

BASIC MATHEMATICS

SKILLS AND STRUCTURE

HALDI

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Preface

There will always be students who need a basic mathematics course. Some of these students are weak in basic skills while others require a review of the fundamentals and a chance to develop their self-confidence. An instructor is thereby challenged with a wide range of student abilities and expectations.

I have developed and refined the material in this text to meet these special needs of basic mathematics students. The material introduces and reviews many useful topics of mathematics, yet requires very little mathematical background. The material in this text has been used in both standard lecture classes and an individualized learning laboratory. The result is a text that is adaptable to either approach.

My concern has been to develop a practical text that includes whole numbers, fractions, decimals, and percentages. Arithmetic skills are emphasized, many patterns and developmental (discovery) exercises show the “whys” of mathematics, and the generalization to algebra is made. Chapters on integers, basic algebra, and square roots cover the fundamental concepts of algebra. Additional chapters on geometry and numerical trigonometry provide for the needs of students enrolled in the wide variety of career fields.

I have included problems on the metric system in some of the early chapters, but care has been taken to ensure that like units have been used so that no conversions are necessary. This is intended to show that no problem exists as long as the same units are used in any system of measure. A more extensive treatment of the metric system occurs in Chapter 6.

Each chapter begins with a list of objectives for and a brief introduction to the chapter. The sections are then explained using examples and occasionally a discovery approach with “developmental exercises” to help you discover mathematical properties. The answers to these developmental exercises follow immediately so that you do not try to discover properties based on incorrect answers. The material is then reinforced with practice problems in the exercises. These exercises include both numerical and word problems, which include a wide variety of real-life situations. The chapter concludes with a summary that includes a list of terms, symbols, properties, and rules introduced in the chapter and a review exercise.

The text contains the answers to all odd-numbered problems. Selected odd-numbered answers immediately follow each exercise with the remaining odd-numbered answers preceding each chapter summary. All odd-numbered answers to the summary review exercise immediately follow the exercise.

The text also includes a glossary, which gives brief definitions for most of the terms listed in the chapter summaries, and a perforated answer strip for those who wish to use it. The answer strip provides space only for answers to even-numbered problems. When answers require number lines, figures, or T-bar proofs, you must use a separate piece of paper.

Most students working in an individualized learning laboratory will start at the beginning of each chapter and do a reasonable number of the problems in each exercise. If you feel you understand this material, skip the regular sections and go directly to the chapter summary. This allows you to progress at your own rate, passing over previously learned material to

proceed to other topics. Use the answers to the odd-numbered problems to verify that you are working the problems correctly. If you make mistakes and have an instructor available, be sure to seek help.

If you want or need more work on some of the topics within the chapters, ask your instructor for the “chapter supplements.” The material included in these supplements builds upon the material in the text, but is not necessary to successfully complete future chapters. The chapter supplements offer additional work on rules of divisibility, prime numbers, factorization, integers, signed numbers, manipulation of fractions and decimals, number bases, ratios, proportions, percentages, conversion between the English and metric systems of measurement, geometric figures, the square root algorithm, the Pythagorean theorem, the square root, and trigonometry tables. The supplement section on the metric system is particularly helpful since the United States, the only major nation still using the English system, is gradually changing over to the metric system. You should know how to translate English measurements into metric system measurements.

I wish to thank the students who have been in my classes and the many instructors who have offered constructive suggestions to help make this book possible. Many helpful suggestions came from David Russel, Calvin Lidstone, Richard Heitman, Stanley Bjorklund, Thomas L. Alexander of University College of the University of Alabama in Birmingham, Donald R. Burleson of Middlesex Community College, John F. Keating of Massasoit Community College, Harvey A. Leboff of Massachusetts Bay Community College, Vincent J. Motto of Monroe Community College, and the staff of Houghton Mifflin Company. Last, but not least, I would like to thank my wife and family for their patience and encouragement throughout this project.

J.F.H.

What About the Pocket Calculator

I have used the following general policy regarding an electronic calculator in my classes:

1. A calculator may be used in conjunction with many problems in the text. In Chapters 1-4, however, use the calculator only to verify manually calculated solutions.
2. By using a calculator in the sections following Chapter 4, you will have time to solve many more practical application problems and gain confidence in the use of a calculator.
3. Do not use a calculator during a quiz or test in Chapters 1-4.

The following paragraphs give brief guidelines for using a calculator as an instructional aid for each chapter in the text.

If you have access to a calculator, use it *to check* your daily work in Chapters 1, 2, and 4. In Chapter 3, use the calculator to help determine common denominators and to obtain equivalent fractions. You can learn a great deal about your calculator as you check your manual calculations in these four chapters. Since these chapters consist of basic skills, I feel that the quizzes and tests should be taken *without* the use of a calculator.

Chapter 5 does not readily lend itself to a calculator, but some of the problems dealing with substitution will allow many more types of formulas to be introduced and save you a great deal of time.

Chapter 6 lends itself very well to the use of a calculator. However, you should first be able to show the proportion you are using to solve the problems. Then, use the calculator to carry out the calculations.

If your calculator has a square root key, use the key *only* to check an answer in Chapter 7. The calculator can be used to help completely factor a number and to carry out the calculations involved when interpolating.

Chapter 8 contains many problems for which you should first set up the correct formula and then carry out the calculations using a calculator. However, answers to problems involving π may vary. The answers in the text were calculated using $\pi = 3.14$. If your calculator has π as a built-in constant, it will use 3.141592654 as the value for π . To obtain the same solutions as those given in the text, you must use 3.14 for π , not the built-in constant.

Chapter 9 does not readily lend itself to the use of a calculator. In Chapter 10 you may use a calculator to perform the necessary calculations — provided you first set up the problems with the correct formula. If your calculator has trigonometric ratios, take advantage of their presence. Although most calculators with the trigonometric ratios have only the sine, cosine, and tangent keys, you should come to understand the reciprocal relationships between the sine and cosecant, the cosine and secant, and the tangent and cotangent. The answers in the text were calculated using the four-place values in Table 10.4 rather than the ten-place values that the calculator contains. Therefore, your answers will be slightly different if the calculator values are used rather than the values in Table 10.4.

A student taking this mathematics course should obtain as much knowledge as possible, and this includes learning how to use an electronic calculator. Your instructor should allow

some flexibility in determining a reliance on the calculator at any particular place in the course. The extent of this reliance must be consistent with the objectives that the student has for enrolling in the course and the objectives of the instructor.

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Chapter 1

Whole Numbers

Students should develop the following skills while studying this chapter.

1. Perform the operations of addition and multiplication on whole numbers.
2. Learn the meaning and use of the symbols \times , \cdot , and $()$ to indicate multiplication.
3. Use parentheses for grouping numbers and know the special properties of 1 and 0.
4. Recognize the terms sum, factor, product, and term.
5. Learn to use the symbols in the set $\{<, =, >, \neq, \approx\}$.
6. Perform the operation of subtraction on whole numbers.
7. Use the borrowing process and understand the concept of place value.
8. Recognize the terms minuend, subtrahend, and difference.
9. Perform the operation of division on whole numbers.
10. Recognize the terms dividend, divisor, quotient, and remainder.
11. Learn to recognize *by sight* if a natural number is divisible by 2, 3, 5, or 10.
12. Recognize and use the terms prime numbers and completely factored form.
13. Recognize and use the terms closure, commutative, associative, identity, and distributive.

Your knowledge of addition, subtraction, multiplication, and division of whole numbers will help determine your “success” as a consumer, citizen, and student. Pocket calculators and published tables can save you a great deal of time computing many everyday calculations. However, you need to know how to apply the basic operations of arithmetic before you can effectively use a pocket calculator and published tables.

This chapter reviews the basic concepts and vocabulary of addition, subtraction, multiplication, and division as they are used with whole numbers. It provides a firm base from which you can study the concepts developed in the chapters that follow.

1.1

Addition and Multiplication of Whole Numbers

The *natural numbers* are the numbers we use when we count. Hence, they are sometimes referred to as the *counting numbers*. This set of numbers gives the well-known pattern:

$$\{1, 2, 3, 4, \dots\}$$

The three dots indicate that the process can be carried on endlessly. When we include the number 0 with the counting numbers we get the set of *whole numbers*. The set of whole numbers forms the well-known pattern:

$$\{0, 1, 2, 3, 4, \dots\}$$

and we will use this set throughout this chapter.

The operation we call addition allows us to combine two whole numbers to arrive at another whole number. We indicate this operation in various ways.

Example 1.1

Horizontally

$$15 + 36 = 51$$

Vertically

$$\begin{array}{r} 15 \\ + 36 \\ \hline 51 \end{array}$$

We can read the statements in example 1.1 as “fifteen plus thirty-six equals fifty-one” or “fifteen plus thirty-six is fifty-one.” The numbers that we add are called *terms*, and the result (answer) of the addition is called the *sum*.

Multiplication can also be shown both horizontally and vertically.

Example 1.2

Horizontally

$$4 \times 6 = 24$$

Vertically

$$\begin{array}{r} 6 \\ \times 4 \\ \hline 24 \end{array}$$

We can read the statements in example 1.2 as “four times six equals twenty-four” or “four times six is twenty-four.” The numbers we multiply are called *factors*, and the result (answer) of the multiplication is called the *product*.

Several symbols can indicate multiplication. The \times used in example 1.2 ($4 \times 6 = 24$) is acceptable. A raised dot between the two numbers ($4 \cdot 6 = 24$) accomplishes the same purpose. Another alternative is to place either one or both of the numbers within parentheses and write the symbols without spaces between them.

Example 1.3

$$(4)6 = 24, 4(6) = 24, \text{ or } (4)(6) = 24$$

When we add or multiply several numbers, we work with only two numbers at a

time. Addition and multiplication are *binary* (*bi* in *binary* means “two”) operations because they combine two numbers at a time to give one answer. The addition in the statement

$$2 + 5 + 4$$

can be performed by adding the first two numbers to obtain 7 and then adding that 7 to the third number to obtain 11.

Example 1.4

$$\begin{aligned} 2 + 5 + 4 &= (2 + 5) + 4 \\ &= 7 + 4 \\ &= 11 \end{aligned}$$

The addition can also be done by adding the last two numbers (instead of the first two numbers) to obtain 9 and then adding that 9 to the first number to obtain 11.

Example 1.5

$$\begin{aligned} 2 + 5 + 4 &= 2 + (5 + 4) \\ &= 2 + 9 \\ &= 11 \end{aligned}$$

We use parentheses to indicate the order in which we want the addition performed. In example 1.4 the *parentheses* tell us to add the 2 and the 5 first and then to add that answer to the 4 to obtain $7 + 4$, which is 11. The parentheses in example 1.5 tell us to add the 5 and the 4 first and then to add that answer to the 2 to obtain $2 + 9$, which is 11. Likewise, parentheses are used with multiplication to indicate the order in which we want the multiplication done.

Example 1.6

$$\begin{aligned} 2 \cdot 5 \cdot 4 &= 2 \cdot (5 \cdot 4) \\ &= 2 \cdot 20 \\ &= 40 \end{aligned}$$

The parentheses tell us to multiply the 5 and the 4 to obtain 20. Then 20 is multiplied by 2 to obtain 40. The answer would be the same if the numbers were grouped as they are in example 1.7.

Example 1.7

$$\begin{aligned} 2 \cdot 5 \cdot 4 &= (2 \cdot 5) \cdot 4 \\ &= 10 \cdot 4 \\ &= 40 \end{aligned}$$

Vertical representation of addition is especially convenient when adding several large numbers. However, vertical representation does not help us when we have several large numbers to multiply.

4 Whole Numbers

Example 1.8

Add

$$\begin{array}{r} 27 \\ 16 \\ \underline{72} \\ 115 \end{array}$$

Example 1.9

Multiply

$$\begin{array}{r} 27 \\ 16 \\ \underline{72} \end{array}$$

We are told to multiply in example 1.9. In this case vertical representation does not allow us to find the product as easily as we could find the sum in example 1.8. The horizontal method shown in example 1.10 is more helpful.

Example 1.10

$$\begin{aligned} 27 \cdot 16 \cdot 72 &= (27 \cdot 16) \cdot 72 \\ &= 432 \cdot 72 \\ &= 31,104 \end{aligned}$$

$$\begin{array}{r} 27 \quad 432 \\ 16 \quad 72 \\ \underline{162} \quad 864 \\ 27 \quad 3024 \\ \underline{432} \quad 31104 \end{array}$$

Before you proceed to exercise 1.1 you must be sure of the addition facts given in Table 1.1 and of the multiplication facts given in Table 1.2.

The shaded area in Table 1.1 shows the addition $6 + 3 = 9$. You find the 6 in the left-hand vertical column and the 3 in the top row. The sum of 6 and 3 is located by the heavily shaded area, which is in the 6 row and the 3 column.

Table 1.1 Addition Table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Table 1.2 shows the multiplication $6 \cdot 9 = 54$. You find the 6 in the left-hand vertical column and the 9 in the top row. The product of 6 and 9 is located by the heavily shaded area, which is in the 6 row and the 9 column.

Table 1.2 *Multiplication Table*

×	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Name _____

Section _____

Date _____

EXERCISE 1.1

2. a.

b.

c.

d.

4. a.

b.

c.

d.

EXERCISE 1.1

1. Add:

a.
$$\begin{array}{r} 82 \\ 49 \\ \hline \end{array}$$

b. $75 + 33$

c.
$$\begin{array}{r} 901 \\ 382 \\ 674 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 1483 \\ 8905 \\ 655 \\ \hline \end{array}$$

2. Add:

a.
$$\begin{array}{r} 74 \\ 22 \\ \hline \end{array}$$

b. $69 + 40$

c.
$$\begin{array}{r} 426 \\ 719 \\ 783 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 5270 \\ 819 \\ 9070 \\ \hline \end{array}$$

3. Multiply:

a.
$$\begin{array}{r} 74 \\ 7 \\ \hline \end{array}$$

b. $(68)(43)$

c. $97 \cdot 403$

d. $65 \cdot 82 \cdot 105$

4. Multiply:

a.
$$\begin{array}{r} 65 \\ 9 \\ \hline \end{array}$$

b. $(44)(50)$

c. $81 \cdot 682$

d. $39 \cdot 84 \cdot 95$

5. Perform the addition problems.

a. $15 + 36$

b. $36 + 15$

c. $302 + 249$

d. $249 + 302$

Name

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EXERCISE 1.1

6. a.

b.

c.

d.

8. a.

b.

c.

d.

10. a.

b.

c.

d.

12. a.

b.

c.

d.

6. Perform the addition problems.

a. $47 + 28$

b. $28 + 47$

c. $709 + 153$

d. $153 + 709$

7. Perform the multiplication problems.

a. $47 \cdot 28$

b. $28 \cdot 47$

c. $709 \cdot 153$

d. $153 \cdot 709$

8. Perform the multiplication problems.

a. $15 \cdot 36$

b. $36 \cdot 15$

c. $302 \cdot 249$

d. $249 \cdot 302$

9. Perform the addition problems.

a. $(91 + 9) + 3$

b. $91 + (9 + 3)$

c. $(7 + 45) + 5$

d. $7 + (45 + 5)$

10. Perform the addition problems.

a. $(2 + 18) + 13$

b. $2 + (18 + 13)$

c. $(21 + 39) + 14$

d. $21 + (39 + 14)$

11. Perform the multiplication problems.

a. $(2 \cdot 5)7$

b. $2(5 \cdot 7)$

c. $(4 \cdot 7)12$

d. $4(7 \cdot 12)$

12. Perform the multiplication problems.

a. $(17 \cdot 5)2$

b. $17(5 \cdot 2)$

c. $(4 \cdot 25)13$

d. $4(25 \cdot 13)$

13. Perform the indicated operations.

a. $12 + 0$

b. $12 \cdot 1$

c. $0 + 419$

d. $1 \cdot 419$

e. $43 + 0$

f. $43 \cdot 1$

g. $0 + 1234$

h. $1 \cdot 1234$

14. Perform the indicated operations.

a. $32 + 0$

b. $32 \cdot 1$

c. $525 + 0$

d. $525 \cdot 1$

e. $35 + 0$

f. $35 \cdot 1$

g. $0 + 2405$

h. $2405 \cdot 1$

15. Perform the indicated operations.

a. $4(6) + 4(4)$

b. $4(6 + 4)$

c. $13(17) + 13(13)$

d. $13(17 + 13)$

16. Perform the indicated operations.

a. $23(431) + 23(569)$

b. $23(431 + 569)$

c. $17(35) + 17(65)$

d. $17(35 + 65)$

17. A typical house contains the following light bulbs: twenty 60 watt, three 15 watt, eight 40 watt, seven 150 watt, and four 100 watt. How many watts are consumed when Carol Smith turns on all the lights?

18. Find the total length of brake lining ordered by a mechanic if he ordered the following amounts.

<i>Number ordered</i>	<i>Length (inches)</i>
1	8
2	9
1	10
3	11
2	12
4	15
3	16

19. When Steven S. Tinker borrowed \$3000 to finance his automobile, his payments were to be \$99 per month for 36 months. How much did he have to repay?

20. A contractor hires 5 carpenters at \$8 per hour, 2 electricians at \$10 per hour, and 1 plumber at \$11 per hour. What is this contractor's payroll for one week if each carpenter worked 40 hours, each electrician worked 38 hours, and the plumber worked 35 hours?

Name

Section

Date

EXERCISE 1.1

14. a.

b.

c.

d.

e.

f.

g.

h.

16. a.

b.

c.

d.

18.

20.

ANSWERS 1.1 Selected Odd-numbered Problems

Throughout this text the answers to selected odd-numbered problems will appear after each exercise. The remaining odd-numbered answers will appear immediately before each chapter review.