Mathematics and Its Applications

Wilhelm Kecs

The Convolution Product

and Some Applications

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To my wife, *Sabina*

Editor's preface

Growing specialization and diversification have brought a host of monographs and textbooks on increasingly specialized topics. However, the "tree" of knowledge of mathematics and related fields does not grow only by putting forth new branches. It also happens, quite often in fact, that branches which were thought to be completely disparate are suddenly seen to be related.

Further, the kind and level of sophistication of mathematics applied in various sciences has changed drastically in recent years: measure theory is used (non-trivially) in regional and theoretical economics; algebraic geometry interacts with physics; the Minkowsky lemma, coding theory and the structure of water meet one another in packing and convering theory; quantum fields, crystal defects and mathematical programming profit from homotopy theory; Lie algebras are relevant to filtering; and prediction and electrical engineering can use Stein spaces.

This programme, Mathematics and Its Applications, is devoted to such (new) interrelations as, exempli gratia:

— a central concept which plays an important role in several different mathematical and/or scientific specialized areas;
— new applications of the results and ideas from one area of scientific endeavor into another;
— influences which the results, problems and concepts of the one field of enquiry have and have had on the development of another.

The Mathematics and Its Applications programme tries to make available a careful selection of books which fit the philosphy outlined above. With such books, which are stimulating rather than definitive, intriguing rather than encyclopaedic, we hope to contribute something

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towards better communications among the practitioners in diversified fields.

Because of the wealth of scholarly research being undertaken in the Soviet Union, Eastern Europe, and Japan, it was decided to devote special attention to the work emanating from these particular regions. Thus it was decided to start three regional series under the umbrella of the main MIA programme.

The present volume, the second to appear in the MIA (Eastern Europe) series, deals with the convolution product and its manifold occurrences and applications. It is surprising, to the non-expert at least, how powerful a tool a product structure can be; in this case the convolution product.

In this book the author rather convincingly illustrates this remark by his applications to linear vibrations, boundary value problems for wave equations, heat transfer equations and equations for networks of power lines, and applications to the theory of visco-elastic solids and inductively coupled circuits.

Krimpen a/d IJssel **Michiel Hazewinkel**
1982

Preface to the English edition

The ideas presented in this book are those discussed in the earlier Romanian version, with several additions and extentions.

Thus, Chapter 2 has been extended by a treatment of Mikusińsky's operational calculus and by presenting new results in connection with the partial convolution product. Chapter 5 has also been considerably improved by the study of the viscoelastic linear solid bodies in order to illustrate the applications of the partial convolution product.

I would like to take this opportunity to express my gratitude to D. Reidel Publishing Company and Editura Academiei for the joint publication of this edition and Dr. Eng. Victor Giurgiuţiu for his English translation of the book. I am also under obligation to those who by reading the text have helped me to improve it.

Professor Dr. Wilhelm Kecs

Petroşani, April 6, 1982

Preface to the Romanian edition

The convolution product holds a central place among the various modes of function composition, due to the important properties it possesses.

The extension of the convolution product in the distribution space created a natural framework for the extension and enrichment of its properties, and it is due to this fact that the convolution product has become a powerful mathematical tool in symbolic calculus, distribution approximation, Fourier series, and the solution of boundary-value problems.

The high effectiveness of the convolution product is especially reflected in its properties with respect to the Fourier and Laplace transforms and in the description of the solutions to linear differential equations with constant coefficients.

Many fundamental results related to the convolution product and its applications are due to the following: L. Ehrenpreis, I. M. Gelfand, L. Hörmander, J. Lions, B. Malgrange, J. Mikusiński, L. Schwartz and G. E. Chilov.

The aim of this work is to present in a systematical manner the fundamental properties of the convolution product for functions and distributions. Additionally, it is shown how the convolution product is used in the study of mathematical physics, deformable solids, mechanical systems, electrical circuits, etc.

Chapter 1 gives the basic notions related to the construction of topological vector spaces, with stress on locally convex spaces, which form the basis for the distribution theory. A fair development of notions and fundamental formulae from the distribution theory is used throughout the book in order to present the properties and applications of the convolution product.

In addition to the theoretical study of the convolution product, an illustration of its use and the importance of the convolution algebra K'_+, were constantly considered in studying the solutions of differential equations, linear vibration systems, behaviour of visco-elastic solids, and inductively-coupled electric circuits. The convolution algebra K'_+, is shown to play a major role in all these developments.

The reference list given at the end of the book is only a part of the consulted literature directly or indirectly related to the subject. Many of author's original developments and results have also been included in the text.

The book is addressed to researchers in pure and applied mathematics, scientists and engineers working in applied mechanics and electrotechnics, as well as to the academic world and to students interested in the topic.

WILHELM KECS

Note to the reader

All theorems, propositions, lemmas, corollaries, observations, examples and equations are numbered consecutively in a single numeration system in each chapter.

References to the results stated in previous chapters are made by the double numeration system, which consists of the number of the chapter, a decimal point, and the primed number of the result within the chapter.

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Chapter 1

Topological vector spaces

1.1. REMARKABLE SETS IN VECTOR SPACES

1.1.1. Introduction

The study of many mathematical problems requires knowledge of special functional classes, the properties of which provide a natural background for theoretical developments and new solutions.

In this book, the properties and applications of the convolution product will be approached in the light of the distribution theory. Since the general properties of normed vector spaces are insufficient for such a theory, account must be taken of several functional spaces whose topology associated with the convergence structure is that of countable normed vector spaces or of reunions of such spaces. These functional spaces are particular types of locally convex spaces.

1.1.2. Remarkable Sets in Vector Spaces

We denote by Γ either the body R of real numbers or the body C of complex numbers. Let E be a set of abstract objects.

DEFINITION 1. The set E is called a vector space with respect to Γ, and is denoted by (E, Γ), if the following two operations are defined: the *sum*, a wapping $(x, y) \to x + y$ from $E \times E$ unto E, and the *product* with scalars from Γ, the mapping $(\lambda, x) \to \lambda x$ from $\Gamma \times E$ into E, having the following properties:

$$1^\circ \quad \forall_{x,y\in E}\ x + y = y + x;$$

$$2^\circ \quad \forall_{x,y,z\in E}\ (x + y) + z = x + (y + z);$$

$$3^\circ \quad \exists_{0\in E}\ \forall_{x\in E}\ x + 0 = x, \quad \text{(0 is the null element)};$$

$$4^\circ \quad \forall_{x\in E}\ \exists_{x'=-x\in E}\ x + (-x) = 0,$$

$$5^\circ \quad \forall_{x\in E}\ 1 \cdot x = x;$$

$$6^\circ \quad \forall_{\lambda,\mu\in\Gamma}\ \forall_{x\in E}\ \lambda(\mu x) = (\lambda\mu)x;$$

$$7^\circ \quad \forall_{\lambda,\mu\in\Gamma}\ \forall_{x\in E}\ (\lambda + \mu)x = \lambda x + \mu x;$$

$$8^\circ \quad \forall_{\lambda\in\Gamma}\ \forall_{x,y\in E}\ \lambda(x + y) = \lambda x + \lambda y.$$

The vector space (E, Γ) is real if $\Gamma = R$, and it is complex if $\Gamma = C$.

DEFINITION 2. The vector space formed by a subset E' of E, with the same body Γ and with the same sum and product operations as for (E, Γ) is called the subspace (E', Γ) of (E, Γ).

DEFINITION 3. The arithmetic sum of two subsets of E, A and B is the set

$$A + B = \{z\in E;\ \exists_{\substack{x\in A\\ y\in B}}\ z = x + y\}.$$

In particular, if A contains a single element, $A = \{a\}$, then

$$\{a\} + B = a + B = \tau_a B,$$

where $a + B$ is called the translation of set B by vector a. The symbol τ_a denotes the translation operator of vector $a\in (E, \Gamma)$ defined by the mapping $\tau_a\colon (E, \Gamma) \to (E, \Gamma)$ with $\tau_a x = x + a$, $x\in (E, \Gamma)$.

The arithmetic subtraction $A - B$ is similarly defined.

DEFINITION 4. The product set λA, where λ is complex and A is a set, is given by $\lambda A = \{z \in E;\ \exists_{x \in A}\ z = \lambda x\}$.

The following properties may be easily proved:

$$A + B = B + A;\ (A + B) + C = A + (B + C) \tag{1}$$

$$\lambda(A + B) = \lambda A + \lambda B;\ \ A \subset B \Rightarrow \lambda A \subset \lambda B \tag{2}$$

$$\lambda(A \cap B) = \lambda A \cap \lambda B;\ \lambda(A \cup B) = \lambda A \cup \lambda B \tag{3}$$

$$(\lambda + \mu)\, A \subset \lambda A + \mu A. \tag{4}$$

For families of such sets the following relations hold:

$$\lambda(\bigcap_i A_i) = \bigcap_i (\lambda A_i);\ \lambda(\bigcup_i A_i) = \bigcup_i (\lambda A_i) \tag{5}$$

$$(\bigcap_i A_i) + (\bigcap_i B_i) \subset \bigcap_i (A_i + B_i) \tag{6}$$

$$A_i \subset B_i \Rightarrow \bigcup_i A_i \subset \bigcup_i B_i \text{ and } \bigcap_i A_i \subset \bigcap_i B_i. \tag{7}$$

For instance, in order to prove (4), we perform

$$\lambda A + \mu A = \{z \in E;\ \exists_{\substack{x \in A \\ y \in A}}\ z = \lambda x + \mu y\}$$

$$(\lambda + \mu)A = \{z \in E;\ \exists_{x \in A}\ z = (\lambda + \mu)x\}.$$

Since for $x = y$, $(\lambda + \mu)x = \lambda x + \mu y$, it follows that

$$(\lambda + \mu)\, A \subset \lambda A + \mu A.$$

Let X_1 be an upper bounded set of real numbers ($\exists_{M \in R}\ \forall_{x \in X_1}\ x \leqslant M$). Then there exists a unique number $M^* = \sup\ X_1$, which is called the lowest upper bound of X_1, such that

i) $\forall_{x \in X_1} x \leqslant M^*$

ii) $\forall_{a \in R} a < M^* \quad \exists_{x \in X_1} x \in (a, M^*]$. (8)

Similarly, if X_2 is a lower bounded set of real numbers, $(\exists_{m \in R} \forall_{x \in X_2} x \geqslant m)$, then there exists a unique number $m^* = \inf X_2$, which is called the greatest lower bound of X_2, such that

$$\text{i}_1)\ \forall_{x \in X_2} x \geqslant m^*$$

$$\text{i}_1\text{i}_1)\ \forall_{b \in R} b > m^* \quad \exists_{x \in X_2} x \in [m^*, b). \tag{9}$$

If A and B are upper (lower) bounded sets, then

$$\sup (A + B) = \sup A + \sup B \tag{10}$$

$$\inf (A + B) = \inf A + \inf B. \tag{10'}$$

Indeed, denoting $\sup A = M_1^*$, $\sup B = M_2^*$, it follows that

$$\forall_{\varepsilon \in R^{+}} \exists_{\substack{x_1 \in A \\ y_1 \in B}} : x_1 > M_1^* - \frac{\varepsilon}{2}, \quad y_1 > M_2^* - \frac{\varepsilon}{2}$$

and, hence,

$$x_1 + y_1 > (M_1^* + M_2^*) - \varepsilon. \tag{11}$$

However, since

$$\forall_{\substack{x \in A \\ y \in B}} : x \leqslant M_1^*, \ y \leqslant M_2^*,$$

it follows that $x + y \leqslant M_1^* + M_2^*$ which, together with (11), shows that $M^* = M_1^* + M_2^*$ is the lowest upper bound of the set $A + B$. The proof of Property (10′) develops analogously.

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