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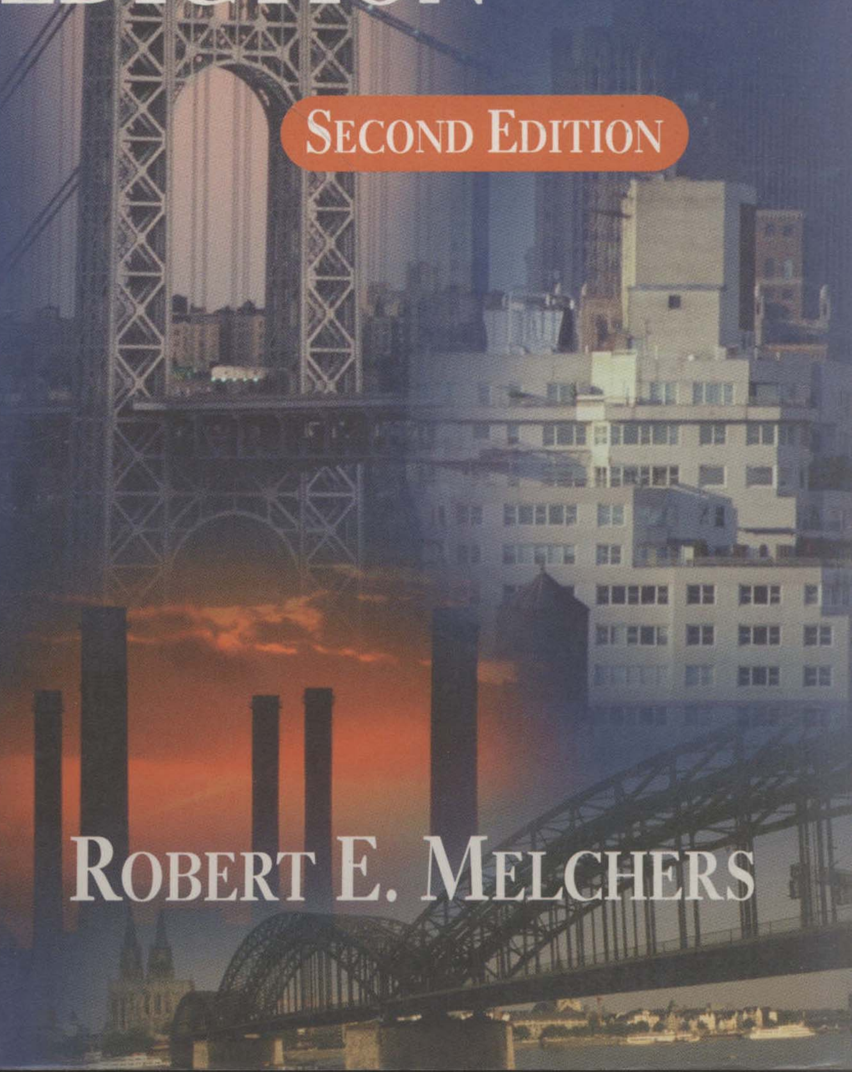

SECOND EDITION

ROBERT E. MELCHERS

Structural Reliability Analysis and Prediction

Second Edition

Robert E. Melchers
The University of Newcastle, Australia

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Structural Reliability Analysis and Prediction

Second Edition

Preface

It is over ten years since the first edition of this book appeared and more than 12 years since the text was written. At the time structural reliability as a discipline was evolving rapidly but was also approaching a degree of maturity. Perhaps it is not surprising, then, that rather little of the first edition now seems out-dated.

This edition differs from the first mainly in matters of detail. The overall layout has been retained but all of the original text has been reviewed. Many sections have been partly rewritten to make them clearer and more complete and many, often small but annoying, errors and mistakes have been corrected. Hopefully not too many new ones have crept in. Many new references have been added and older, now less relevant, ones deleted. This is particularly the case in referring to applications, in which area there has been much progress.

The most significant changes in this edition include the up-dating of the sections dealing with Monte Carlo simulation, the addition of the Nataf transformation in the discussion of FOSM/FORM methods, some comments about asymptotic methods, additional discussion of structural systems subject to multiple loads and a new chapter devoted to the safety checking of existing structures, an area of increasing importance.

Other areas in which there have been rapid developments, such as simulation of random processes and random fields, applications in structural dynamics and fatigue and specialist refinements of theory are all of interest but beyond the scope of an introductory book. Readers might care to refer to the specialist literature, proceedings of conferences such as the ICASP, ICOSSAR and IFIP series and to journals such as Structural Safety, Probabilistic Engineering Mechanics and the Journals of Engineering Mechanics and Structural Engineering of ASCE. Overviews of various aspects of applications of structural reliability are given also in Progress in Structural Engineering and Mechanics. There are, of course, other places to look, but these should form a good starting point for keeping in touch with theoretical developments and applications.

In preparing this edition I had the good furtune to have at hand a range of comments, notes and advice. I am particularly indebted to my immediate colleagues Mark Stewart and Dimitry Val for their critical comments and their assistance with some of the new sections. Former research students have also contributed and I mention in this regard particularly H.Y. Chan, M. Moarefzadeh and X.L. Guan. Naturally, I owe a very significant debt to the international structural reliability community in general and to some key people in particular, including Ove Ditlevsen, Rudiger Rackwitz, Armen Der Kiureghian and Bruce Ellingwood—they, and many others, will know that I appreciate their forebearance and friendship.

The encouragement and generous comments from many sources is deeply appreciated. It has contributed to making the hard slog of revision a little less painful. Sometimes a beach run or a surf seemed a better alternative to spending an hour or so making more corrections to the text... As before, the forebearance of my family is deeply appreciated. Like many academic households they have learnt that academics are their own worst enemies and need occasionally to be dragged away from their Macintoshes to more socially acceptable activities.

Bar Beach, Newcastle Rob Melchers
August, 1998

PREFACE TO THE FIRST EDITION

The aim of this book is to present a unified view of the techniques and theory for the analysis and prediction of the reliability of structures using probability theory. By reliability, in this context, will be understood not just reliability against extreme events such as structural collapse or facture, but against the violation of any structural engineering requirements which the structure is expected to satisfy.

In practice, two classes of problems may arise. In the first, the reliability of an existing structure at the 'present time' is required to be asssessed. In the second, and much more difficult class, the likely reliability of some future, or as yet uncompleted, structure must be predicted. One common example of such a requirement is in structual design codes, which are essentially instruments for the prediction of structural safety and serviceability supported by previous experience and expert opinion. Another example is the reliability assessment of major structures such as large towers, offshore platforms and industrial or nuclear plants for which structural design codes are either not available or not wholly acceptable. In this situation, the prediction of safety both in absolute terms and in terms of its interrelation to project economics is becoming increasingly important. This class of assessment relies on the (usually reasonable but potentially dangerous) assumption that past experience can be extrapolated into the future.

It might be evident from these remarks that the analysis (and prediction) of structural reliability is rather different from the types of analysis normally performed in structural engineering. Concern is less with details of stress calculations, or member behaviour, but rather with the uncertainties in such behaviour and how this interacts with uncertainties in loading and in material strength. Because such uncertainties cannot be directly observed for any one particular structure, there is a much greater level of abstraction and conceptualization in reliability analysis than is conventionally the case for structural analysis or design. Modelling is not only concerned with the proper and appropriate representation of the physics of any structural engineering problem, but also with the need to obtain realistic, sufficiently simple and workable models or representations of both the loads and the material strengths, and also their respective uncertainties. How such modelling might be done and how such models can be used to analyse or predict structural reliability is the central theme of this book.

In one important sense, however, the subject matter has a distinct parallel with conventional structural engineering analysis and its continual refinement; that is, that ultimately concern is with costs. Such costs include not only those of design, construction, supervision and maintenance but also the possible cost of failure (or loss of serviceability). This theme, although not explicitly pursued throughout the book, is nevertheless a central one, as will become clear in Chapter 2. The assessment or predictions obtained using the methods outlined in this book have direct application in decision-making techniques such as cost-benefit analysis or, more precisely when probability is included, risk-benefit analysis. As will be seen in Chapter 9, one important area of application for the methods presented here is in structural design codes, which, it will be recognized, are essentially particular (if perhaps rather crude and intuitive) forms of risk-benefit methodology.

A number of other recent books have been devoted to the structural reliability theme. This book is distinct from the others in that it has evolved from a short course of lectures for undergraduate students as well as a 30-h graduate course of lectures which the author has given periodically to (mainly) practising structural engineers during the last 8 years. It is also different in that it does not attempt to deal with related topics such as spectral analysis for which excellent introductory texts are already available.

Other features of the present book are its treatment of structural system reliability (Chapter 5) and the discussion of both simulation methods (Chapter 3) and modern second-moment and transformation methods (Chapter 4). Also considered is the important topic of human error and human intervention in the relationship between calculated (or 'nominal') failure probabilities and those observed in populations of real structures (Chapter 2).

The book commences (Chapter 1) by reviewing traditional methods of defining structural safety such as the 'factor of safety', the 'load factor', 'partial factor' formats (i.e. 'limit state design' formats) and the 'return period'. Some consistency aspects of these methods are then presented and their limited use of available data noted, before a simple probabilistic safety measure, the 'safety margin' and the associated failure probability are introduced. This simple one-load one-resistance model is sufficient to introduce the fundamental ideas of structural reliability assessment. Apart from Chapter 2, the rest of the book is concerned with elaborating and illustrating the reliability analysis and prediction theme.

While Chapters 3, 4 and 5 deal with particular calculation techniques for time-independent situations, Chapter 6 is concerned with extending the 'return period' concept introduced in Chapter 1 to more general formulations for time-dependent problems. The three principal methods for introducing time, the time-integrated approach, the discrete time approach and the fully time-dependent approach, are each outlined and examples given. The last approach is considerably more demanding than the other two (classical) methods since it is necessary to introduce elements of stochastic process theory. First-time readers may well decide to skip rather quickly through much of this chapter. Applications to fatigue problems and structural vibrations are briefly discussed from the point of view of probability theory, but again the physics of these problems is outside the scope of the present book.

Modelling of wind and floor loadings is described in Chapter 7 whilst Chapter 8 reviews probability models generally accepted for steel properties. Both load and strength models are then used in Chapter 9. This deals with the theory of structural design codes and code calibration, an important area of application for probabilistic reliability prediction methodology.

It will be assumed throughout that the reader is familiar with modern methods of structural analysis and that he (or she) has a basic background in statistics and probability. Statistical data analysis is well described in existing texts; a summary of probability theory used is given in Appendix A for convenience.

Further, reasonable competence in applied mathematics is assumed since no meaningful discussion of structural reliability theory can be had without it. The level of presentation, however, should not be beyond the grasp of final-year undergraduate students in engineering.

Nevertheless, particularly difficult theoretical sections which might be skipped on a first reading are marked with an asterisk (*).

For teaching purposes, Chapters 1 and 2 could form the basis for a short undergraduate course in structural safety. A graduate course could take up the topics covered in all chapters, with instructors having a bias for second-moment methods skipping over some of the sections in Chapter 3 while those who might wish to concentrate on simulation could spend less time on Chapter 4. For an emphasis on code writing, Chapters 3 and 5 could be deleted and Chapters 4 and 6 cut short.

In all cases it is essential, in the author's view, that the theoretical material be supplemented by examples from experience. One way of achieving this is to discuss particular cases of structural failure in quite some detail, so that students realize that the theory is only one (and perhaps the least important) aspect of structural reliability. Structural reliability assessment is not a substitute for other methods of thinking about safety, nor is it necessarily any better; properly used, however, it has the potential to clarify and expose the issues of importance.

Acknowledgements

This book has been a long time in the making. Throughout I have had the support and encouragement of Noel Murray, who first started me thinking seriously about structural safety, and also of Paul Grundy and Alan Holgate. In more recent times, research students Michael Harrington, Tang Liing Kiong, Mark Stewart and Chan Hon Ying have played an important part.

The first (and now unrecognizable) draft of part of the present book was commenced shortly after I visited the Technical University, Munich, during 1980 as a von Humboldt Fellow. I am deeply indebted to Gerhart Schueller, now of Universitat Innsbruck, for arranging this visit, for his kind hospitality and his encouragement. During this time, and later, I was also able to have fruitful discussions with Rudiger Rackwitz.

Part of the last major revision of the book was written in the period November 1984-May 1985, when I visited the Imperial College of Science and Technology, London, with the support of the Science and Engineering Research Council. Working with Michael Baker was a most stimulating experience. His own book (with Thoft-Christensen) has been a valuable source of reference.

Throughout I have been extremely fortunate in having Mrs. Joy Helm and more recently Mrs. Anna Teneketzis turn my difficult manuscript into legible typescript. Their cheerful co-operation is very much appreciated, as is the efficient manner with which Rob Alexander produced the line drawings.

Finally the forbearance of my family was important, many a writing session being abruptly concluded with a cheerful 'How's Chapter 6 going, Dad?'

Monash University
December 1985

Robert E. Melchers

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