

ENCYCLOPAEDIC DICTIONARY
OF PHYSICS

SUPPLEMENTARY VOLUME 4

SUPPLEMENTARY VOLUME 4

ENCYCLOPAEDIC DICTIONARY OF PHYSICS

GENERAL, NUCLEAR, SOLID STATE, MOLECULAR
CHEMICAL, METAL AND VACUUM PHYSICS
ASTRONOMY, GEOPHYSICS, BIOPHYSICS
AND RELATED SUBJECTS

EDITOR-IN-CHIEF

J. THEWLIS

FORMERLY OF HARWELL

Associate Editors

R.C. GLASS

LONDON

A.R. MEETHAM

TEDDINGTON



PERGAMON PRESS

OXFORD · NEW YORK · TORONTO
SYDNEY · PARIS · FRANKFURT

OXFORD	Pergamon Press Ltd., Headington Hill Hall, Oxford OX3 0BW, England
U. S. A.	Pergamon Press Inc., Maxwell House, Fairview Park, Elmsford, New York 10523, U.S.A.
CANADA	Pergamon of Canada Ltd., P.O. Box 9600, Don Mills M3C 2T9, Ontario, Canada
AUSTRALIA	Pergamon Press (Aust.) Pty. Ltd., 19a Boundary Street, Rushcutters Bay, N.S.W. 2011, Australia
FRANCE	Pergamon Press SARL, 24 rue des Ecoles, 75240 Paris, Cedex 05, France
WEST GERMANY	Pergamon Press GmbH, 6242 Kronberg-Taunus, Pferdstasse 1, Frankfurt-am-Main, West Germany

Copyright © 1971 Pergamon Press Ltd.

All Rights Reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means: electronic, electrostatic, magnetic tape, mechanical, photocopying, recording or otherwise, without permission in writing from the publishers

First edition 1971

Reprinted 1976

Library of Congress Catalog Card No. 60-7069

LIST OF CONTRIBUTORS TO THIS VOLUME

- | | | |
|--|---|--|
| AGGARWAL A. K. (<i>New Delhi</i>) | DEWEY M. A. P. (<i>Stoke Poges, Bucks.</i>) | HEARLE J. W. S. (<i>Manchester</i>) |
| ALBERT R. D. (<i>Berkeley, Calif.</i>) | DODD D. S. (<i>Stoke on Trent, Staffs.</i>) | HILLS P. R. (<i>Harwell</i>) |
| ALDER B. J. (<i>Berkeley, Calif.</i>) | DUGDALE R. A. (<i>Harwell</i>) | HOLBROOK K. A. (<i>Hull</i>) |
| ARRIDGE R. G. C. (<i>Oxford</i>) | DUNNING W. J. (<i>Bristol</i>) | HONIG A. (<i>Syracuse, N. Y.</i>) |
| BAGNOLD R. A. (<i>Edenbridge, Kent</i>) | DYCE R. B. (<i>Stanford, Calif.</i>) | JAEGER R. R. (<i>Ohio</i>) |
| BAKER R. M. L. JR. (<i>El Segundo, Calif.</i>) | ELION H. A. (<i>Cambridge, Mass.</i>) | JAGGER J. (<i>Dallas, Texas</i>) |
| BARKLA H. M. (<i>St. Andrews, Scotland</i>) | ESIN A. (<i>Ankara, Turkey</i>) | JAIN S. C. (<i>Delhi, India</i>) |
| BEHRMAN H. P. (<i>Stamford, Conn.</i>) | FINE N. J. (<i>Pennsylvania</i>) | JAMES J. F. (<i>Manchester</i>) |
| BELL W. W. (<i>Aberdeen, Scotland</i>) | FJELDBO G. (<i>Stamford, Calif.</i>) | JERRARD H. G. (<i>Southampton</i>) |
| BERGER H. (<i>Argonne, Illinois</i>) | FOLKES M. J. (<i>Somerset</i>) | JONES E. (<i>Groningen, Holland</i>) |
| BLOCH A. (<i>Wembley, Middx.</i>) | FROGGATT C. D. (<i>Harwell</i>) | JUETT D. A. (<i>Cambridge</i>) |
| BOWEN E. G. (<i>Sydney, Australia</i>) | GALE A. J. (<i>Lexington, Mass.</i>) | KIRKWOOD D. H. (<i>Sheffield</i>) |
| BREINER S. (<i>Stanford, Calif.</i>) | GARDNER J. W. (<i>Leicester</i>) | KLASS D. L. (<i>Chicago, Illinois</i>) |
| CARLEY-MACAULY K. W. (<i>Harwell</i>) | GAZEY B. K. (<i>Birmingham</i>) | KREJCIK S. (<i>Montreal, Canada</i>) |
| CARLSON H. W. (<i>Virginia</i>) | GERICKE O. R. (<i>Watertown, Mass.</i>) | LANE R. K. (<i>Ontario, Canada</i>) |
| CHAMBERLAIN J. (<i>Teddington</i>) | GOLDSMID H. J. (<i>Bath</i>) | LARGE M. I. (<i>Sydney, Australia</i>) |
| COHEN E. (<i>Manchester</i>) | GOODMAN P. (<i>Victoria, Australia</i>) | LEE W. (<i>New York</i>) |
| COLEMAN K. R. (<i>Aldermaston</i>) | GORINGE M. J. (<i>Oxford</i>) | LONGSTAFF R. M. (<i>Wantage, Berks.</i>) |
| COMPTON-COLLINS (<i>San Francisco, Calif.</i>) | GOSSICK B. R. (<i>Lexington, Kentucky</i>) | LUNDQUIST C. A. (<i>Washington D.C.</i>) |
| COPELAND J. A. (<i>Murray Hill, N.J.</i>) | GRAY G. W. (<i>Hull</i>) | MARCH N. (<i>Sheffield</i>) |
| COWLARD F. C. (<i>Towcester, Northants</i>) | GRIEVESON M. H. (<i>Northwood, Middx.</i>) | MARTIN A. E. (<i>Newcastle upon Tyne</i>) |
| COX L. G. (<i>Ottawa, Ontario</i>) | HADFIELD D. (<i>Sheffield</i>) | MASSEY G. A. (<i>Calif.</i>) |
| CRAWFORD J. H. (<i>Oak Ridge, Tennessee</i>) | HADNI A. (<i>Nancy, France</i>) | MCCARTNEY B. S. (<i>Godalming, Surrey</i>) |
| CROMPTON R. W. (<i>Canberra, Australia</i>) | HAGSTRUM H. D. (<i>Murray Hill, N.J.</i>) | MCMILLAN P. W. (<i>Stafford</i>) |
| CUTHILL J. R. (<i>Washington, D.C.</i>) | HAMILTON E. I. (<i>Reigate, Surrey</i>) | MILLER A. (<i>Oxford</i>) |
| DAVIDGE R. W. (<i>Harwell</i>) | HARRIS F. E. (<i>New York</i>) | MOODIE A. F. (<i>Victoria, Australia</i>) |
| DE JAGER (<i>Macclesfield, Cheshire</i>) | HARRIS L. A. (<i>Shenectady, N.Y.</i>) | NABARRO F. R. N. (<i>Johannesburg, S. Af.</i>) |
| | HAWKINS G. S. (<i>Cambridge, Mass.</i>) | NARAYAN V. (<i>Delhi, India</i>) |
| | | NAYLER J. L. (<i>Claygate, Surrey</i>) |
| | | NELSON R. S. (<i>Harwell</i>) |

-
- | | | |
|---|---|---|
| PAIGE E. G. S. (<i>Great Malvern, Worcs.</i>) | ROWLAND T. J. (<i>Urbana, Illinois</i>) | SUTCLIFFE R. C. (<i>Reading</i>) |
| PASHLEY D. W. (<i>Saffron Walden, Essex</i>) | RYCROFT M. J. (<i>Southampton</i>) | TARRANT A. W. S. (<i>Surbiton, Surrey</i>) |
| PETLEY B. W. (<i>Teddington</i>) | SANDERS L. G. (<i>Dorchester on Thames</i>) | T'JAMPENS G. (<i>Turnhout, Belgium</i>) |
| PUGH S. F. (<i>Harwell</i>) | SCHALLAMACH A. (<i>Herts.</i>) | VILLA F. (<i>Stanford, Calif.</i>) |
| PURCHAS J. G. (<i>Harwell</i>) | SHARPE J. W. (<i>Glasgow</i>) | WAGNER P. E. (<i>Baltimore, Maryland</i>) |
| PURSEY H. (<i>Teddington</i>) | SHARROCK S. J. (<i>London</i>) | WALKDEN A. J. (<i>Wembley, Middx.</i>) |
| | SHAW W. (<i>Middletown, N.J.</i>) | WARD I. M. (<i>Leeds</i>) |
| | SHULL C. G. (<i>Cambridge, Mass.</i>) | WASSE M. P. (<i>Harlow, Essex</i>) |
| RAWLINGS R. (<i>Cardiff</i>) | SMITH G. C. (<i>Cambridge</i>) | WHEELER L. K. (<i>London</i>) |
| REDWOOD M. (<i>London</i>) | SMITH R. G. (<i>Murray Hill, N.J.</i>) | WILLIAMSON R. M. (<i>Rochester, Michigan</i>) |
| REES D. J. (<i>Northwood, Middx.</i>) | SODHA M. S. (<i>New Delhi, India</i>) | WYSLOUZIL W. (<i>Ottawa, Canada</i>) |
| REES J. A. (<i>Liverpool</i>) | SOOTHA G. D. (<i>Delhi, India</i>) | |
| REIN R. (<i>Amberst, N.Y.</i>) | SPICER W. E. (<i>Stanford, Calif.</i>) | |
| RENNER B. (<i>Pasadena, Calif.</i>) | SPRING K. H. (<i>London</i>) | |
| RICHARDS T. L. (<i>Ontario, Canada</i>) | STEVENS K. N. (<i>Cambridge, Mass.</i>) | |
| RITCHIE I. G. (<i>Cardiff</i>) | | |
| ROSENSWEIG R. E. (<i>Burlington, Mass.</i>) | | |
| | | ZIJP W. L. (<i>Petten, Holland</i>) |

51-66
72

FOREWORD

IN THE Foreword to the first of this series of Supplementary Volumes I wrote:

"Of its very nature, a work of the size and scope of the Encyclopaedic Dictionary of Physics can never reach ultimate completion. However, by issuing a continuous series of Supplementary Volumes, we shall strive to keep it as up to date and comprehensive as we can (having regard to the inevitable time lapse between writing and publication), and as free from errors as may be.

"The volumes in this series are intended to form part of a unified whole, and are numbered accordingly. They are designed to deal with new topics in physics and related subjects, new development in topics previously covered and topics which have been left out of earlier volumes for various reasons. They will also contain survey articles covering particularly important fields falling within the scope of the Dictionary.

"The contents of these volumes will be arranged alphabetically, as in the previous volumes. Articles will be reasonably short and will be signed. Cross references to other articles will be incorporated as necessary, and bibliographies will be included as a guide to further study. Each volume will have its own index, prepared on the same generous scale as before; and, in addition, it is intended to issue a cumulative index every five years. Errata and addenda lists will be published, referring to the original Encyclopaedic Dictionary of Physics, and to those Supplementary Volumes which will already have been published.

"In preparing the Supplementary Volumes regard will be had to the changing emphasis in many branches of physics—the invasion of the biological sciences by physics, the possibilities opened out by the increasing use of computers in all branches of science and technology, the ever-increasing scope of theoretical physics, the progress in high energy physics, the emergence of new instrumental techniques etc.; and, at the same time the authors of previously published articles will be given the opportunity of bringing those articles up to date. Naturally there are many articles for which this will not be necessary, and it is certainly not intended that new articles shall be written if there is no need for them. In short, it is our intention to produce a series of volumes in which the high standard already achieved in the Encyclopaedic Dictionary of Physics is fully maintained."

The continuing success of the Supplementary Volumes would seem to indicate that our efforts have not been unavailing. I hope and trust that the present volume, the fourth in the series, will maintain the standard already set. However, in the knowledge that mistakes are inevitable in a work of this nature, I shall be grateful if readers will let me know of any errata or omissions, so that appropriate amendments may be made at a suitable stage.

Once again I should like to express my gratitude to the publishers, and particularly to Mr. Robert Maxwell and Mr. E. J. Buckley for their constant support; to my Associate Editors, Dr. A. R. Meetham, of the National Physical Laboratory, and Mr. R. C. Glass, of the City University, for their help and counsel; to Mr. S. Crimmin, the Assistant Editor at the Pergamon Press, and his successor Mr. M. S. Gale, for their solid work behind the scenes; and to the referees who, as in the past, have helped to ensure the quality of the published articles. Last but not least it is a pleasure to acknowledge the debt I owe to my wife, who has been of invaluable assistance at all stages of the work.

J. THEWLIS
Editor-in-Chief

ARTICLES CONTAINED IN THIS VOLUME

Acoustic holography	G. A. MASSEY
Acoustic waveguides	M. REDWOOD
Acousto-electric effects	E. G. S. PAIGE
Airborne collision prevention devices	S. KREJOIK
Ambient noise in the sea	B. S. MCCARTNEY
Astro-archaeology	G. S. HAWKINS
Astrodynamics	R. M. L. BAKER, Jr.
Auger emission spectroscopy	L. A. HARRIS
Auroral electrons	R. D. ALBERT
Biological pressure transensors	C. COMPTON COLLINS
Bordoni peak	F. R. N. NABARRO
Bosons, intermediate	W. LEE
Brittleness in ceramics	R. W. DAVIDGE
Cardiac pacemaker, external	D. A. JUETT
Ceramics production, physics in	D. S. DODD
Charge conjugation	C. D. FROGGATT
Chart of the nuclides—trilinear	D. J. REES
Conductivity at high temperatures	S. C. JAIN and V. NARAYAN
Current algebras and their applications	B. RENNER
Daylight, power distribution in	A. W. S. TARRANT
Diffusion in non-uniform media	D. H. KIRKWOOD
Dislocation kinks	F. R. N. NABARRO
Dispersion and fibre strengthening	G. C. SMITH
Dynamic electron diffraction, applications of	P. GOODMAN
Dynamical n -beam theory of electron diffraction	A. F. MOODIE
Electric dipole moment of the neutron	C. G. SHULL
Electroaerodynamics	L. G. SANDERS
Electrohydraulic crushing	K. W. CARLEY-MACAULY
Electrohydraulic forming of metals	K. W. CARLEY-MACAULY
Electron beam machining	J. G. PUBCHAS
Electron diffraction from periodic magnetic fields	M. J. GORINGE
Electron metallography, applications of	M. A. P. DEWEY
Electroviscous fluid	D. L. KLASS
Electroviscous fluids, applications of	D. L. KLASS
Elementary particles, nomenclature of	W. W. BELL
Emissivity at high temperatures	S. C. JAIN and V. NARAYAN
Epitaxy, recent work on	D. W. PASHLEY
Far infra-red lasers	A. HADNI
Far infra-red spectroscopy	A. HADNI
Fast passage in paramagnetic resonance	P. E. WAGNER
Ferrohydrodynamics	R. E. ROSENSWEIG
Fluidics	J. L. NAYLER

Fourier transform spectroscopy	J. CHAMBERLAIN
Fracture of brittle materials	S. F. PUGH
Freeze-coating	R. G. C. ARRIDGE
Geochronology (radiometric dating)	E. I. HAMILTON
Glandless reciprocating-jet pump	A. J. WALKDEN
Glass-ceramics	P. W. McMILLAN
Glow discharge electron and ion guns for material processing	R. A. DUGDALE
Gyroscopic coupler	A. BLOCH
Hall effect, applications of	E. COHEN
Hamilton's principle	B. R. GOSSICK
Helicity	W. W. BELL
High intensity pulsed radiation sources	A. J. GALE
High-lift devices	J. L. NAYLER
High polymers, mechanical properties of	M. J. FOLKES and I. M. WARD
High resolution electron microscopy	J. W. SHARPE
Holography	H. G. JERRARD
Hydrologic cycle	T. L. RICHARDS
Hydrometeorology	T. L. RICHARDS
Impatt diode microwave generators	M. P. WASSE
Infra-red cameras and infra-red TV	A. HADNI
Infra-red spectroscopy, recent developments in	A. E. MARTIN
Interferometric spectrometers	A. E. MARTIN
Interplanetary dust	J. F. JAMES
Ionization and attachment cross-sections of atoms and molecules by electron impact	J. A. REES
Ion-neutralization spectroscopy	H. D. HAGSTROM
Ionosphere, recent observations of	M. J. RYCOFT
Ionospheres of Mars and Venus	G. FJELDEO
Jet flap	J. L. NAYLER
Josephson effect	B. W. PETLEY
Kirchoff's law (electrical circuits)	A. BLOCH
Laser-induced sparks	J. W. GARDNER
Lasers, applications of	H. A. ELION
Lasers, tunable	R. G. SMITH
Law enforcement science and technology	W. SHAW
Liquid crystals and molecular structure	G. W. GRAY
Liquid metals	N. H. MARSH
Luminosity of a spectrometer	J. CHAMBERLAIN
Magnetic thin films	M. J. FOLKES
Magnetoresistivity	E. COHEN
Melting, modern theories of	B. J. ALDER
Metal film preparation by electroless (chemical) reduction	K. A. HOLBROOK
Metallic colloids in non-metallic crystals	S. C. JAIN and G. D. SOOTHA
Microplasticity	A. ESIN
Microwave solid-state oscillators, recent	J. A. COPENLAND
Mirage and looming, optics of	M. S. SODHA and A. K. AGGARWAL
Model ships, photoelectric tracking of	W. WYSLOUZIL
Momentum transfer cross-section for low energy electrons: determination by swarm techniques	R. W. CROMPTON
Multiplex principle (of Fellgett)	J. CHAMBERLAIN
Neutretto	W. W. BELL
Nuclear quadrupole resonance	T. J. ROWLAND

Optical information processing	K. R. COLEMAN
Peierls stress	F. R. N. NABARRO
Peltier refrigerators	H. J. GOLDSMID
Permanent magnet technology	D. HADFIELD
Phonon avalanche and bottleneck	P. E. WAGNER
Photoemission and band structure	W. E. SPIERRE
Physical limnology	R. K. LANE
Piezomagnetic effect	S. BRENNER
Pockels effect	H. PURSEY
Polarized proton targets	A. HONIG
Polymer physics	J. W. S. HEARLE
Proteins, structure of	A. MILLER
Proton scattering microscopy	R. S. NELSON
Pulse code modulation	L. K. WHEELER
Pyroelectric detector for infra-red radiation	H. P. BEERMAN
Q-switching	H. A. ELION
Quantum mechanics in biochemistry	F. E. HARRIS and R. REIN
Quantum metrology	B. W. PETLEY
Quarks	S. J. SHARROCK
Radar astronomy	R. B. DYCE
Radiation chemistry, industrial applications of	P. R. HILLS
Radiation quantities, systematics of	W. L. ZIJP
Radioemissions from interstellar hydroxyl radicals	G. DE JAGER
Radioisotopes in food processing	R. M. LONGSTAFF
Radio telescopes, the present position	M. I. LARGE
Refractories, ultra-purification of	S. C. JAIN and V. NARAYAN
Reverberation under water	B. K. GAZEY
Rubber physics	A. SCHALLAMACH
Sailing, physics of	H. M. BARKLA
Satellites in soft X-ray spectra	J. R. CUTHILL
Semiconductors, effects of radiation on	J. H. CRAWFORD, JR.
Shock pressure in meteorites	R. R. JAEGER
Singing sands	R. A. BAGNOLD
Snoek spectra	I. G. RITCHIE and R. RAWLINGS
Soft X-ray spectroscopy	J. R. CUTHILL
Solar cell	K. H. SPRING
Solar wind, planetary interactions with	E. G. BOWEN
Solid nucleus formation from solution	W. J. DUNNING
Sonic boom, generation and suppression of	H. W. CARLSON
Space power supplies	K. H. SPRING
Speaking machines	K. N. STEVENS
Split-beam microwave beacon	L. G. COX
Sputtering, theories of	R. S. NELSON
Standard Earth, geodetic parameters for	C. A. LUNDQUIST
Streamer chambers	F. VILLA
Submillimetre radiation, applications of	J. CHAMBERLAIN
Submillimetre-wave lasers	J. CHAMBERLAIN
Submillimetre-wave methods	J. CHAMBERLAIN
Temperature-resistant polymers	E. JONES
Textile industry, physics in	J. W. S. HEARLE
Thermionic generation of electricity	K. H. SPRING
Thermoelectric cooling of Infra-red detectors	H. P. BEERMAN

Thermoelectric generation of electricity	K. H. SPRING
Thermomagnetic gas torque (the Scott effect)	R. M. WILLIAMSON
Thin films, nucleation and growth of	M. J. FOLKES
Tungsten halogen lamps	T. JACOBS and G. T'JAMPENS
Ultrasonic amplification in semiconductors	E. G. S. PAIGE
Ultrasonic imaging	H. BERGER
Ultrasonic spectroscopy	O. R. GERICKE
Ultra-violet radiation, some biological effects of	J. JAGGER
Vitreous carbons, structure and properties of	F. C. COWLARD and J. C. A. LEWIS
Walsh functions	N. J. FINE
Weather control	R. C. SUTCLIFFE
Whole body counting	M. H. GRIEVESON and D. J. REES

ACOUSTIC HOLOGRAPHY

1. Introduction

It is possible to produce high-resolution, three-dimensional images of objects exposed to acoustic radiation by applying holographic techniques to the detection and display of sound waves. The use of sound instead of light makes available an enormous and continuously variable range of wave-lengths which may be produced by high-powered sources with excellent temporal coherence. Because linear transducers for the conversion of sound energy to electrical energy are readily available, some methods for making acoustic holograms exist without an analogue in conventional optical holography, which must depend on nonlinear electro-optical or photographic detection. However, the absence of a convenient, sensitive medium for recording two-dimensional acoustic fields has led to extensive research in the area of scanning acoustic detectors, sampled arrays, and other devices which correspond to photographic film in optical holography.

Because the subject of acoustic holography is new and rapidly developing, comprehensive survey articles do not exist. This article describes the theory of acoustic holography, with and without an acoustic reference beam, and presents the problems of wave-length scaling, motion of the acoustic medium, and three-dimensional imaging at low resolution. References to a representative sampling of papers in this field are given, but the list is necessarily far from complete.

2. Basic Imaging Theory

2.1. Holography with Offset Acoustic Reference Beam

This method can be developed by direct analogy with the offset reference method of optical holography developed by Leith and Upatnieks (1962, 1963). We assume the scalar theory of diffraction by Huygens' principle applies, and we take the time variation at a single temporal acoustic frequency as understood. Figure 1 shows the basic configuration. Since we are concerned with the details of the reconstruction process for a wide variety of objects which are to be imaged, we can consider only the field present at the hologram recording plane. Using phasor notation, let this field be $U_{\text{sig}}(x, y)$ and let the field from the reference source be $Ae^{-j2\pi f_0 x} = U_{\text{ref}}(x, y)$. Here f_0 is the spatial frequency produced on the recording plane

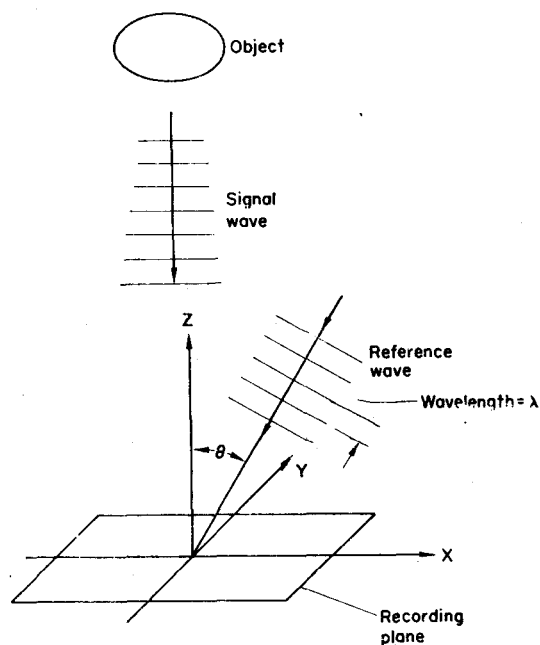


Fig. 1.

by tilting the reference wave off-normal by an angle $\theta = \sin^{-1}(\lambda/f_0)$. Now suppose that in the x - y plane there is a recording medium such as a scanning detector which records at each position, with a square-law characteristic, a quantity $I(x, y)$ proportional to the square of the magnitude of the field incident at that point. Then

$$\begin{aligned} I(x, y) &= K_1 [U_{\text{ref}}(x, y) + U_{\text{sig}}(x, y)] [U_{\text{ref}}^*(x, y) + U_{\text{sig}}^*(x, y)] \\ &= K_1 [U_{\text{ref}}(x, y) U_{\text{ref}}^*(x, y) + U_{\text{sig}}(x, y) U_{\text{sig}}^*(x, y) \\ &\quad + U_{\text{sig}}(x, y) U_{\text{ref}}^*(x, y) + U_{\text{ref}}(x, y) U_{\text{sig}}^*(x, y)] \\ &= K_1 [A^2 + |U_{\text{sig}}(x, y)|^2 + A^* U_{\text{sig}}(x, y) e^{j2\pi f_0 x} + A U_{\text{sig}}^*(x, y) e^{-j2\pi f_0 x}]. \end{aligned}$$

For reconstruction of the recorded image, the $I(x, y)$ will ordinarily be recorded as a photographic transparency, which can be placed in an optical beam for

viewing. If we assume a positive transparency is made, the film recording process can be approximated by the following model:

1. The exposure E of an area of film is proportional to incident intensity and duration of the applied light signal. Therefore define $E = tI$, where t is the exposure time and I is the light intensity.

2. Over a reasonable range of exposure, the intensity transmission τ_i of the positive transparency, as a function of E , is given by:

$$D \equiv \log \frac{1}{\tau_i} = -\gamma \log E + D_0$$

or

$$\tau_i = e^{-D_0 E^\gamma}$$

where γ and D_0 are functions of the film type, development process, and exposure level.

The amplitude transmission is therefore

$$\tau_a = \sqrt{\tau_i} = K_2 E^{\gamma/2} = K_3 I^{\gamma/2}$$

where

$$K_2 = e^{-D_0/2} \quad \text{and} \quad K_3 = t^{\gamma/2} e^{-D_0/2}.$$

Now if the hologram information $I(x, y)$ is made equal to I in the exposure relation above, for example, by writing on film with a modulated light beam, the processed transparency can be placed in a beam of collimated light. At normal incidence, with the incident amplitude set equal to unity for convenience, the optical field directly behind the illuminated transparency is simply equal to $\tau_a(x, y) = K_3[I(x, y)]^{\gamma/2}$. Thus if processing is done so that $\gamma = 2$, the reconstructed field $U'(x, y)$ is:

$$U'(x, y) = K_1 K_3 [|A|^2 + |U_{\text{sig}}(x, y)|^2 + A^* U_{\text{sig}}(x, y) e^{j2\pi f_0 x} + A U_{\text{sig}}^*(x, y) e^{-j2\pi f_0 x}].$$

Since the reference beam is a plane wave, the quantity $|A|^2$ represents an undeviated transmitted wave normal to the hologram plane. For objects of limited angular extent, or for a field of view much less than 180° , the spatial frequency spectra of $U_{\text{sig}}(x, y)$, $U_{\text{sig}}^*(x, y)$, and $|U_{\text{sig}}(x, y)|^2$ are relatively narrow. Under this assumption, and with the further assumption that f_0 is greater than the spatial bandwidth of the signal, the quantity $|U_{\text{sig}}(x, y)|^2$ is then a group of waves propagating near the normal and the two remaining terms, containing $U_{\text{sig}}(x, y)$ and $U_{\text{sig}}^*(x, y)$, are off-axis groups centered about the angles $\pm \sin^{-1}(\lambda f_0)$ away from the normal. The coefficients A and A^* represent only a constant gain and optical phase shift, and the exponential phase factors imply off-axis propagation. Thus, the original wavefront $U(x, y)$ is reconstructed with only a possible change in signal strength and direction, and an observer viewing the hologram at an angle corresponding to the direction of the term $A^* U_{\text{sig}}(x, y) e^{j2\pi f_0 x}$ will see a virtual image of the object which produced the original field. The term $A U_{\text{sig}}^*(x, y) e^{-j2\pi f_0 x}$ has phase variations opposite to the original wave and

corresponds to light converging to a real image behind the hologram. This can be understood intuitively by decomposing the object and image into points and considering the phase of the spherical wavefronts corresponding to light from these points. The phase change corresponding to taking the complex conjugate transforms diverging fronts to converging ones; thus the complex conjugate term represents the real image. By offsetting the angle of the reference wave, we have made it possible to separate the light from real and virtual images as well as the direct on-axis component consisting of distortion products. The wavefronts over the aperture of the hologram can be perfectly reproduced; thus high resolution three-dimensional imaging is often possible, depending, of course, on the wavelength and recording geometry.

2.2. Holography without an Acoustic Reference Beam

A fundamental difference between the techniques available for making optical and acoustical holograms is the fact that the acoustic beam can be converted to a useful electrical signal directly with linear transducers, whereas optical detection is always square-law and requires a reference beam to determine phase. As a result, it is possible to avoid the need for an acoustic reference beam by suitable electronic processing of the signal after linear detection.

To illustrate this concept with an example, consider the system diagrammed in Fig. 2. The normal to the

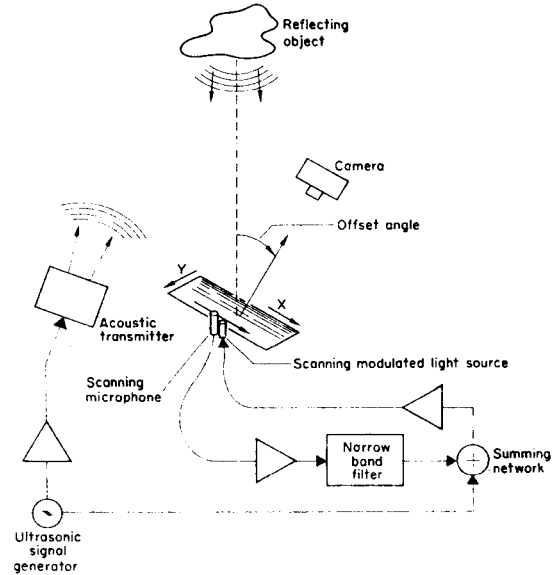


Fig. 2.

hologram plane has been rotated through an angle θ away from the general direction of the object, and the linear transducer is scanned across the tilted

plane. The output signal voltage from the transducer will then be

$$U(x, y) = K_4 U_{\text{sig}}(x, y) e^{j2\pi f_0 x}$$

where $U_{\text{sig}}(x, y)$ is the acoustic field that would be present without the tilt, and K_4 is the transfer characteristic of the transducer. A voltage of complex amplitude A is also obtained from the signal generator used to produce the original acoustic wave transmitted to the object. The two voltages are added linearly and used to modulate a light source whose output intensity I is proportional to input electrical power, that is:

$$I = K_5 |v|^2.$$

Then the time average over a few cycles of the carrier frequency is

$$\bar{I}(x, y) = K_5 [A + K_4 U_{\text{sig}}(x, y) e^{j2\pi f_0 x}] [A^* + K_4^* U_{\text{sig}}^*(x, y) e^{-j2\pi f_0 x}]$$

$$\bar{I}(x, y) = K_5 [|A|^2 + |K_4 U_{\text{sig}}(x, y)|^2 + A^* K_4 U_{\text{sig}}(x, y) e^{j2\pi f_0 x} + A K_4^* U_{\text{sig}}^*(x, y) e^{-j2\pi f_0 x}]$$

This expression is identical in form to the one derived above using a reference beam. Clearly then, if the average intensity $\bar{I}(x, y)$ is then used to expose film, a hologram can be produced in the same manner as in the reference beam method described earlier.

The tilt could be eliminated if the direct voltage from the oscillator were phase shifted with transducer location so that

$$A \rightarrow A e^{-j2\pi f_0 x}.$$

Since it may be more difficult to produce this spatially controlled phase than to tilt the hologram recording plane, the tilted plane technique may be more practical.

3. Wave-length and Hologram Scaling Considerations

The wave-length scaling from acoustical to optical and the hologram size scaling from the original scan area to the film recording affect the size and location of the reconstructed images because of the diffraction angle relations. Obviously if the hologram size scaling is exactly the same as the wave-length scaling, the image size and position are also scaled in the same way, so that all quantities diminish in the ratio optical wave-length/acoustic wave-length. Sometimes this choice of hologram scaling may be inconvenient for viewing, however, because the resulting optical hologram may be extremely small. In general it is desirable to view a scaled hologram that is smaller than the original sound field and larger than that area scaled down by the ratio of the reconstruction and recording wave-lengths.

By considering the hologram as a complex diffraction grating or superposition of Fresnel zone plates, and by applying lowest-order diffraction theory, one can easily show that the distance from the hologram

to the reconstructed image of the object varies according to the relation

$$d_2 = \frac{h_2^2 \lambda_1}{h_1^2 \lambda_2} d_1$$

where d is the object-(or image-)to-hologram distance, h is the width of the hologram, λ is the wave-length, and the subscript "1" refers to the recording parameter values while "2" refers to values of quantities after scaling in the reconstruction process. From similar considerations it may also be shown that the physical size of the image is given approximately by

$$w_2 = \frac{h_2}{h_1} w_1$$

where w_2 is a dimension across the reconstructed image, and w_1 is the corresponding dimension of the original object.

Because d_2 and w_2 scale differently, various distortions will be present in general. Obviously, for ratios of acoustic wave-length/optical wave-length of the order of 10^3 or so, values of the scaling parameter h_2/h_1 may be chosen to give severe distortion of depth-to-transverse width ratios in the reconstructed image. An additional fact, not considered here in any detail, is the introduction of aberrations into the imaging process. Fortunately, as the resolution of the acoustic hologram is increased by enlarging the w_1/λ_1 ratio, it is usually practical to scale the hologram down by a corresponding greater factor also. This approaches the ideal condition

$$\frac{h_2}{h_1} = \frac{\lambda_2}{\lambda_1}$$

more closely and reduces the distortions and aberrations mentioned above.

A related consideration is the amount of depth illusion present in the reconstructed image. In viewing an ordinary optical hologram, the "three-dimensional" quality of the image is present because the hologram width-to-optical wave-length ratio is sufficiently large to provide good resolution for an observing pupil much smaller than the hologram. Thus the observer may move his point of observation across the hologram aperture and easily resolve substantial differences in the perspective of the image. Putting this in more quantitative terms, the object must lie well within the limit of the near field of the hologram, or

$$d_1 < \frac{w_1^2}{\lambda_1}$$

if any depth illusion is to be obtained. Beyond this limit, the object is imaged essentially as though it were at infinity. Because of coherence problems and the small wave-length, this limit is seldom approached in optical holograms; however, it is easily reached in acoustic work if w_1 is not sufficiently large.

With acoustic holograms it is also possible to realize a condition in which the object is near the recording area but the wave-length is so large that the entire hologram aperture must be covered by the viewing pupil in order to achieve even the minimum required resolution. Depth information remains in this situation in the form of a focusing requirement; in fact, it may be difficult for the observer to determine whether or not the image is correctly focused in the presence of such prominent diffraction effects. A similar situation can exist in the case of high-magnification optical microscopy if the significant object detail becomes comparable to an optical wave-length.

4. Phase Stability Requirements and Effects of Movement of the Acoustic Medium

In any holographic process the relative phases of the acoustic signal and the real or synthetic reference wave must remain constant to within a fraction of a cycle during the recording process. For cases in which all of the waves of interest pass through essentially the same medium, this condition may not be difficult to meet; however, for objects of considerable extent compared to the scale of turbulence in the medium, phase shifts due to motion and thermal inhomogeneities of the medium can be significant. Also, in the case of systems using electronically synthesized reference waves, the medium acts as a delay line many wave-lengths long, and the hologram phase condition may be difficult to achieve in many practical cases. For example, if the total acoustic path difference is $N\lambda$, then a change in acoustic velocity producing an average shift $\Delta\lambda$ will result in a phase shift of $2\pi N \frac{\Delta\lambda}{\lambda}$.

If the phase tolerance is set arbitrarily at π radians, we have the requirement

$$\frac{\Delta\lambda}{\lambda} < \frac{1}{2N}.$$

Thus the tolerance on acoustic velocity and oscillator drift is set by the above requirement. Thermal variations in acoustic velocities of water and air under ordinary conditions are of the order of 10^{-3} per deg Centigrade. Thus a 1°C average change along the path would set a path length limit of the order of a few hundred wave-lengths.

In water and air under ordinary conditions the phase disturbances due to motion along the acoustic path can produce significant phase shifts also. The velocity of the medium adds to the sonic velocity to give

$$\Delta\Phi = \frac{2\pi d}{\lambda} \left(\frac{v}{c+v} \right)$$

where the phase shift $\Delta\Phi$ depends on the path length d , the acoustic wave-length λ , and the medium velocity v compared to the acoustic velocity c . As an example, if $d = 10^3\lambda$ and $c = 1450$ m/sec, as in water, a phase shift of π would result from a relative velocity of

the medium of 0.7 m/sec. From this it is clear that holography under uncontrolled conditions with large object distances and high resolution (small wave-lengths) may be very difficult. In the case of synthetic reference holography, it may prove necessary to shift the phase of the reference with time to maintain constant phase at one point in the hologram. Such a scheme does not avoid the problems of localized turbulence and temperature variations, however.

5. Conclusion

Thus far, most acoustic holography experiments have been limited to air (Massey, 1967; Metherell *et al.*, 1967) and water (Preston and Kreuzer, 1967; Thurstone, 1967) as the acoustic media, with mechanically scanned linear transducers used to record the sound field on photographic film by means of a modulated optical source. A method of holographic recording which avoids the scanning transducer by reflecting light from a liquid-gas interface (Mueller and Sheridan, 1966) under the influence of acoustic radiation pressure has been demonstrated also. Research for many years in the area of ultrasound image conversion has produced a variety of alternate methods (Kennedy and Muenow, 1967; Berger and Dickens, 1963) which may be useful in acoustic holography. It is hoped that further development of one or several of these approaches will lead to significant applications in the areas of medical diagnosis, underwater imaging, materials inspection, and perhaps to underground mapping of geological features.

See also: Holography.

Bibliography

- BERGER H. and DICKENS R. E. (1963) A review of ultrasonic imaging methods with a selected annotated bibliography, Argonne National Laboratory publication No. ANL-6680.
- KENNEDY JOHN A. and MUENOW RICHARD (1967) Practical improvements and applications for the ultrasonic image converter, *IEEE Transactions on Sonics and Ultrasonics*, SU-14, No. 2, 47-52.
- LEITH E. N. and UPATNIEKS J. (1962) Reconstructed wavefronts and communication theory, *Journal of the Optical Society of America*, 52, No. 10, 1123-1130.
- LEITH E. N. and UPATNIEKS J. (1962) Wavefront reconstruction with continuous-tone objects, *Journal of the Optical Society of America*, 53, No. 12, 1377-1381.
- MASSEY G. A. (1967) Acoustic holography in air with an electronic reference, *Proceedings of the IEEE*, 55, No. 6, 1115-1117.
- METHERELL A. F., EL-SUM H. M. A., DREHER J. J. and LARMORE L. (1967) Introduction to acoustic holography, *Journal of the Acoustical Society of America*, 42, 733.
- MUELLER R. K. and SHERIDON N. K. (1966) Sound holograms and optical reconstruction, *Applied Physics Letters*, 9, No. 9, 328-329.

- PRESTON K. JR. and KREUZER J. L. (1967) Ultrasonic imaging using a synthetic holographic technique, *Applied Physics Letters*, 10, No. 5, 150-152.
- THURSTONE F. L. (1967) Three-dimensional imaging by ultrasonic holography, *Digest of the 7th International Conference of Medical and Biological Engineering* (Stockholm, Sweden), p. 313.

G. A. MASSEY

ACOUSTIC WAVEGUIDES. Simple examples of the acoustic waveguide are the air-filled "speaking tube", allowing verbal communication between different parts of a building or ship, and the child's "mechanical telephone" made of a stretched string linking two cans. In the speaking tube a *fluid*, air, carries the acoustic waves, while the relatively rigid solid walls of the tube guide the waves over their route by preventing appreciable radiation of energy into the surrounds. In the mechanical telephone a *solid*, the material of the string, carries the energy, and again guides the waves, there being little radiation into the surrounding air. In both these waveguides there is a large change in both density and velocity (in "acoustic impedance") at the boundary—between air and tube wall, or between string and air—and this has the consequence that any waves in the guide striking the boundary will be reflected into the guide again with only a small loss of energy to the surrounds. As a consequence of this acoustic signals may be guided over appreciable distances. Another example, at much higher frequencies (perhaps 1-10 MHz), is provided by certain types of ultrasonic delay line used in radar and computers. These are used to delay a signal for times of the order of 100 μ sec, achievable with relative ease using the travel time of an ultrasonic signal, but difficult to achieve electrically because of the much higher velocity of electromagnetic waves.

In discussing the theory of acoustic waveguides it is usual at first to idealize the boundary conditions, assuming that no energy is transmitted across the boundary. In practice this condition is difficult to achieve completely at lower acoustic frequencies (of 0-100 kHz say) since for no transmission of sound there must be no motion of the tube walls, but the assumption of rigid walls provides a reasonable starting point for first analysis. At ultrasonic frequencies of 1 MHz and above, however, as commonly used in ultrasonic delay lines, it is much simpler to obtain boundary conditions closely approximating to the ideal. A wave guided along a metal wire or strip may be used; ideal boundary conditions would be achieved if the metal were situated in vacuum, but in practice the presence of air has usually a negligible effect. The boundary is here free to move but there is zero stress, in contrast to the fluid waveguide with zero motion but finite pressure. A third possible idealized boundary condition is sometimes found with a liquid in a tube at ultrasonic frequencies; here the boundary of the liquid sometimes behaves as if

it were free to move, with zero pressure. The reasons are not fully understood, but in certain instances it is believed to be associated with very short waves propagating in a liquid which does not wet the walls of its container; some early delay lines using mercury in steel containers appeared to behave in this way. There are three cases of ideal boundaries to be considered, then, (i) fluid with boundaries not free to move (zero displacement), (ii) fluid with boundaries completely free to move (zero pressure), (iii) solid with boundaries free to move (zero stress). The fourth possibility, solid with boundaries not free to move, is rarely, if ever, met with, and will not be discussed.

Before treating these waveguides in more detail it is appropriate to point out that guided waves may also occur in systems with boundaries far removed from the ideal forms mentioned. For example, waves may be guided over appreciable distances through the air or through the sea in a layer whose boundaries are formed by gradual changes in density or temperature; ill-defined but nevertheless effective. Other extremely complex waveguide systems may be found in seismology, where guided-wave propagation may be found in structures composed of layers of different solids and liquids.

It is also important to point out that there is little clear distinction between the *waveguide* and the *resonator*. If some degree of reflection of waves occurs at the two ends of a waveguide, the multiple reflections may reinforce to produce marked resonance at certain frequencies. This is, of course, the principle used in all musical instruments, whether they use the simple resonances of waves travelling in air-filled tubes (organs, woodwind, brass) or the more complex resonances of flexural waves travelling on stretched strings, skins, or metal and wood objects of various shapes (violin, keyboard, percussion families). An example at much higher frequencies is the quartz crystal used in filters and frequency control in electronic equipment, making use of the resonances of ultrasonic waves in a quartz plate at frequencies as high as 100 MHz. Many other examples of resonators are found in the mechanical systems studied in vibration analysis, for example the resonant cantilever. Although studies of such resonators frequently commence by analysis of the standing wave system and do not specifically deal in terms of guided travelling waves, it can sometimes be useful to a physical understanding if the problem is thought of in terms of the reflections of guided waves.

We now describe some of the important properties of guided waves in the three idealized systems previously listed.

Fluid Waveguides with Rigid Boundaries

Many of the features of acoustic waveguides are illustrated by this idealized system—approximated to by the air-filled tube. The wave in a fluid is "compressional" or "longitudinal" in nature, transmitted

through fluctuations in pressure and in particle velocity (the velocity with which small portions of the fluid move as they are displaced from the position they occupy in the absence of the wave). The pressure at the boundary may take any value, but the particle displacement and particle velocity normal to the boundary must be zero since the boundary cannot move. Analysis shows that many modes of propagation are possible. The most commonly analysed waveguides have circular or rectangular cross-section; for both these curves of the general form illustrated by Fig. 1 are obtained. These display the variation of the phase and group velocities for each mode with the frequency of the sinusoidal wave assumed to be propagating.

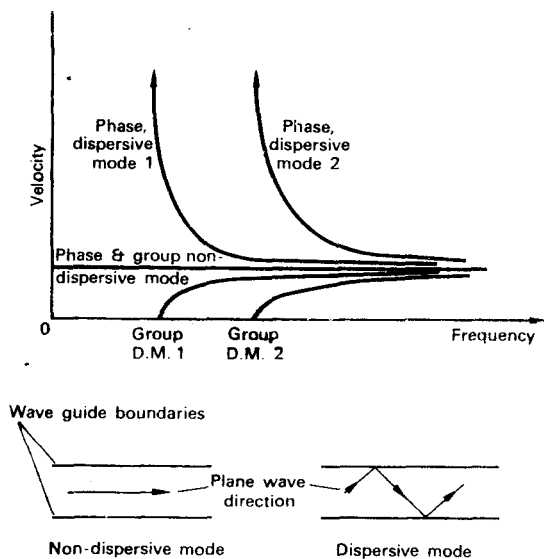


Fig. 1. Waveguide modes in (a) fluid with rigid boundaries, (b) fluid with free boundaries (non dispersive mode absent), (c) shear waves in a solid plate, polarized with motion parallel to plate surfaces, (d) torsional modes in a solid cylinder.

The simplest and most important mode is the "non-dispersive" mode whose phase and group velocities are at all frequencies equal to the velocity of a compressional wave in an unbounded fluid. This mode is made up of a plane wave travelling directly along the waveguide (as indicated by the arrow in Fig. 1); pressure, particle velocity and displacement are all uniform over the cross-section of the waveguide.

In each of the "dispersive" modes (of which the first two are shown, but there are an infinite number), it is possible to regard the mode as a set of plane

waves travelling a zig-zag path by successive reflections between the boundaries. The angle of the zig-zag depends on the frequency and on the mode. In the first dispersive mode propagation is only possible if the wave-length is less than about twice the cross-sectional dimension (a more precise statement involves considering the shape of the cross-section). At lower frequencies than this "cut-off" frequency the mode is "evanescent" and waves will not propagate freely, but are attenuated, usually rapidly. At the cut-off frequency the angle of the zig-zag path becomes such that the wave strikes the boundaries at normal incidence and makes no progress; the group velocity (which is the velocity of energy transfer along the waveguide) falls to zero; while the phase velocity (which is the velocity of intersection of the wavefront and boundary) rises to infinity. At high frequencies and short wave-lengths, the angle of the zig-zag path becomes such that the waves travel almost directly along the guide, striking the boundaries at near-grazing incidence; here the group and phase velocities both tend asymptotically to the velocity of waves in an unbounded material. The set of plane waves travelling a zig-zag path combine to produce the cross-sectional variation in pressure, particle velocity and displacement typical of the mode; in this first mode in a system of rectangular geometry, one half-cycle of a sinusoid between boundaries. (Or a Bessel function in a cylindrical system.)

Higher dispersive modes travel a more zig-zag path and show higher cut-off frequencies, corresponding in a system with very wide parallel boundaries to the cross-sectional dimension being an integral number of half-wave-lengths. The variation in pressure, etc., over the cross-section also possesses an integral number of half-cycles of a sinusoid; the higher the mode, the larger this integral number. In other ways their characteristics are similar to those of the first dispersive mode.

The non-dispersive mode of this system is of considerable importance since in it waves of different frequencies travel in the same way, and there is no distortion of a signal of wide frequency spectrum. It is this mode that is used in the speaking tube, since at voice frequencies all other modes are well below cut-off. At much higher frequencies it might be possible to excite also the higher modes, and a signal travelling in any one of these might suffer considerable distortion since its velocity depends on frequency. In addition if several modes were simultaneously excited, even with a single frequency interference between modes would be possible since the velocity in each mode is different.

The extent to which each mode is excited depends on the configuration of the source of waves. Uniform vibration over the whole cross-sectional area should produce only the non-dispersive mode, while another pattern might produce several modes simultaneously; their relative amplitudes can be deduced using Fourier analysis.

Fluid Waveguides with Free Boundaries

If the fluid has free boundaries, with zero pressure, a new feature appears; the waveguide will not sustain a non-dispersive mode. Dispersive modes are again possible, and their general form is much the same as in Fig. 1, but the mode with constant phase and group velocity disappears. Again the dispersive modes may be thought of in terms of reflecting plane waves, and again sinusoidal distributions of pressure, particle velocity, and displacement are found. In such a waveguide, however, unattenuated propagation is not possible at all below a certain minimum frequency—in marked contrast to the fluid with rigid boundaries—and even above this frequency only dispersive propagation is possible.

The characteristics of these modes are somewhat similar to those found in the propagation of electromagnetic waves in hollow electrical waveguides of rectangular or circular cross-section, where again no non-dispersive mode is possible.

Solid Waveguides with Free Boundaries

Reflection of a plane wave at the free boundary of a solid is often considerably more complex than reflection at the boundary of a fluid. The fluid can carry only compressional waves, but the solid will sustain both compressional and shear waves, travelling with different velocities (the shear wave is slower) and with markedly different characteristics. The most important difference is that in the shear wave the particle motion is at right angles ("transverse") to the direction of propagation, and the "polarization" of the motion of the wave with respect to the boundary must therefore be taken into account. Reflection is only simple with a shear wave whose particle motion is parallel to the boundary; this wave is reflected without loss of amplitude, as in the reflection of a compressional wave at a fluid boundary. With any other polarization the shear wave may produce both another (weaker) shear wave and a compressional wave; though for some angles of incidence the latter degenerates into an evanescent disturbance and does not (in theory) withdraw energy from the incident shear wave. In the reflection of a compressional wave both a weaker compressional wave and a shear wave are always produced. Further, in the presence of a free boundary a Rayleigh surface wave may exist; this has a velocity slightly less than that of plane shear waves in an unbounded medium, and amplitude falling exponentially with distance from the surface.

All these factors combine to produce waveguide modes which may be far more complex than those found in fluids, with two important exceptions, which are treated first.

Simple shear and torsional modes

If a shear wave with particle motion parallel to the waveguide boundaries is launched, relatively simple waveguide modes might be expected. There are two

practical cases of importance. (i) In the first propagation takes place in a flat strip of width much greater than thickness; here it is often possible to ignore the effects of the small side boundaries and obtain modes in which the particle displacement is indeed parallel to the major boundaries. (ii) In the second propagation of a torsional mode in a cylinder of circular cross-section leads to simple modes; these are analogous to (i) because in torsion the particle motion may again be parallel to the circular boundary. In both cases a set of modes very similar to those of Fig. 1 is obtained, including the important non-dispersive mode. This mode is made use of in ultrasonic delay lines employing (i) shear wave propagation in thin, relatively wide, strips of metal or (ii) torsional waves on wires.

Higher, dispersive, modes again exist, and may be launched from sources of certain geometries. Unlike the air-filled tube, the ultrasonic delay line often operates at frequencies when many modes may be above their cut-off frequencies; the aim then is usually to design a source whose vibration produces only the required mode.

"Longitudinal" and "flexural" modes

If the shear motion is not parallel to the boundary of the waveguide, or if a compressional wave is launched, another set of modes may be excited. The phase-velocity/frequency characteristics of these are shown in Fig. 2; curves of this type would be found in cylinders of circular cross-section and in plates whose width is much greater than thickness, but the situation with rods of rectangular cross-section is more complicated and is not treated here.

Two modes exist at low frequencies and are therefore of prime importance, though there are many additional modes showing cut-off frequencies. One of these low-frequency modes is the well-known "flexural" wave possible in a plate or in a cylinder. Its particle displacement is antisymmetrical with respect to its central plane; its overall "snaking" motion in both plate and cylinder is well known. Its phase velocity rises from zero for very long waves towards the velocity of surface waves at short wavelengths. The other low-frequency mode is given various names: "extensional", "longitudinal", "compressional", or "Young's Modulus", the latter because in the cylinder the low frequency velocity tends to $(\text{Young's Modulus} \div \text{Density})^{1/2}$. With wavelengths much greater than the cross-sectional dimensions the whole cross-section moves with almost equal amplitude and the wave propagates with a "concertina" motion, but with short waves a marked exaggeration of motion at the surface is observed and the velocity falls towards the Rayleigh surface wave velocity. The motion characterizing this mode is symmetrical about the central plane of the plate or axis of the cylinder.

In both sets an infinite number of higher modes, exhibiting cut-off frequencies, are possible. It should be particularly noted that all modes, including the