Walter G. Kropatsch Robert Sablatnig Allan Hanbury (Eds.)

Pattern Recognition

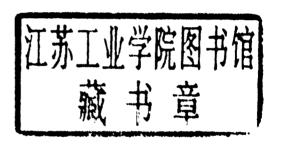
27th DAGM Symposium Vienna, Austria, August/September 2005 Proceedings



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Pattern Recognition

27th DAGM Symposium Vienna, Austria, August 31 – September 2, 2005 Proceedings





Volume Editors

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Preface

It is both an honor and a pleasure to hold the 27th Annual Meeting of the German Association for Pattern Recognition, DAGM 2005, at the Vienna University of Technology, Austria, organized by the Pattern Recognition and Image Processing (PRIP) Group. We received 122 contributions of which we were able to accept 29 as oral presentations and 31 as posters. Each paper received three reviews, upon which decisions were made based on correctness, presentation, technical depth, scientific significance and originality. The selection as oral or poster presentation does not signify a quality grading but reflects attractiveness to the audience which is also reflected in the order of appearance of papers in these proceedings. The papers are printed in the same order as presented at the symposium and posters are integrated in the corresponding thematic session.

In putting these proceedings together, many people played significant roles which we would like to acknowledge. First of all our thanks go to the authors who contributed their work to the symposium. Second, we are grateful for the dedicated work of the 38 members of the Program Committee for their effort in evaluating the submitted papers and in providing the necessary decision support information and the valuable feedback for the authors. Furthermore, the Program Committee awarded prizes for the best papers, and we want to sincerely thank the donors.

We were honored to have the following three invited speakers at the conference:

- Jan P. Allebach (School of Electrical and Computer Engineering, Purdue University): Digital Printing – A Rich Domain for Image Analysis and Pattern Recognition.
- Sven Dickinson (Department of Computer Science, University of Toronto):
 Object Categorization and the Need for Many-to-Many Matching.
- Václav Hlaváč (Center for Machine Perception, Czech Technical University):
 Simple Solvers for Large Quadratic Programming Tasks.

We are grateful for economic support from the Austrian Computer Society, Microsoft Europe, IBM, Advanced Computer Vision, and the Vienna Convention Bureau. Many thanks to our local support team, Karin Hraby, Ernestine Zolda and Patrizia Schmidt-Simonsky, who made this symposium possible and took care of all practical tasks involved in planning DAGM 2005. Special thanks go to Martin Kampel, who wrote and maintained the symposium website and supported the organization of the review process. We hope that these proceedings, following the tradition of all DAGM symposiums, will not only impact on the current research of the readers but will also represent important archival material.

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Since 1978 DAGM (German Association for Pattern Recognition) has organized annual scientific conferences at various venues. The goal of each DAGM symposium is to inspire conceptual thinking, support the dissemination of ideas and research results from different areas in the field of pattern recognition, stimulate discussions and the exchange of ideas among experts, and support and motivate the next generation of young researchers.

DAGM e.V. was founded as a registered research association in September 1999. Until that time, DAGM had been comprised of the following support organizations that have since become honorary members of DAGM e.V.:

- 1. DGaO, Deutsche Arbeitsgemeinschaft für angewandte Optik (German Society for Applied Optics)
- 2. GMDS, Deutsche Gesellschaft für Medizinische Informatik, Biometrie und Epidemiologie (German Society for Medical Informatics, Biometry, and Epidemiology)
- 3. GI, Gesellschaft für Informatik (German Informatics Society)
- 4. ITG, Informationstechnische Gesellschaft (Information Technology Society)
- 5. DGN, Deutsche Gesellschaft für Nuklearmedizin (German Society for Nuclear Medicine)
- 6. IEEE, Deutsche Sektion des IEEE (Institute of Electrical and Electronics Engineers, German Section)
- DGPF, Deutsche Gesellschaft für Photogrammetrie und Fernerkundung (German Society for Photogrammetry, Remote Sensing and Geo-information)
- 8. VDMA, Fachabteilung industrielle Bildverarbeitung/Machine Vision (VDMA Robotics + Automation Division)
- 9. GNNS, German Chapter of the European Neural Network Society
- 10. DGR, Deutsche Gesellschaft fur Robotik (German Robotics Society)

¹ DAGM e.V.: Deutsche Arbeitsgemeinschaft für Mustererkennung (German Association for Pattern Recognition).

Prizes 2004

Olympus Prize

The Olympus Prize 2004 was awarded to:

Daniel Cremers

for his significant contributions in the research area of Image Segmentation.



DAGM Prizes

The main prize was awarded to:

Bastian Leibe and Bernt Schiele

Scale-Invariant Object Categorization Using a Scale-Adaptive Mean Shift Search.

Further DAGM prizes for 2004 were awarded to:

Volker Roth and Tilman Lange

Adaptive Feature Selection in Image Segmentation.

Michael Felsberg and Gösta Granlund

POI Detection Using Channel Clustering and the 2D Energy Tensor.

Daniel Keysers, Thomas Deselaers and Hermann Ney

Pixel-to-Pixel Matching for Image Recognition Using Hungarian Graph Matching.

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On Determining the Color of the Illuminant Using the Dichromatic Reflection Model

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Abstract. The human visual system is able to accurately determine the color of objects irrespective of the spectral power distribution used to illuminate the scene. This ability to compute color constant descriptors is called color constancy. Many different algorithms have been proposed to solve the problem of color constancy. Usually, some assumptions have to be made in order to solve this problem. Algorithms based on the dichromatic reflection model assume that the light reflected from an object results from a combined matte and specular reflection. This assumption is used to estimate the color of the illuminant. Once the color of the illuminant is known, one can compute a color corrected image as it would appear under a canonical, i.e. white illuminant. A number of different methods can be used to estimate the illuminant from the dichromatic reflection model. We evaluate several different methods on a standard set of images. Our results indicate that the median operator is particularly useful in estimating the color of the illuminant. We also found that it is not advantageous to assume that the illuminant can be approximated by the curve of the black-body radiator.

1 Motivation

A white wall illuminated by yellowish light reflects more light in the red and green part than in the blue part of the spectrum. If we use a camera to take an image of the wall, the sensor of the camera will measure the light reflected from the wall. Thus, a photograph of the wall will have a yellow cast. A human observer, however, is able to somehow discount the illuminant. He will perceive the wall as being white irrespective of the type of illuminant used. This ability to compute color constant descriptors is known as color constancy [1]. Developing algorithms for color constancy is obviously very important for consumer photography. Another area where color constancy algorithms may be used is machine based object recognition. In this paper, we will be looking at several different methods on how to estimate the color of the illuminant from a color image. Once the illuminant is known, it can be used to compute a color corrected image under a canonical, i.e. white illuminant. The different methods will be evaluated on a standard set of test images.

W. Kropatsch, R. Sablatnig, and A. Hanbury (Eds.): DAGM 2005, LNCS 3663, pp. 1–8, 2005. © Springer-Verlag Berlin Heidelberg 2005

2 The Dichromatic Reflection Model

The dichromatic reflection model assumes that object color is a result of a matter effection in combination with a specular reflective component [2,3,4]. The overall color of the object is determined by the matter effection whereas specular highlights are caused by the specular reflection. These highlights occur whenever the light is reflected such that it directly enters the camera. Since the light from the light source is reflected directly into the camera it can be used to estimate the color of the illuminant.

Let $\mathbf{S}(\lambda)$ be the vector with the response functions of the sensor. For an RGB-sensor, we have $\mathbf{S} = [S_r(\lambda), S_g(\lambda), S_b(\lambda)]$ where the functions $S_i(\lambda)$ with $i \in \{r, g, b\}$ specify the sensor's response characteristics to light in the red, green, and blue part of the spectrum. Let $E(\lambda)$ be the light falling into the sensor, then the response of the sensor is given by

$$\mathbf{I} = \int_{-\infty}^{+\infty} E(\lambda) \mathbf{S}(\lambda) d\lambda.$$

Under the dichromatic reflection model, the response of the sensor is given by

$$\mathbf{I} = \int_{-\infty}^{+\infty} \left(s_M R_M(\lambda) E(\lambda) + s_S R_S(\lambda) E(\lambda) \right) \mathbf{S}(\lambda) d\lambda$$

where $R_M(\lambda)$ is the object reflectance with regard to the matte reflection, $R_S(\lambda)$ is the object reflectance with regard to the specular reflection, s_M and s_S are two scaling factors which depend on the object geometry and $E(\lambda)$ is the irradiance falling onto the object [4].

Let us now assume that the response functions are very narrow, i.e. they can be modeled by delta functions $S_i(\lambda) = \delta(\lambda - \lambda_i)$. Such ideal sensors only respond to a single wavelength λ_i with $i \in \{r, g, b\}$. This gives us

$$I_i = s_M R_{M,i} E_i + s_S R_{S,i} E_i.$$

Assuming that the specular reflection behaves like a perfect mirror, i.e. $R_{S,i}=1$, we obtain

$$I_i = s_M R_{M,i} E_i + s_S E_i.$$

Let $\mathbf{C}_M = [R_{M,r}E_r, R_{M,g}E_g, R_{M,b}E_b]$ be the measured matte color of the object and let $\mathbf{C}_S = [E_r, E_g, E_b]$ be the color of the illuminant. We now see that the color measured by the sensor is restricted to the linear combination of the matte color of the object point \mathbf{C}_M as seen under illuminant E and the color of the illuminant \mathbf{C}_S . The two vectors \mathbf{C}_M and \mathbf{C}_S define a plane inside the RGB color space [3].

By computing chromaticities, the three-dimensional data points are projected onto the plane r+g+b=1. This gives us a line in chromaticity space. The two points which define the line are the chromaticities of the measured object color $[r_O, g_O]^T$ and the chromaticities of the color of the illuminant $[r_E, g_E]^T$

$$\begin{pmatrix} r \\ g \end{pmatrix} = s \begin{pmatrix} r_O \\ g_O \end{pmatrix} + (1-s) \begin{pmatrix} r_E \\ g_E \end{pmatrix}$$

for some scaling factor s. The data points which belong to a uniformly colored surface will all be distributed along this, so called, dichromatic line. We now assume that the illuminant is uniform over the entire scene. In this case, all dichromatic lines will have one point in common, the color of the illuminant.

3 Natural Illuminants

If we know the correspondence between data points and surfaces, we can compute the dichromatic line for each surface. The dichromatic line can be found by doing a linear regression on the data. Alternatively we can also compute the covariance matrix and then locate the eigenvector which corresponds to the largest eigenvalue to determine the orientation of the dichromatic line. According to Finlayson and Schaefer [4] the algorithms based on the dichromatic reflection model perform well only under idealized conditions. The estimated illuminant turns out not to be very accurate. If small amounts of noise are present in the data then the computed intersection may be very different from the actual intersection. Finlayson and Schaefer note that the method works well for highly saturated surfaces under laboratory conditions but does not work well for real images. In their work, they assumed the images to be pre-segmented.

They suggest to compute the intersection of the dichromatic lines with the curve of the black-body radiator in order to make the method more robust. Many natural light sources can be approximated by a black-body radiator. The power spectrum $E(\lambda, T)$ of a black-body radiator depends on the temperature T. It can be described by the following equation [5,6]

$$E(\lambda,T) = rac{2hc^2}{\lambda^5} rac{1}{(e^{rac{hc}{k_BT\lambda}} - 1)}$$

where T is the temperature of the black-body measured in Kelvin, $h=6.626176\cdot 10^{-34}Js$ is Planck's constant, $k_B=1.3806\cdot 10^{-23}\frac{J}{K}$ is Boltzmann's constant, and $c=2.9979\cdot 10^8\frac{m}{s}$ is the speed of light. Many natural light sources such as the flame of a candle, light from a light bulb or sunlight can be approximated by the power spectrum of the black-body radiator. The chromaticities of daylight also follows the curve of the black-body radiator closely [7]. Plotting the chromaticities of the black-body radiator in CIE XYZ color space, one obtains a curve which can be approximated by a quadratic equation.

Using this approximation, we can compute the intersection between the dichromatic line and the curve of the black-body radiator. As a result, one either obtains none, one or two points of intersection. If the dichromatic line does not intersect the curve of the black-body radiator, then one can locate the closest point between the line and the curve of the black-body radiator. If two intersections are found, one can use some heuristics to select one of the two as the correct intersection. Using the constraint that the illuminant can be approximated by the curve of the black-body radiator, in theory it is possible, to determine the color of the illuminant from a single surface. Algorithms based on the gray world assumption [8,9,10] in contrast, require that the scene be sufficiently diverse.

4 Estimating the Color of the Illuminant by Segmentation and Filtering

Risson [11] extended the algorithm of Finlayson and Schaefer by also addressing the segmentation problem. Risson proposed to determine the illuminant by first segmenting the image and then filtering out regions which are not in line with the dichromatic reflection model. As a first step, noise is removed by pre-filtering the image using a Gaussian or median filter. Then the image is segmented. Regions which do not agree with the dichromatic reflection model, such as achromatic regions or regions which belong to the sky, are removed. In order to compute the direction of the dichromatic line reliably, the region has to have a certain size. Risson [11] suggested to remove all regions with a saturation less than 12%. For each remaining region, the dichromatic line is computed.

The dichromatic line can be computed by performing a linear regression on the x- and y-coordinates in CIE XYZ chromaticity space. We can also compute the covariance matrix for the pixel colors which belong to a single region. Using singular value decomposition, the largest eigenvalue tells us the direction of the dichromatic line. Let \mathbf{e}_i be the normalized eigenvector which corresponds to the largest eigenvalue obtained for region j. The dichromatic line \mathcal{L}_j of region j is then given by

$$\mathcal{L}_j = \{\mathbf{a}_j + s\mathbf{e}_j | \text{with } s \in \mathbb{R}\}$$

where \mathbf{a}_j is the average chromaticity of the region. In theory, the illuminant is located at the point where all dichromatic lines \mathcal{L}_j intersect. In practice, however, the dichromatic lines do not intersect in a single point because of noise in the data. It may also be that some of the computed lines are not caused by pure matter reflections in combination with specular reflections.

It may be possible to develop a classifier to rule out lines which are not in agreement with the dichromatic reflection model. A simpler method is to use the large number of dichromatic lines obtained from the image and to gather statistical evidence for the actual point of intersection. The exact method on how to determine the location of the point of intersection is not specified by Risson [11]. In finding the point of intersection, the curve of the black-body radiator may or may not be used to constrain the set of possible illuminants.

A simple method with no constraints on the color of the illuminant would be to compute the intersection for all possible combinations between two dichromatic lines. This gives us a set of intersections [12]. Let n be the number of dichromatic lines of the image. This gives us $\frac{1}{2}n(n-1)$ points of intersection $\mathbf{p}_i = [x_i, y_i]$ where x_i and y_i are the chromaticities in CIE XYZ color space.

$$\{\mathbf{p}_{i}|\text{with }i\in[1,...,\frac{1}{2}n(n-1)\}$$

From this set we can estimate the actual point of intersection by computing the average of the points of intersection. In this case, the position of the illuminant \mathbf{p} is given by

$$\mathbf{p} = \frac{2}{n(n-1)} \sum_{i} \mathbf{p}_{i}.$$