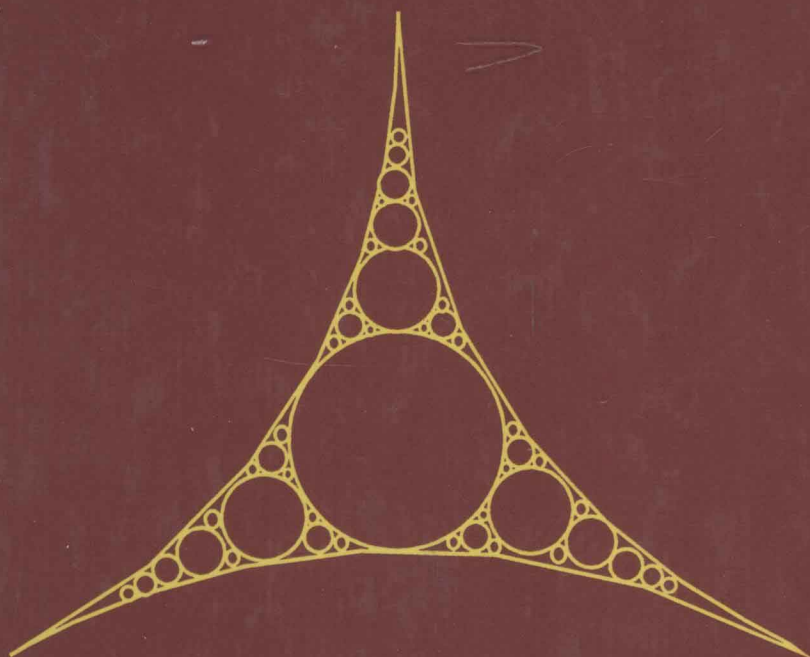


Jacques Duran

Sands, Powders, and Grains

An Introduction to the Physics
of Granular Materials



Springer

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of Granular Materials

Foreword by Pierre-Gilles de Gennes

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Sands, Powders, and Grains: An Introduction to the Physics of Granular Materials

Jacques Duran

Foreword

The physics of granular materials comes from an illustrious lineage. It includes names like Coulomb during the reign of Louis XVI, Faraday and Reynolds in the nineteenth century, and Bagnold, a remarkable Englishman who became enthralled by the sands of the desert, perhaps even more so than T. E. Lawrence, to the point that he resolved to understand its laws.

In spite of such great pioneers and their valiant efforts, the mechanics of powders remains largely a mystery. Even seemingly trivial questions lack clear answers. What is the weight distribution of a pile of sand dumped on the floor of an apartment under construction? Does it depend on the way it was dumped? No one really knows.

Such questions happen to be quite important in a variety of technical fields. They have applications to situations ranging from agricultural facilities storing corn in silos to the space exploration of Saturn's rings—this “fascinating merry-go-round of rocks,” to quote Guyon and le Troadec.

I have just mentioned two contemporary authors. They recently wrote a book entitled *Le sac de billes* (The bag of marbles), published in 1994 by Odile Jacob. It is a fascinating introduction to these problems, as well to somewhat unusual objects like porous materials. Yet, there is a need for a more advanced textbook for the benefit of our science students. For the last few years, Jacques Duran has taught a course in granular materials aimed at undergraduates in their senior year. He exposed them to a few general principles—for instance, the existence of many equilibrium states in the presence of friction. He was able to describe some key factors with the help of simple experiments. Or perhaps I ought to say “almost simple,” for granular media can be deceptive. A naive theoretician like myself may think that all the answers can be worked out with a couple of pounds of sand,

a funnel, a few hoses, and a glass container. As it turns out, a thousand technical details can upset an experiment and lead to nonsensical results. The physics of granulars may not require costly hardware, but it demands enormous care. Nor is it exempt of real dangers: Industrial reactors can generate violent explosions, even when they are ostensibly empty.

I am thrilled to see French physics actively engaged in the field of grains and powders, where almost everything is yet to be discovered. Within the last decade, a number of highly creative teams have popped up in places like Paris, Lyon, and Rennes. Jacques Duran leads one of those teams at the Pierre and Marie Curie University in Paris. This book draws on his first-rate expertise. I personally had the opportunity to read various drafts as it was taking shape. I learned a great deal from them in the process. There was a time when we had to steer anyone interested in sand to the classic text by Bagnold. From now on, Duran will be high on the required reading list. This book will fill a very real need. I sincerely wish it great success.

Pierre-Gilles de Gennes
Paris, France

Preface

“A body is liquid when it is divided into several smaller parts that move separately, and it is solid when all its parts are in contact.”

René Descartes, *Principles of Philosophy* (1644–1647)

Granular materials were probably not on Descartes’s mind when he wrote the sentence quoted above, but in some sense they might as well have been. They are indeed “divided into several smaller parts that move separately,” although they do not enjoy—far from it—the properties of a conventional liquid. And “all its parts are in contact,” although that does not confer on them the type of solidity envisioned by Descartes. This ambiguity illustrates well the lack of understanding, which has persisted until quite recently, of the properties of granular media, in spite of their prevalence not only in nature but in many human endeavors as well. Recent progress in several different areas of physics has fostered a critical reassessment of the concepts passed on to us by the scientists and engineers of generations past. As a result, a number of research groups dedicated to working on granular materials have sprouted all over the world in the last decade. They have produced a harvest of experimental observations, numerical simulations, novel concepts, and theoretical models.

Faced with such an onslaught of new facts, a newcomer to the field is likely to feel a bit overwhelmed. To make matters worse, this growing area of science has yet to benefit from any standardization, even in matters of terminology. I myself came to that realization while collecting material to teach an advanced course in the physics of liquids. The purpose of this text is to provide a smoother introduction to this exciting new area of research. Meant primarily as a teaching vehicle, it should answer the needs of fourth-year college students, researchers just entering the field, and young engineers interested in process engineering dealing with granular materials. More generally, it is aimed at anyone with a sufficient theoretical basis who is interested in learning how to start from quite simple experiments and, from there, build

up concepts and theories about a medium whose behavior is most often surprising and counterintuitive. My goal has been to gather, in as self-contained a manner as possible and in a unified language, the background necessary to understand the latest developments, as well as to present the rudiments of granular physics.

Given the current status of this young discipline, any progress is likely to come from experiments. Henri Poincaré once said: “It has been quite some time since anyone has thought of getting ahead of experimentation or to construct the world on the basis of a few hasty hypotheses.” The remark is particularly relevant to the physics of granular media. This book is rooted in that very principle. In this spirit, it contains many descriptions of experimental devices and accounts of experimental results obtained under conditions that are as controlled as possible. These provide the backdrop for concepts, models, and ideas that are pushed to levels that some might find too speculative. The adopted strategy, whose limitations must be borne in mind, goes with the very nature of this discipline, which is still very much in its infancy. In this context, this book discusses concepts and results as they are known to us *at the present moment*. As such, it is only a snapshot, as our understanding of this topic is bound to evolve, perhaps even undergo major revisions.

Lest it become a dry compilation of facts, writing a book of this kind entails somewhat arbitrary choices that remain the sole responsibility of the author. On more than one occasion, I felt compelled to leave out this particular result or that, even though I may come to regret it in the future. I did so simply because it did not readily fit in with the flow of the presentation or because it would have taken me too far afield. I beg for those whose work is mentioned too cursorily or not at all to forgive me. This by no means implies a negative judgment on my part. My choice was guided exclusively by pedagogical concerns and my own concept of the logical flow of the material. My decisions were often agonizingly difficult.

My thanks go to my colleagues and friends in my own laboratory (Eric Clément, Jean Rajchenbach, and Touria Mazozi). I am also grateful to the members of the Physics of Heterogeneous and Complex Matter Research Group affiliated with the National Center of Scientific Research (CNRS) and with the European HCM (Human Capital and Mobility) Network. I am enormously indebted to them not only for much of the technical material in the book but also for infusing in it their irrepressible enthusiasm which I hope will brighten the experience of the reader. I acknowledge the careful reading of the french manuscript and comments by Ko Okumura and VJ.-P. Faroux. Axel Reiseinger successfully carried out the translation from the French edition. He kept within the spirit of the text while suggesting numerous improvements and clarifications.

Finally, I owe a large debt of gratitude to Pierre-Gilles de Gennes and Etienne Guyon, both of whom played an instrumental role in the early stages of the research discussed here. I was fortunate to have their unrelenting support and encouragement to write and publish the material in this book. It borrows a great deal from their own work.

Jacques Duran
Paris, France

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1

Introduction

1.1. Some Orders of Magnitude Defining the Problem

The physics of granular materials deals primarily with macroscopic objects. The term “macroscopic” means here that the objects making up such materials must at the very least be visible to the naked eye. This is in contrast to mesoscopic or microscopic media. As it turns out, the very concept of granular material imposes some further restrictions, raising the low end of the range somewhat above the limits of our visual acuity. It is important to realize from the outset that the physics of such objects has in most cases nothing whatsoever to do with the notion of temperature, at least not as we normally understand it. To appreciate this, we need only calculate the kinetic energy E_k of a small bead of silicate glass—a basic component of river sand. As we will see shortly, the physics of dry granular materials of the type we are interested in involves objects that are typically 100 microns (μm) in size or larger. For typical translation velocities of the order 1 cm/s, we find that $E_k = \frac{1}{2}mv^2 \approx 10^{-12}$ Joule. If this kinetic energy were due entirely to thermal agitation, it would correspond to a temperature of 10^{11} K!

Furthermore, the loss ΔE_p of potential energy experienced by such a particle, as it drops by a height equal to its own diameter d , is given by $\Delta E_p = mgd$ (this corresponds to a situation where particles slide down past each other while remaining in contact). We find that ΔE_p and E_k are roughly equal.

The conclusion is that the collective behavior of such a granular medium simply cannot be described in terms of traditional Brownian motion. Some other mechanism responsible for generating fluctuations is required if we insist on defining a temperature of some kind [1], [2]. We will encounter many such examples in the

course of this book. Incidentally, it is instructive to calculate what the diameter of a particle of this same material should be if it is to be the seat of any significant thermal agitation at ordinary temperature (in the sense of $E_k \approx kT$). The result is about $1\ \mu\text{m}$, or some two orders of magnitude smaller than the size of the smallest particles of interest here. This one observation alone constitutes a serious obstacle to modeling the properties of granular piles by means of conventional thermal or hydrodynamic variables. Except in a few very special cases,¹ it is clear that the behavior of granular materials will be determined primarily by the more or less preferential and collective displacement of the particles involved.

The fact that Brownian motion is inconsequential explains why granular materials almost invariably resist any attempt at mixing. As we will see, shaking a collection of grains typically results in segregation, in other words, in separation according to size, precisely the opposite to what is being sought. By contrast, it is well known that in liquids, Brownian motion is the key mechanism responsible for intimate mixing of the various components. The phenomenon of granular segregation can be viewed as further evidence of the lack of thermal agitation with a mean free path characteristic of the average distance between particles.

Both nature and industry make heavy use of a variety of granular materials covering an extremely broad range of shapes, sizes, micromechanical, and chemical properties. In Chapter 2, we will mention a standard classification scheme based on size, but we will not concern ourselves with shapes, nor will we discuss the distribution laws of dissimilar media and associated measurement or identification techniques. Readers interested in such issues are referred elsewhere [5], [6]. What we will do is try to highlight *the fundamental principles governing the physics of these materials*, and to that end we will focus on simple and well-characterized objects. For reasons that will become clear later on, we will restrict our attention almost exclusively to the physical properties and behavior of so-called *dry granular materials*. Granted, a number of concepts that will come up during the course of this book can, with some precautions, apply to more complicated objects, such as pastes, muds, and other concentrated mixtures, in which particles interact with an ambient fluid. Be that as it may, we will largely confine our study to particles with idealized shapes (generally spherical or cylindrical) and whose micromechanical properties are well understood. In order to convince the reader that granulars present a serious challenge both from an economic perspective and on a more fundamental physics level, we shall briefly review a few practical applications of these materials as well as some basic problems which we will examine in more detail in later chapters. In a somewhat different vein—of considerable importance to human affairs, we might add—granulars of various degrees of complexity are at the heart of the geophysical sciences. In fairness, geophysicists have often beaten “conventional” physicists in defining many of the concepts to be discussed in these pages.

¹One such case is the effect of a uniform vertical air flow, which keeps particles in suspension and imparts to them random movements mimicking a classical Brownian motion [3]. It is also possible to model the system by way of a thermal agitation of sorts when the particles undergo repeated mutual collisions, as in a rapid flow [4].

The physical laws governing the behavior of granular media actually apply to objects whose dimensions cover *several orders of magnitude*. From grains, a few hundred microns each, to ice floes drifting across the polar seas (over distances of 1000 km),² not forgetting Saturn's rings (made of icy particles about 1 cm wide distributed in a band roughly 1 km thick), the science of granulars covers at least twelve decades of sizes.

The fact that aggregates seem to obey *universal laws* applicable over such a wide range of dimensions and characteristics is a strong incentive to pursue fundamental studies in that area. For instance, phenomena of segregation and intermittent blockages are pervasive in numerous industrial processes involving granular materials. Accordingly, the remainder of this chapter will be devoted to a brief and selective review of a few techniques and processes that in one way or another are subject to phenomena such as convection, segregation, blockages by arching, all of which are routinely encountered in industry. These effects will be examined in greater depth in subsequent chapters.

1.2. Economic Implications and Industrial Problems

As we have already pointed out, granular materials occupy a prominent place in our culture. The worldwide annual production of grains and aggregates of various kinds is gigantic, reaching approximately ten billion metric tons. Coal accounts for about 3.5 billion tons of that total, cements and ordinary construction materials for one billion tons, to which we can add equal amounts of sand and gravel. These constitute what is generally referred to as low-cost raw materials. The processing of granular media and aggregates consumes roughly 10% of all the energy produced on this planet. As it turns out, this class of materials ranks second, immediately behind water, on the scale of priorities of human activity. As such, any advance in understanding the physics of granulars is bound to have a major economic impact.

The industrial technology used in the treatment of granular materials involves a number of processes. First comes the extraction of ores, sands, and gravel, which often relies on dredging. Next comes crushing and grinding, followed by separation, all of which are commonly used with low-value-added materials. Since raw materials often account for more than 85% of the total cost, it is easy to understand why so little effort has been expended to improve the basic technology, which often dates back to the nineteenth century.³ Not much has been optimized, despite the fact that methods of transport (fluidized beds, conveyor belts), storage (silos), and mixing (e.g., cement trucks) figure in all stages of the industrial processing of aggregates. Problems have received primitive solutions at best. The more specialized and developed arena of high-value-added materials include the cosmetic

²Studies of the movements of icebergs, notably in the vicinity of port facilities, has been the object of several research contracts with the Canadian Navy. It turns out to be closely related to the physics of granular materials.

³A recent report by a US government agency stresses the obsolescence of the techniques used to process granular materials. Significantly, it is entitled "Granular Materials: A Legacy of Neglect."