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Monique Pavel

Fundamentals of Pattern Recognition

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Preface

Pattern recognition (P.R.) is no longer in its infancy, yet one can picture this field as a building built on an insecure base. Concerning the notion of pattern or shape, for instance, one may notice that many "applied" publications in P.R. deal with shape in an empirical manner, particularly in shape analysis, where shape is decomposed, reconstructed, transformed, analyzed, and even recognized, but never defined. What is essentially missing today in P.R. is a well-grounded base or, more precisely, its mathematical formalization.

To better explain what we mean by "base," let us also take the notions of classification and recognition. Whereas they arise from different methodologies (i.e., associating two objects with the same class, without necessarily specifying which class, respectively associating an object with a representative of a class, pattern, or structure), the essential principle that they have in common and that directs the resolution of these two problems (classification and recognition) is their definition, which can be given in mathematical terms such as isomorphism (homeomorphism when the objects are topological).

Modeling is a basic activity of applied mathematics and is necessary in such fields as P.R. In mathematics in general, the essence of innovation and discovery is the formulation of concepts and the elucidation of structure; mathematics has thus constantly narrowed the gap between itself and the rest of science, and its use is almost pervasive within the physical sciences and their engineering applications. There are also unpredictable applications of mathematics, which are harder to characterize and will spring from mathematics' universality and vitality. Looking to the future, we see a challenge in technology which P.R. must meet. History teaches that new technology will require new mathematics; that excellence in technology will require excellence in mathematics is a corollary of this lesson.

The question is: Which mathematics to use? Mathematics thrives on the interaction of independent viewpoints and different approaches. With luck, and taste, the choices lead in time to unpredictable applications, but it is already obvious that a theory analogous to that of vector spaces already led to the modeling of the syntactic approach in P.R.

It is said that mathematics is both the handmaiden of science and its queen, and it is for the most part driven by goals of its own choosing; but again, luck, taste, and knowledge have made possible recent predictable applications of recent developments of mathematics: the theory of categories, which has already modeled grammars, languages, and machines; and homotopy theory, which has already been used, on one hand, to classify defects in crystals and, on the other hand, for engineering applications and economics, for which algorithms and codes have recently been written. It is one of our goals to show how homotopy theory and its generalization, the theory of shape, formalize the notion of shape or pattern for regular spaces such as polyhedra, respectively for very irregular spaces, such as those that we encounter in P.R., and to show that the theory of categories does formalize the notions of classification and recognition.

These considerations created the contents of this book, which we hope is a contribution to the saga of making P.R. a discipline in its own right in applied mathematics and in computer science.

This book is intended for both theorists and practitioners in P.R. Its mathematical aspect proceeds, again, from the interaction of independent viewpoints and different approaches, and the explanation stems from the fact that mathematics is to a large extent theoretical. It is not exclusively theoretical because computers give it an experimental side. Also, because some mathematical notions, such as homotopy, certain topologies on a finite set, and so on, have already been programmed, the application of the categorical formalism is just starting to be applied in P.R. Some other aspects suggested by this book—such as the shape-theoretical approach—should also give rise to algorithms and computer codes.

This book is self-contained, in that the necessary preliminary notions of the theory of homotopy, the theory of shape, and the theory of categories will be given at the beginning of the chapters where they will first be used.

This book originated from the lecture notes of the graduate course on the mathematics of P.R. which I teach at the Université de Paris VI (formerly known as the Faculté des Sciences de Paris) since 1967. This course has gradually been completed from my own original findings: in 1967, I pointed out the analogy between (syntactic) recognition and the decision problem for finite automata; later I enriched the course with my categorical results in P.R., and recently with the initiation to the topological theory of shape and its possible application to P.R. Actually, my oral course follows the structural–discrete topological–categorical–shape-theoretical guideline. That is the history of my own thoughts on the field and also on the sense of the growing mathematical complexity needed for the comprehension of the course by my students. This book could be used as a graduate textbook on the mathematics of P.R., following either the presentation used throughout this book or the sequence noted above.

Monique Pavel

Contents

1
Introduction

1.1 ORGANIZATION OF THE BOOK

Our major concern in the development of pattern recognition is to give precise (mathematical) definitions to basic terms frequently used in this field and to follow mathematical methods and their ordering in fundamental research.

Chapter 2 deals with the fundamental problems of pattern recognition: classification and recognition. These terms can be stated generally and defined by mathematical notions. This process raises the problem of decidability. Because we must decide, and that by automatic means, we need to be aware of the approximations that we create by substituting a decidable and automatically computable problem for the given one.

Chapter 3 deals with images and patterns or shapes. It is therefore necessary to define mathematically what we mean by shape, and this notion can be expressed by using topological equivalence; this is the object of Section 3.3. But because, in automatic pattern recognition, images always present themselves under a discrete form, we first deal, in Sections 3.1 and 3.2, with the discretization problem of images. Given the existence of different topologies on a finite space, we focus our attention on discrete spaces generated by finitely presented Abelian groups, which formalize the standard discretization of the Euclidean plane and three-dimensional space by regular rectangular grids.

Concerning Chapter 4, recall that reality can be grasped in all its complexity only by a descriptive approach, so we must make use of a structural or syntactic formalism. We also define the vocabulary used in the categorical framework.

Finally, because all these notions, whether algebraic or topological, can be unified by powerful, general mathematical tools which set forth the generality and unity of different concepts and allow us to come closer to reality (less restrictive hypotheses), we present in Chapter 5 the categorical formalization of pattern classification and recognition. One of the important roles of mathematics in society is not only the ability to solve problems, but the ability to invent, or use if already invented, languages in which one can express problems (for example, category theory). This formalism also allows us to give a structure to classes of images, their projections, and their skeletons.

What has happened in modern mathematics is that one cannot do algebra without knowing geometry and one cannot do geometry without knowing analysis; what has happened in our century's science and technology is that one cannot work in any of these fields by ignoring mathematics. It is our belief that pattern recognition does not constitute an exception to this general law, and therefore cannot afford to ignore the more and more intertwined branches of mathematics. This book could have been entitled Math Anxiety in Pattern Recognition: Response to Questions from the Floor or The Mathematical Challenge in Pattern Recognition. It is in this spirit that we give an overview of the book and the field in Chapter 6.

1.2 MATHEMATICAL METHODS AND THEIR ORDERING IN FUNDAMENTAL RESEARCH, PARTICULARLY IN PATTERN RECOGNITION

If one compares the syntactic or structural, the topological (discrete or continuous), and the categorical approaches in pattern recognition and the mathematical techniques used in all of them, one notices that i) there is an effort to study a whole class of images by associating with it one of its representatives, which is equivalent to and "simpler" than the original one; ii) only some fundamental tools, such as equivalence, or invariance of some shape properties with respect to a set of transformations are used; iii) these tools, taken from algebra (and category theory) and geometry (and topology) have been, or have to be, adapted to the needs of automatic pattern recognition.

These techniques are currently used in fundamental research, whether mathematical or nonmathematical. The process is as follows (see Pavel [1.1]):

Define a set of objects or images, a property or "point of view," and a set of transformations.

Define a certain equivalence relation on the whole class of objects with respect to the set of transformations.

Derive a partition into equivalence classes.

Study representatives of these classes, "prototypes," "canonical forms," patterns, shapes, or structures, which, though resembling from the adopted point of view the initially given object, are more "regular" or "simpler" (think of skeletons, of images with "regular" perimeters that are connected-component-similar with a given image, and so on).

One can consider all present and future approaches to pattern recognition in this general methodological framework. This statement is true not only for the classical algebraic and discrete topological approaches but, perhaps even more, for the shape-theoretical and the categorical approaches. We intend to show the use of this methodology in Chapters 3, 4, and 5.

The advantages of using methods of fundamental research in different approaches in pattern recognition is considerable, since, on one hand, it situates pattern recognition in science, particularly in fundamental computer science and, on the other hand, it allows a uniform and general perception of different notions and results. But this general way of proceeding leads naturally to a certain number of (open) research and development problems when applied to practical pattern recognition. We shall deal with some of them. We point out that i) a single approach of this general kind is not sufficient by itself to provide all the information we need, whence the necessity of combining several or all of them for image analysis and recognition; ii) we need to evaluate the threefold approximation that one commits by associating with a given real image its structure, prototype, or shape (in the eventually strict sense of the theory of shape), plus by discretizing afterward if we use continuous topology, for example, plus by processing information automatically.

We start by giving mathematical definitions to Pattern or Shape and to Recognition so that we deliberately choose general methods and general tools, all the more that our objects are almost arbitrary sets of points.

Furthermore, if thinking and formalizing is needed before computing, there is still another question left open before deciding and computing: is pattern recognition a decidable problem? If yes, is it decidable by automatic means and in which cases?

If no, how can we approximate the given real problem by a decidable and computable one. This seems to us to be the fundamental problem of pattern recognition, and we shall deal with it in the next chapter.