also the centripetal force you experience. When you ride one of the outer horses, your speed is larger, which means that your acceleration and the centripetal force on you must also be larger. The illusion of a centrifugal force resulting from some unseen agent may then be quite compelling.

Ferris Wheel*

A Ferris wheel also creates illusions. In this ride, you sit in a cage that is suspended on the rim of a vertical structure (Fig. 2). When the structure rotates around its center, you move around a large vertical circle. Throughout the circle your cage is free to rotate around its own axis so that you remain vertical. When you move through the bottom of the circle, you feel especially heavy whereas when you move through the top of the circle, you feel light but not quite weightless. The ride produces two illusions: the sensation that your weight varies and the sensation that you are subject to a

* The original Ferris wheel was created for the 1893 World's Columbian Exposition in Chicago by George Washington Gale Ferris, a civil-engineering graduate of Rensselaer Polytechnic Institute. The original wheel was 250 ft in diameter and could carry 2000 passengers at one time in its 36 cars.

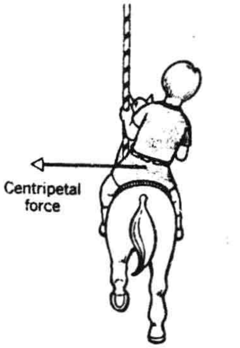

Figure 1 Centripetal force on a merry-go-round.

centrifugal force. The Ferris wheel is scarier than the merry-go-round not only because it generates a fear of falling but also because the forces on you vary with time.

Consider the phase when you move through the bottom of the circle (Fig. 3*a*). The apparent centrifugal force on you seems to press you into the seat more than normal, creating the illusion that your weight is greater than normal. Of course, there is no unseen agent forcing you radially outward from the center of rotation and the earth's gravitational attraction on you is unchanged. You feel especially heavy because the force from the seat is larger than it would be were the Ferris wheel stationary. Let **W** be your weight (a downward vector) and **P** be the force you feel from the seat (an upward vector). Since you accelerate toward the center of the rotation, you are undergoing a positive (upward) acceleration. The magnitude of the centripetal force on you is

$$F_c = \frac{mv^2}{r},$$

where m is your mass, v is your speed and the speed of the cage you are in, and r is the radius of the Ferris wheel and thus your distance from the ride's axis of rotation. The centripetal force is the vector sum of **W** and **P**:

$$-W + P = \frac{mv^2}{r},$$

which yields P as

$$P = W + \frac{mv^2}{r}.$$

Figure 2 Ferris wheel.

In addition to supporting you, the seat must also supply the centripetal force necessary for you to continue moving in the circle. You misinterpret the larger than normal force on you from the seat as evidence that your weight has increased.

When you move through the top of the circle, your acceleration is again toward the center of the rotation, but this time the vector is downward and thus negative (Fig. 3*b*):

$$-W + P = -\frac{mv^2}{r},$$

which yields P as

$$P = W - \frac{mv^2}{r}.$$

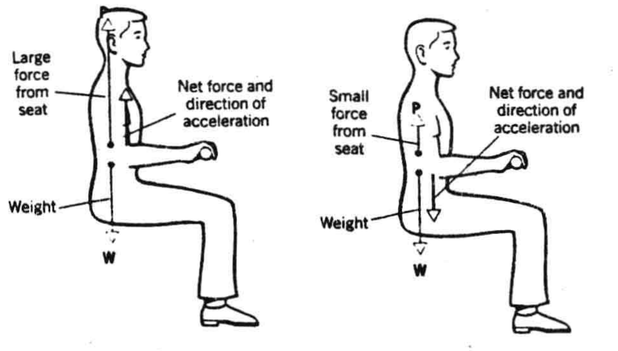

Figure 3 Forces on passenger in rotating Ferris wheel: (*a*) at bottom of circle and (*b*) at top of circle.

Figure 4 Tilted Roundup ride.

The force on you from the seat is smaller than it would be were the Ferris wheel stationary. You misinterpret the reduction in the force as evidence that you are lighter than normal.

Roundup

The Roundup is a ride that initially appears to be similar to the Rotor (see Sample Problem 9 in Chapter 6). It consists of a wide horizontal disk that has a sturdy network of metal bars fastened to its perimeter (Fig. 4). You stand on the disk next to a solid wall that is fastened to the bars. A constraining bar lies in front of you, locked into place. When the disk spins around its center, the wall presses against your back, providing centripetal force. As the speed builds, the force grows stronger until it is quite large when the disk reaches its final, constant speed. You feel as though an unseen agent is pushing you radially outward with an irresistible force, pinning you against the wall like some butterfly specimen.

While the disk continues to spin, a machine lifts and tilts it so that you then rotate around an axis that is 60° off the vertical. The ride is then similar to a Ferris wheel, except that a wall and disk push against you instead of a seat. However, it differs from a Ferris wheel in that when you are at the top of the rotation, the centripetal acceleration on you is greater than the acceleration of free fall. Your sensation of weight there, as evidenced by the downward force on you from the wall, may seem normal, even though you are upside down. When you pass through the lowest portion of the circle, you seem to be heavier than normal because the force from the wall is then greatest. The continuous variations in the force you feel and the sensation of your weight make the tilted phase of the ride scarier than when the spinning disk is horizontal and the forces on you are constant.

Roller Coaster

One of the most popular rides at an amusement park is a roller coaster (Fig. 5). The coaster, which consists of several cars with passengers, runs along a track over hills and through valleys. To get it started, a chain driven by a machine pulls the coaster up the first hill, which is the highest of all the hills. The track that descends from the hill on the far side is steep. When the first cars reach the descending track, the chain disengages and the front cars drag the rest of the coaster into the descent. Speed builds, as does fear in the passengers.