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Geometry Seminar "Luigi Bianchi" II – 1984

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Authors

Mauro Meschiari
Dipartimento di Matematica Pura ed Applicata, Università di Modena
Via G. Campi 213/B, 41100 Modena, Italy

John H. Rawnsley
Mathematics Institute, University of Warwick
Coventry CV4 7AL, England

Simon Salamon
Mathematical Institute, University of Oxford
24–29 St. Giles', Oxford OX1 3LB, England

Editor

Edoardo Vesentini
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CONTENTS

SIMON SALAMON: HARMONIC AND HOLOMORPHIC MAPS

MAURO MESCHIARI

A CLASSIFICATION OF REAL AND COMPLEX FINITE DIMENSIONAL J*-ALGEBRAS

PREFACE

The purpose of this paper is to give a complete classification of real and complex finite dimensional J*-algebras.

First of all we consider real finite dimensional J*-algebras. We characterize twelve infinite families of real matrix J*-algebras and prove that any finite dimensional real J*-algebra is J*-isomorphic to a finite direct sum whose summands are chosen in the above families.

Next, using the above classification devised for real J*-algebras, we obtain a complete classification for finite dimensional complex J*-algebras. To this end we characterize four families of complex matrix J*-algebras and prove that any finite dimensional complex J*-algebra is J*-isomorphic (as a complex J*-algebra) to a finite direct sum whose summands belong to any of the above families.

The classification of finite dimensional complex J*-algebras obtained in this way happens to coincide with the classification given by S.A. Harris [2], in a completely independent way. At the same time we solve, negatively, the problem of the existence of a complex J*-algebra whose associated unitary disc is one of the two exceptional domains in the Cartan classification. This same problem was solved independentely by O. Loss and K. McCrimmon [3].

For convenience we divide this paper in four parts:

Part one. Definitions and elementary properties are exposed. We give some relevant properties of some J*-algebras and prove that any finite dimensional J*-algebra is the direct sum of finite dimensional irreducible ones.

Part two. We give some example of matrix J*-algebras and determine which ones are J*-isomorphic.

Part three. First we classify some particular J*-algebras (J*-algebras of height 2), then we extend this classification to all finite dimensional real J*-algebras.

Part four. We study some properties characterizing complex finite dimensional indecomposable J*-algebras (considered as real J*-algebras).

Then, using the above properties, we sort out four finite families of real matrix J*-algebras which provide all the models for complex finite dimensional indecomposable J*-algebras. Finally we prove that the existence of a real J*-isomorphism between two complex J*-algebras implies the existence of a complex J*-isomorphism.

I thank warmly prof. Edoardo Vesentini whose patience in discussing this paper with me helped me greatly in attaining the solution.

PART 1.

In what follows $\mathbb{K}$ will be either the real number field $\mathbb{R}$ or the complex number field $\mathbb{C}$. Moreover, whenever V is an Hilbert vector space on the field $\mathbb{K}$, let $\langle , \rangle_{\mathbb{K}}$ ($\langle , \rangle$) denote the inner product in V and $| \ |$ the relative norm.

Let V and W be two Hilbert vector spaces on $\mathbb{K}$ and $f : V \to W$ a bounded $\mathbb{K}$-linear operator of V into W. $f^* : W \to V$ will be the $\mathbb{K}$-linear bounded operator defined by

$$\langle f(x), y \rangle_{\mathbb{K}} = \langle x, f^*(y) \rangle_{\mathbb{K}} \text{ , whenever } x \in V \text{ and } y \in W.$$

A J*-algebra over $\mathbb{K}$, or a $\mathbb{K}$-J*-algebra, is a closed $\mathbb{K}$-linear subspace j of the space $L(V, W)$, of all bounded $\mathbb{K}$-linear operators of the $\mathbb{K}$-Hilbert space V into the $\mathbb{K}$-Hilbert space W (endowed with the operator norm), such that

$$aa^*a \in j, \text{ whenever } a \in j.$$

From now on, J*-algebra will always mean finite dimensional J*-algebra, i.e. a J*-algebra of finite dimension as a vector space.

Let V be a complex Hilbert space. V is naturally a real Hilbert space too and the two inner products on V are bounded by the identities:

$$\langle x, y \rangle_{\mathbb{R}} = \frac{1}{2}(\langle x, y \rangle_{\mathbb{C}} + \langle y, x \rangle_{\mathbb{C}}), \quad \text{for all } x, y \in V;$$

$$\langle x, y \rangle_{\mathbb{C}} = \langle x, y \rangle_{\mathbb{R}} + i\langle x, iy \rangle_{\mathbb{R}} , \quad \text{for all } x \in V.$$

Hence, whenever $f : V \to W$ is a bounded linear operator, we have

$$\langle f(x), y \rangle_{\mathbb{R}} = \langle x, f^*(y) \rangle_{\mathbb{R}} \text{ , for all } x \in V \text{ and } y \in W,$$

and any $\mathbb{C}$-J*-algebra is naturally a $\mathbb{R}$-J*-algebra too.

Let j_1 and j_2 be two $\mathbb{K}$-J*-algebras; a bounded $\mathbb{K}$-isomorphism $L : j_1 \to j_2$ is a $\mathbb{K}$-J*-isomorphism if

$$L(aa^*a) = L(a)\, L(a)^*\, L(a), \quad \text{for all} \quad a \in j_1.$$

It is trivial to prove that any $\mathbb{C}$-J*-isomorphism of the $\mathbb{C}$-J*-algebra j_1 onto the $\mathbb{C}$-J*-algebra j_2 is a $\mathbb{R}$-J*-isomorphism of j_1 onto j_2 (considered as $\mathbb{R}$-J*-algebras) too.

In the following we shall omit the prefixes $\mathbb{K}$-, $\mathbb{R}$- and $\mathbb{C}$- whenever possible.

Given a,b, and c in any J*-algebra j, then

$$\{abc\} = \frac{1}{6}(ab^*c + cb^*a + ac^*b + bc^*a + ba^*c + ca^*b)$$

is in j.

LEMMA 1. Let j be a finite dimensional J*-algebra of bounded linear operators of the Hilbert space V into the Hilbert space W. For any $a \in j$ there exist an orthonormal basis $\mathcal{B}$ of V and an orthonormal basis $\mathcal{R}$ of W such that for any $u \in \mathcal{B}$, $a(u)$ is proportional to an element of $\mathcal{B}$.

PROOF. Let $I_p(a^*a)$ denote the point spectrum of the bounded endomorphism $a^*a : V \to V$, i.e.

$$I_p(a^*a) = \{ \lambda : \lambda \in \mathbb{K} \text{ such that } a^*a - \lambda 1_V \text{ is not injective} \}.$$

We shall prove that:

i) $I_p(a^*a) \subset \mathbb{R}$;

ii) $V_\lambda = \mathrm{Ker}(a^*a - \lambda 1_V) = \mathrm{Im}(a^*a - \lambda 1_V)^\perp \neq \{0\}$, whenever $\lambda \in I_p(a^*a)$; (if U is a subspace of V, $U^\perp$ is the orthogonal complement of U in V)

iii) V_λ and V_μ are orthogonal ($V_\lambda \perp V_\mu$) whenever $\lambda, \mu \in I_p(a^*a)$ with $\lambda \neq \mu$;

iv) $V = \Sigma_\lambda V_\lambda$ $(\lambda \in I_p(a^*a))$.

Let $\lambda \in I_p(a^*a)$ and $x \in \mathrm{Ker}(a^*a - \lambda 1_V) - \{0\}$; we have

$$\lambda |x|^2 = \langle \lambda x, x\rangle = \langle a^*a(x), x\rangle = \langle x, a^*a(x)\rangle = \langle x, \lambda x\rangle = \bar{\lambda}|x|^2$$

and hence i).

Let $\lambda \in I_p(a^*a)$, $x \in \mathrm{Ker}(a^*a - \lambda 1_V)$, $y \in \mathrm{Im}(a^*a - \lambda 1_V)^\perp$ and $z \in V$. We have

$$0 = \langle (a^*a - \lambda 1_V)(x), z\rangle = \langle x, (a^*a - \lambda 1_V)(z)\rangle$$

and

$$0 = \langle y, (a^*a - \lambda 1_V)(z)\rangle = \langle (a^*a - \lambda 1_V)(y), z\rangle.$$

Then ii) follows trivially.

Let $\lambda, \mu \in I_p(a^*a)$, with $\lambda \neq \mu$, $x \in \mathrm{Ker}(a^*a - \lambda 1_V)$ and $y \in \mathrm{Ker}(a^*a - \mu 1_V)$. We have

$$\lambda \langle x, y\rangle = \langle \lambda x, y\rangle = \langle a^*a(x), y\rangle = \langle x, a^*a(y)\rangle = \langle x, \mu y\rangle = \mu \langle x, y\rangle$$

and hence

$$(\lambda - \mu)\langle x, y\rangle = 0.$$

which proves iii).

Assume now that $M = (\Sigma_\lambda V_\lambda)^\perp \neq \{0\}$ $(\lambda \in I_p(a^*a))$ and let $x \in M - \{0\}$, $p = a^*a - 2|a^*a|1_V$. Clearly $p : V \to V$ has a bounded inverse p^{-1} since the spectral radius of a^*a, $\rho(a^*a)$, is $|a^*a|$. Now let us consider the set $I = \{p^m(x) : m \in \mathbb{Z}\}$. We have

$$\langle p^m(x), y\rangle = \langle x, p^m(y)\rangle = (\lambda - 2|a^*a|)^m \langle x, y\rangle = 0,$$

whenever $\lambda \in I_p(a^*a)$, $y \in V_\lambda$ and $m \in \mathbb{Z}$, and hence $I \subset M$.

Since j is finite dimensional, there exist a natural number n

and n scalar $\lambda_0,..,\lambda_{n-1}$ such that

$$a(a^*a)^n = \lambda_0 a + \lambda_1 a(a^*a) + \ldots + \lambda_{n-1} a(a^*a)^{n-1}$$

and hence

$$(a^*a)^{n+1} = \lambda_0 (a^*a) + \lambda_1 (a^*a)^2 + \ldots + \lambda_{n-1} (a^*a)^n .$$

Therefore we obtain

$$(p + 2|a^*a|1_V)^{n+1} = \lambda_0 (p + 2|a^*a|1_V) + \ldots + \lambda_{n-1} (p + 2|a^*a|1_V)^n$$

and this implies that I generates a finite dimensional invariant (for a^*a) subspace Z of V. Since $I_p(a^*a/_Z)$ is a finite subset of $I_p(a^*a)$ and $Z = \Sigma_\lambda V_\lambda \cap Z\,(\lambda \in I_p(a^*a)/_Z))$ we have the contradiction $y \in \Sigma_\lambda V_\lambda\ (\lambda \in I_p(a^*a))$. Thus iv) is proved.

Now let B_λ denote and arbitrary orthogonal basis of V_λ, $\lambda \in I_p(a^*a)$, and

$$B = \cup_\lambda B_\lambda \qquad (\lambda \in I_p(a^*a)).$$

Whenever $x,y \in B$ with $x \neq y$, we have

$$\langle a(x), a(y)\rangle = \langle x, a^*a(y)\rangle = k\langle x,y\rangle = 0$$

(y is supposed to be an element of V_k). It follows that $a(B)$ is a subset of W whose elements are mutually orthogonal. Then there exists an orthogonal basis of W, R, such that

$$R \supset \{\frac{u}{|u|} : u \in a(R) - \{0\}\}.$$

COROLLARY 1. For any element a of the J*-algebra j, the norm $|a|$ is given by:

$$|a| = \sqrt{\rho(a^*a)} .$$