ELMER B. MODE

# *Elements of* STATISTICS

*Third Edition*

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*third edition*

***Elmer B. Mode***

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# *Elements of*
# *STATISTICS*

*"The time has come," the Walrus said,*
*"To talk of many things:*
*Of shoes—and ships—and sealing-wax—*
*Of cabbages—and kings—"*

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# *PREFACE*

It has been said that the peculiar characteristic which sets man apart from the other animals is his capacity for capitalizing the achievements of his ancestors. Surely statistics offers a striking example of man's attempt to employ the records of the past in solving the problems of the present and in plotting the course of the future.

Much of the field of elementary statistics has been pretty well organized, so that the general route of progress for the beginner can be mapped out with little difficulty. Yet this route, plainly marked as it is, may be traversed with such facile mathematical skill—and nothing else—as to cause the novice to lose some of the most valuable lessons in simple critical analysis which statistics can teach. There is a peculiar logic associated with the study of statistics. Some call it common sense; others prefer a more exact characterization. In any case, it is difficult to dissociate the purely mathematical aspects from those which are not exactly mathematical but which do require a good brand of clear thinking. It is hoped that this book, dealing as it does with the elementary phases of statistics, will develop on the part of the reader an appropriate attitude of critical reasonableness toward the subject. There is no reason why sound mathematical conclusions should not be accompanied by a healthy appreciation of what such conclusions do *not* assert.

A large amount of important work can be done without extensive preparation in mathematics. Most persons who study elementary statistics are motivated by the fact that it has a direct bearing on a related field of interest:

business, economics, sociology, biology, psychology, education, and others. They want to acquire "sufficient statistical terminology and technique" to enable them to read intelligently and somewhat critically the statistical content of the literature in these fields. If called upon to perform a simple statistical analysis, they wish to be able to handle the fundamental procedures.

The present book favors no particular field of interest. The subjects enumerated above are well represented. There is an unusual abundance and a wide variety of exercises, none of which has been borrowed from other textbooks.

The arrangement of material allows much flexibility in the choice of topics. Many sections and some chapters may be eliminated, if so desired, without affecting the continuity of the book. Those who desire a brief course on methodology can omit such proofs as are present, although these cast valuable light on the concepts and processes involved. In particular, the writer has aimed to describe the full purport of each distinctive statistical concept. A list of references appropriate to an introductory study of this kind will be found at the end of the book.

The present edition, the third, presents a considerable revision of the earlier texts. In a field whose rate of extension and clarification is very rapid, fairly frequent changes in concepts and method of presentation are inevitable. The chief features of this revision are (1) a separate chapter on probability—the cornerstone upon which statistics rests; (2) the exposition of the normal distribution following that of the binomial; (3) a more systematic treatment of the testing of hypotheses; (4) some diminution in the computational aspects of statistics; (5) greater emphasis on the interpretation of results; (6) more up-to-date tables and charts of critical values; and (7) a brief optional chapter on nonparametrics.

The newer expositions have proved to be teachable at the level of the earlier editions of this book, and it has been the author's experience that the more challenging chapters excite the student's interest most of all.

The author is indebted to Sir Ronald A. Fisher and to Messrs. Oliver and Boyd, Ltd., Edinburgh, for permission to reprint Tables III, IV, and Va, from their book, *Statistical Methods for Research Workers*. The author is also grateful to Professor George W. Snedecor and the Iowa State College Press, to Professor Egon S. Pearson and the *Biometrika* Trustees, to Professors Wilfrid J. Dixon and Alexander M. Mood, to Drs. F. Swed and Churchill Eisenhart, and to Professor E. G. Olds, for the use of Tables appearing in the back of the book.

The author's thanks are due his kind colleague Professor Albert Morris, for helpful comments made after reading the manuscript of the first edition, and especially to Professor Albert A. Bennett, whose constructive criticisms

and invaluable suggestions added greatly to the interest of the first two editions. In particular the author is deeply in debt to Professor Elizabeth A. Shuhany for her many recommendations for the improvement of the presentation of the material in this, the third edition.

E.B.M.

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# INTRODUCTION

# 1

"In 1786, I found that in Germany they were engaged in a species of political inquiry, to which they had given the name of Statistics... an inquiry for the purpose of ascertaining the political strength of a country, or questions respecting matters of state."
SIR JOHN SINCLAIR
*The Statistical Account of Scotland*, Vol. 21 (1791–1799)

## 1.1. The Origin of Statistics

It is curious that the modern science of statistics traces its origin to two quite dissimilar human interests, political states and games of chance. In the mid-sixteenth century Girolamo Cardano, the Italian mathematician, physician, and gambler wrote his *Liber de Ludo Aleae* (The Book on Games of Chance) in which appeared the first known study of the principles of probability. About a hundred years later, the gambler Chevalier de Méré proposed to Blaise Pascal the famous "Problem of the Points," which may be described as follows: Two men are playing a game of chance. The one first gaining a certain number of points wins the stake. They are forced to quit before the game is completed. Given the number of points each has won, how should the stake be divided? This problem offered a real challenge to the wits of the two astute French mathematicians, Pascal and Fermat. A lengthy correspondence between the two men led to solutions, not only of the problems proposed, but of more general ones. The methods employed by Cardano and Pascal may be said to represent the beginnings of the mathematics of probability, about which modern statistical theory centers today. The publication by Laplace in 1812 of the epoch-making *Théorie Analytique des Probabilités* laid a firm foundation for this theory.

In the mid-eighteenth century, *statistics* itself was born as a word describing the study of "the political arrangement of the modern states of

the known world." The description of states was at first verbal, but the increasing proportion of numerical data in the descriptions gradually gave the new word the quantitative connotation that is associated with it now. From the rather restricted study of data pertaining to a state, statistics branched out into other fields of investigation.

Between 1835 and 1870, the Belgian astronomer Quetelet was applying the theory of probability to anthropological measurements. His conclusions may be summarized and extended by stating that the same general laws of variation governing gambler's luck may be discovered in the statures of soldiers, the intelligence quotients of children, the blood pressures of adults, the speeds of molecules of a gas, and innumerable other aggregates of observations.

In more recent times, an English school of statisticians under the leadership of Karl Pearson (1857–1936) and Ronald A. Fisher, have made notable contributions to both theoretical and applied statistics. The power of general methods based on probability concepts became more clearly perceived, and as a result, applications have been made to many diverse fields of inquiry. An appreciation of the importance of the statistical method in man's attempt to come to grips with a marvelously complex physical and social world is a serious and legitimate aim of any educated person.

## 1.2. The Meaning of Statistics

The layman frequently conceives of statistics as a mass of figures or a collection of data such as we might find in the publications of the United States Census Bureau, among the records of a school principal, or in the files of a large hospital. The often repeated phrase "Statistics show ... " is likely to imply that a given mass of figures contains salient and unalterable characteristics that can easily be discerned among the mass by any person of normal intelligence. That the word *statistics* may apply to certain aggregates of figures is not to be denied, but that important facts contained therein are easily detected is by no means always true.

A second meaning of *statistics* is simply the plural of *statistic*, where a *statistic* is a certain kind of measure used to evaluate a selected property of the collection of items under investigation. The average weight of a football squad, for example, may be found by adding the weights of the individual players and dividing by their number. The average thus obtained is a *statistic*.

A third meaning of *statistics* is of prime concern to us in this book. It is the science of assembling, analyzing, characterizing, and interpreting collections of data. In this sense, statistics is a field of study, a doctrine concerned with mathematical characterizations of aggregates of items.

Statistics, as a science, is fundamentally a branch of applied mathematics, just as mechanics is mathematics applied to problems connected with bodies

subjected to forces. In statistics, the applications may be made to almost any aggregate of observations or measurements. For this reason it is useful in business, economics, sociology, biology, psychology, education, physics, chemistry, agriculture, and related fields.

### 1.3. Sample and Population

A *population* is a totality of all actual or conceivable objects of a certain class under consideration. More precisely, a population consists of numerical values connected with these objects. Head lengths of criminals, test scores of pupils, thicknesses of washers, lengths of life of electric light bulbs, or numbers of negative replies on a questionnaire may constitute populations of measurements or observations. Such aggregates may be finite or infinite, real or fictitious, but in this book we shall assume all populations to be essentially infinite. Problems connected with finite populations will not be considered here.

A *sample* is a finite number of objects selected from the population. If these are chosen in such a manner that one object has as good a chance of being selected as another, we say that we have a *random sample.* In this connection we should note that some objects may have identical measurements, so that one *measurement* is not necessarily as likely to occur as another.

A group of 10 washers taken at random from a barrel of them constitutes a sample from a larger aggregate or population, from, say 1000 washers, or from the even larger potential product of the machine manufacturing the washers.

If we know the average thickness of the 10 washers, what conclusions can we safely draw concerning the average thickness of all the washers in the barrel? From the statistical characteristics of the sample, what deductions may we make about the number of washers smaller than a desired dimension? This type of problem is basic in statistics and is one to which notable contributions have been made in recent years.

The greatest care must be exercised in selecting a sample that is truly random. The 10 washers should be selected from different parts of the barrel, not from a particular part such as the top. If a sample of student records is to be selected from an alphabetically arranged card file, it would be risky to select all the records from the same portion of the file. Student names might reflect racial factors. A sample of soil should be synthesized from several areas of the garden plot.

In more elaborate analyses, samples must be scientifically constructed so as to include appropriate numbers of different representative groups. Thus, the sample taken in an election poll must contain different economic, geographic, social, racial, or other groups if it is to reflect the voting tendencies of the country as a whole. The problem of obtaining an adequate sample is

**Table 1–1** A PAGE OF RANDOM DIGITS

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 70079 | 99064 | 97423 | 68793 | 91763 | 14940 | 55550 | 19900 | 36879 | 27718 |
| 74372 | 99540 | 00119 | 55063 | 97512 | 73665 | 45331 | 93614 | 49512 | 08359 |
| 43658 | 71456 | 63894 | 28132 | 98307 | 83300 | 08001 | 11186 | 21446 | 35864 |
| 72448 | 27714 | 10704 | 36331 | 68905 | 18477 | 42727 | 72133 | 25167 | 41601 |
| 43269 | 47963 | 88026 | 79532 | 82919 | 03920 | 10924 | 02018 | 13708 | 05281 |
| 66360 | 47852 | 32769 | 59586 | 00133 | 72584 | 26480 | 00245 | 48371 | 37526 |
| 22043 | 77224 | 26075 | 68778 | 87332 | 83287 | 54373 | 96391 | 82132 | 89338 |
| 78519 | 43251 | 18412 | 30777 | 14380 | 13550 | 37902 | 46169 | 27785 | 10488 |
| 58454 | 13026 | 26618 | 18537 | 44015 | 73261 | 42001 | 06096 | 21918 | 94440 |
| 00666 | 78245 | 32662 | 03375 | 54485 | 89848 | 90606 | 55556 | 49481 | 35329 |
| 80043 | 26080 | 72508 | 53576 | 49390 | 35273 | 86769 | 07108 | 66688 | 24636 |
| 53787 | 10007 | 66163 | 88811 | 21977 | 92078 | 95503 | 43655 | 57975 | 25768 |
| 88907 | 42653 | 05541 | 13459 | 89731 | 89459 | 98306 | 55222 | 32363 | 68675 |
| 76654 | 24020 | 67332 | 62362 | 65014 | 18061 | 92185 | 08657 | 92167 | 47793 |
| 11675 | 96819 | 10965 | 31214 | 39215 | 29883 | 34235 | 27113 | 22919 | 31278 |
| 90066 | 91253 | 59174 | 58312 | 84990 | 52539 | 64054 | 34864 | 00483 | 17913 |
| 29480 | 78114 | 48305 | 67868 | 85176 | 50048 | 62792 | 82816 | 52055 | 93273 |
| 93992 | 71132 | 91042 | 96303 | 11372 | 13817 | 15490 | 19452 | 08265 | 57612 |
| 79938 | 37498 | 27019 | 18573 | 88617 | 31245 | 60208 | 53962 | 52981 | 04301 |
| 20506 | 31384 | 51173 | 33453 | 93156 | 43166 | 33599 | 98112 | 09422 | 48744 |
| 43006 | 16020 | 49784 | 09917 | 50236 | 59837 | 18739 | 85767 | 49111 | 51512 |
| 45186 | 04205 | 76923 | 06181 | 81538 | 68226 | 73500 | 60779 | 65584 | 24305 |
| 49966 | 94867 | 62902 | 43090 | 37205 | 72584 | 78048 | 98669 | 83267 | 13303 |
| 62224 | 77713 | 14540 | 24003 | 20499 | 32752 | 42271 | 75891 | 45681 | 44445 |
| 73217 | 21643 | 46106 | 73942 | 02936 | 45948 | 74850 | 17297 | 44957 | 31068 |
| 11219 | 20296 | 59367 | 31426 | 31166 | 66247 | 54764 | 91861 | 83130 | 37507 |
| 02164 | 54666 | 21868 | 65824 | 97370 | 23627 | 39822 | 29285 | 31387 | 17045 |
| 73171 | 27920 | 41254 | 60089 | 00693 | 58712 | 88187 | 56810 | 92728 | 07894 |
| 48435 | 58944 | 61989 | 84538 | 67060 | 69031 | 28814 | 31405 | 82384 | 77694 |
| 45687 | 46494 | 61920 | 26751 | 54241 | 09903 | 71831 | 98113 | 33094 | 99925 |
| 64573 | 28270 | 63695 | 16900 | 25980 | 61906 | 38832 | 44327 | 01141 | 37889 |
| 36345 | 24793 | 88754 | 95921 | 99442 | 30336 | 07705 | 41314 | 53028 | 07381 |
| 37402 | 15236 | 64920 | 25909 | 25085 | 85456 | 00198 | 32419 | 54583 | 83635 |
| 27358 | 35142 | 91012 | 35570 | 50420 | 30509 | 44150 | 99868 | 77894 | 05250 |
| 17222 | 24172 | 26021 | 79527 | 44721 | 19041 | 04399 | 74266 | 15134 | 17952 |
| 48436 | 19800 | 03441 | 60218 | 83099 | 10869 | 27264 | 06777 | 70388 | 34992 |
| 08752 | 26430 | 45080 | 80472 | 35599 | 34343 | 90581 | 46482 | 13441 | 74151 |
| 79075 | 92335 | 12474 | 33423 | 72174 | 02953 | 37198 | 97172 | 98019 | 92623 |
| 73073 | 26360 | 19111 | 65852 | 87760 | 41988 | 77620 | 83328 | 24394 | 23932 |
| 48418 | 80642 | 09023 | 48310 | 25218 | 79006 | 12709 | 39456 | 02883 | 83600 |
| 01362 | 30222 | 93728 | 16044 | 23187 | 40562 | 71067 | 13330 | 11022 | 17378 |
| 38148 | 24320 | 87981 | 57518 | 37136 | 04182 | 67913 | 88235 | 61865 | 24638 |
| 27411 | 82008 | 23860 | 45246 | 03403 | 97639 | 28686 | 67623 | 00542 | 63666 |
| 48322 | 46340 | 31022 | 55657 | 58297 | 36244 | 25091 | 75297 | 14695 | 75932 |
| 38823 | 78043 | 75095 | 58043 | 95125 | 74783 | 24693 | 06360 | 66853 | 66663 |
| 87891 | 01449 | 19122 | 70232 | 38118 | 30249 | 76453 | 20802 | 76374 | 83474 |
| 11627 | 55036 | 51014 | 95142 | 41014 | 28968 | 77021 | 79801 | 95957 | 87132 |
| 43277 | 09284 | 89837 | 17654 | 84726 | 49893 | 29601 | 02749 | 77246 | 21271 |
| 18946 | 64377 | 60317 | 28724 | 82044 | 03820 | 25767 | 53052 | 43304 | 70629 |
| 04996 | 65987 | 16738 | 51367 | 54872 | 93628 | 69984 | 29220 | 58652 | 06087 |

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