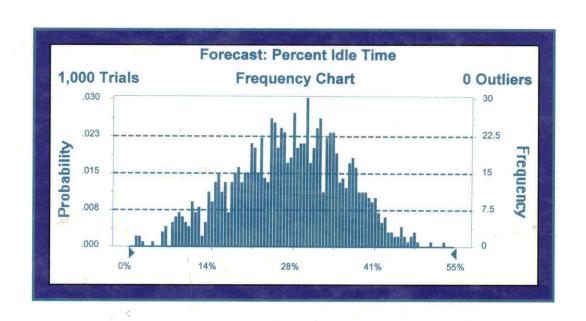


INTRODUCTION TO SIMULATION AND RISK ANALYSIS



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Crystal Ball, is introduced as an alternative.

A student version of the Crystal Ball software is included with this book.

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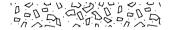
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Preface

The purpose of this book is to provide an introduction to the concepts, methodologies, and applications of simulation in business. One of the difficulties in teaching and learning simulation is the choice of a software platform. There exists a wide variety of excellent simulation software packages and languages, most of which require significant startup on the part of the student (and in many cases, the instructor as well). A course can easily deteriorate into a language course, and students can easily lose sight of the basic concepts and principles. Moreover, these packages are generally oriented toward engineering applications and are not suitable for applications of risk analysis, a subject that is being used increasingly in practical business applications. We avoid these problems by using spreadsheets as the principal means to illustrate simulation models and computational issues.

Spreadsheets provide the ideal environment with which to introduce simulation to business students. First, spreadsheets are nearly as common as calculators, and provide a way to convey quantitative methodologies in a language that business students can most easily understand. Second, spreadsheets allow one to address both risk analysis and systems simulation approaches in a common framework. For these reasons, spreadsheets are used exclusively in this book as the means of implementing simulation models. Although we do not advocate using spreadsheets for complex systems simulation models, spreadsheets do provide a very convenient way of conveying basic principles and allow students considerable hands-on experience with minimal frustration. With such a foundation, the advanced student can more easily learn to use commercial simulation software.

This book is aimed at upper-level undergraduate and beginning graduate students in business administration and related disciplines. Microsoft Excel is used throughout the book, although most models can easily be translated into other spreadsheet formats. The book is logically divided into three parts. Part I, "Foundations of Simulation," consists of three chapters that provide the basic concepts of simulation. In chapter 1 we describe the nature of simulation models, provide examples of pure Monte Carlo (repeated sampling) approaches, and introduce the concept of systems (time/event driven) simulation. The simulation process and benefits and limitations of simulation are also discussed. In chapter 2 we describe how to implement simple simulation models on spreadsheets. Methods for generating probabilistic outcomes and performing simple Monte Carlo simulations are introduced. The Excel add-in, Crystal Ball, is introduced as an alternative to brute-force spreadsheet replication. Crystal Ball is a powerful tool for performing Monte Carlo simulation. It is used throughout the book, and a student

version of the software is included with the book. In the appendix to chapter 2 we present optional material about random number generation techniques. Chapter 3 focuses on probability and statistics in simulation. It provides a comprehensive review of probability distributions commonly used in simulation, issues related to modeling probabilistic inputs, random variates and their generation, and statistical issues of analyzing the output from Monte Carlo simulations. In an appendix to the chapter we review the necessary prerequisite concepts from probability and statistics. We assume that students will have had at least a basic course in business statistics.

Part II, "Simulation and Risk Analysis," consists of two chapters that focus exclusively on risk analysis. Chapter 4 provides a quasi-users' manual for Crystal Ball as well as an original application developed by Cinergy Corporation, a major midwestern gas and electric utility. Chapter 5 presents a variety of applications in operations management, finance, and marketing. These examples show the variety of uses of Monte Carlo simulation as well as the flexibility of Crystal Ball in addressing risk.

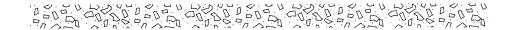
Part III, "Systems Simulation," consists of four chapters that deal with systems simulation. In chapter 6 we describe the fundamentals of simulating inventory and queueing systems, including a review of essential analytical models, simulation model development from a process or activity-scanning view, spreadsheet implementation, and model experimentation to address fundamental business issues. Chapter 7 focuses on event-driven simulation models and extends the examples presented in chapter 6. This leads naturally to a discussion of simulation software. Continuous simulation modeling is also described. In chapter 8 we discuss output analysis and experimentation in systems simulation, including issues of transient behavior and statistical methods for comparing different systems. In chapter 9 we present realistic applications in operations scheduling, information systems, and medicine. Finally, a comprehensive bibliography on simulation is included at the end of the book.

Several features have been designed into this book to improve pedagogy. First, cell formulas and detailed explanations are presented for each spreadsheet model. Second, each chapter has at least two "Simulation in Practice" features that describe real applications of simulation in various businesses. Finally, each chapter has numerous questions and problems that provide a means of review of important concepts and allow students to work with and extend models in the chapter or apply the concepts to new situations.

We would like to thank the following reviewers for their helpful comments: Vaidyanathan Jayaraman, University of Southern Mississippi; Ralph Badinelli, Virginia Tech; Arnold Buss, U.S. Naval Academy, Postgraduate School; and Linda Friedman, Baruch College, CUNY.

A special note of thanks goes to Eric Wainwright and Decisioneering, Inc., for developing a student version of Crystal Ball and for helping to improve the manuscript. Finally, we would like to thank our editor, Tom Tucker, for his help and guidance throughout this process.

James R. Evans David L. Olson



Crystal Ball Software and Excel Example Files

This text includes a disk with the student version of Crystal Ball. Crystal Ball is a sophisticated forecasting and risk analysis add-in program for spreadsheet users. Taking full advantage of the Microsoft Windows environment, Crystal Ball offers a unique combination of advanced simulation techniques in a graphical package that's easy to learn and use.

Decisioneering, in cooperation with Prentice Hall Publishing, has made this special student version available to you as an added tool to assist you in understanding, and putting into practice, the principles and methodologies described in the accompanying textbook.

As you will notice, some of the functionality of the commercial version of Crystal Ball is inaccessible in this version. A complete listing of the limitations may be found below. If you would like to purchase a fully functional copy of Crystal Ball, please contact Decisioneering.

Over 75 companies in the Fortune 100 have found Crystal Ball to be an invaluable tool in managing risk and making critical decisions. We hope that you too find Crystal Ball to be a valuable asset.

Please verify that your computer system meets the necessary requirements listed below. Then, follow the setup instructions below to install the student version on your hard drive. Once the installation is completed, double-click on the Crystal Ball icon to start Excel and Crystal Ball. From there, you may consult the accompanying textbook and view the on-line help for further information.

System Requirements

To run Crystal Ball on your computer, you have have:

- Microsoft Excel version 4.0 or later
- Microsoft Windows 3.x, Windows 95, or Windows NT
- PC with 386/486/Pentium processor with 8 MB RAM
- Hard disk drive, 3.5" 1.44-MB floppy drive, mouse

Setup Instructions

- 1. Run the SETUP.EXE program from the CD ROM.
- 2. In the dialog box that is displayed, select either the **Auto** or **Manual** startup option (we suggest you use the **Auto** option).
- 3. You will next be asked to enter the location of Excel on your system. The default location for Excel is:
 - For Excel 4.0 or 5.0: C\Excel
 - For Office 95: C:\Msoffice\Excel
 - For Office 97: C:\Program Files\Microsoft Office\Office where C: is the drive containing Excel.
- **4.** Next, locate the directory in which you want Crystal Ball installed. You may install Crystal Ball into any directory on your hard drive.

The Examples folder, which will be installed in the Crystal Ball directory, contains spreadsheet files for all major examples in this book. The files are named by their corresponding figure numbers in the book.

Student Version Limitations

- Maximum of 6 assumption cells and 6 forecast cells.
- Probability distribution types are limited to: normal, triangular, uniform, custom, Poisson, and exponential.
- Only one pair-wise correlation can be defined per model.
- No overlay charts are available.
- Maximum of 1.000 trials.
- Maximum of 100 data points can be fitted to a distribution.
- The word "Student" will appear in report titles and on all printed or copied charts.
- Special licensing language appears during startup.

How to Contact Decisioneering

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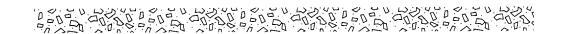
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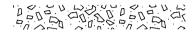
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Introduction to Simulation

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Questions and Problems

Management scientists use a wide variety of tools and techniques to model, analyze, and solve complex decision problems. These tools, many of which you may be familiar with at a basic level, include linear programming, decision analysis, queueing (waiting-line) theory, forecasting, and simulation. Many of these tools often require the analyst to make some highly simplifying model assumptions. For instance, many queueing models assume that arrival rates follow a Poisson probability distribution, leading to elegant mathematical solutions. Other tools, such as linear programming, apply only to restricted types of problems. Linear programming applies to well-structured situations that can be modeled with a linear objective function and linear constraints and do not include probabilistic elements. Furthermore, we typically assume that all data are known with certainty. Unfortunately, this is seldom true in practice.

Real situations rarely meet the assumptions of analytical models. For example, market uncertainties relative to competition may make predicting unit profit very difficult; the rate at which resources are consumed may vary; availability of resources from suppliers may not be assured; and clearly, demand is almost always uncertain. Often, the more elegant the mathematical formulation of a problem is, the less it matches reality. Finally, techniques such as queueing theory and linear programming do not incorporate the dynamics of the real business environment and provide little information about the *behavior* of the systems they model. Thus, although queueing theory, linear programming, and other management science techniques have many important practical applications, particularly as planning tools, they may not always be effective analysis tools.

For situations in which a problem does not meet the assumptions required by standard analytical modeling approaches, simulation can be a valuable approach to modeling and solving the problem. Simulation is the process of building a mathematical or logical model of a system or a decision problem, and experimenting with the model to obtain insight into the system's behavior or to assist in solving the decision problem. The two key elements of this definition are model and experiment. The principal advantage of simulation lies in its ability to model any appropriate assumptions about a problem or system, making it the most flexible management science tool available. Of course, it usually takes considerable effort to develop the model. Throughout this book we will see many examples of different types of simulation models. A model is worthless unless it provides some insight to the user. Thus, a major focus of simulation is conducting experiments with the model and analyzing the results. This requires some basic knowledge of statistics, which we review in Chapter 3 and use throughout the book.

Simulation is particularly useful when problems exhibit significant uncertainty, which generally is quite difficult to deal with analytically. In fact, a recent survey of management science practitioners showed that simulation and statistics have the highest rate of application over all other tools by over a 2:1 margin. Simulation use was even higher than statistics. We do caution you, however, that simulation should not be used indiscriminately in place of sound analytical models. Many situations exist when approaches such as linear programming or queueing theory are more appropriate. The task of the modeler is to understand the pros and cons of different approaches and use them appropriately.

In this chapter we:

- 1. Discuss the nature of simulation models and their role in management science.
- 2. Illustrate basic concepts of simulation by presenting some simple examples.
- 3. Discuss some benefits and limitations of simulation as a problem-solving tool.

The Nature of Simulation

Throughout history, simulation has been used for analyzing systems and decision problems. The Prussian army used to simulate wars by holding field exercises, making soldiers march through the woods of central Europe in all forms of wind and weather at the whims of their general staff (a tradition maintained by armies throughout the world today). Troop leaders would maneuver their units over the

¹L. Leon, Z. Przasnyski, and K. C. Seal, Spreadsheets and OR/MS models: An end-user perspective, *Interfaces*, **26**(2), 1996, 92–104.

countryside to simulate war, with officers serving as umpires to interpret outcomes.

We often see examples of simulation all around us. Simulation is used to forecast the weather and develop the graphical (and now three-dimensional) weather maps we see on television. Pilots use simulators that show how an airplane would react in specific conditions to pilot actions. This helps pilots learn how to cope with emergency situations without the need for traumatic experience. The movie Apollo 13 illustrated the use of simulation to train astronauts as well as to solve the problem of finding the best "power-up" sequence within the electrical system limitations that the astronauts faced before their critical reentry. (In fact, the launch sequence in the movie was itself a simulation.) NASA also uses simulations to predict rocket and satellite trajectories. Simulation can be observed in popular board games. For example, Monopoly simulates the old Atlantic City real estate market, using dice as a means to identify random events. Several companies sell simulations of major league baseball, which is one of the most statistically analyzed social environments in existence.

Manual simulations, such as moving troops through the field, operating a space shuttle simulator, or playing a board game, are very time consuming. People tire easily, and one or two simulated outcomes provide very little information on which to base a decision. However, if the simulation can be implemented on a computer, thousands of replications can be made in a matter of seconds, providing a wealth of information. With the increasing availability of faster and more powerful computers, and better understanding of quantitative modeling because of extensive spreadsheet use, simulation has become a very popular approach in recent years among practicing managers for the analysis of business problems.

Today, simulation is widely accepted in the world of business to predict, to explain, to train, and to help identify optimal solutions. Simulation is used extensively in manufacturing to model production and assembly operations, develop realistic production schedules, study inventory policies, analyze reliability, quality, and equipment replacement problems, and design material handling and logistics systems. It is used in designing and evaluating computer and communication networks and scheduling resources in complex projects. Simulation also finds extensive application in both profit-seeking service firms such as financial and retail companies, and in nonprofit service organizations such as health care, government, and education. These applications might involve the study of customer waiting-line behavior, evaluating surgical schedules, and designing efficient work flows in offices. For instance, simulation models can be used by a bank to help identify the number of tellers required to maintain a specified level of customer service as measured by waiting time or line length. Rather recently, due to increased availability and power of personal computers, simulation capabilities have been linked with spreadsheets to allow managers to evaluate risks of financial investment, marketing, real estate, and other common types of business decisions.

MODELS AND SIMULATION

Like all management science approaches, simulation revolves around *models*. A **model** is an abstraction or representation of a real system, idea, or object. Models in management science take many different forms. Some models are *prescriptive*; that is, they determine an optimal policy. Linear programming models are prescriptive because the solution to a linear program suggests the best course of action that a decision maker should take. Other models are *descriptive*; they simply

describe relationships and provide information for evaluation. Queueing models, which provide measures of system performance such as the average number in the queue and the average waiting time, are an example of descriptive models. Descriptive models are used to explain the behavior of systems, to predict future events as inputs to planning processes, and to assist decision makers in choosing the best solution or systems design. For example, a model of a factory operation can help production managers understand why bottlenecks occur. It can also be used to predict factory output as resources are added or changed in the system. By experimenting with different factory configurations, descriptive models can assist managers in selecting the best design to meet system objectives.

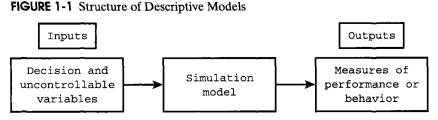
Models may also be *deterministic* or *probabilistic*. In a deterministic model, all data are known, or assumed to be known, with certainty. In a probabilistic model, some data are described by probability distributions. Using this classification, linear programming models are deterministic whereas queueing models are probabilistic.

Finally, models may be discrete or continuous. In mathematical programming, this dichotomy refers to the types of variables in the model. For example, linear programming models are continuous, whereas integer programming models are discrete. It may also refer to how model variables change over time. For instance, in multiperiod production planning models, we often assume that demand takes place at discrete points in time, such as the beginning of every week; this would classify the model as discrete. In modeling a chemical process, variables such as temperature and pressure would change continuously over time, and it would be important to capture this in the model.

How does simulation fall within these model classification schemes? Simulation models are *descriptive*; they simply evaluate measures of performance or the behavior of a system for a specific set of inputs, as shown in Figure 1-1. Model inputs include the controllable (decision) variables specified by the user, and uncontrollable variables or constants that capture the problem's environment. The simulation model itself is literally a set of assumptions that define the system or the problem. Probably the closest analogy with which you are familiar is a spreadsheet. A *spreadsheet* is a descriptive model in which the assumptions are the formulas entered in the cells. For any set of inputs, the spreadsheet calculates some output measures of interest.

Figure 1-2 shows an example of a financial spreadsheet. The model assumptions are given in rows 4 through 9. The key inputs are the base values in year 1 (column C) and inflation factors for subsequent years (column B). Formulas for computing the model outputs are entered in the lower section of the spreadsheet. Calculating the outputs basically consists of "stepping through" the formulas. Thus, a spreadsheet is essentially a deterministic simulation. The user may experiment with the model using different assumptions, for example, changing inflation factors or baseline values to answer a variety of what-if questions.

The major difference between an ordinary spreadsheet model and a simulation model is that a simulation model generally includes one or more probabilistic



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