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Wolfgang Ebeling

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For Bettina, Bastian, and Mirja

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INTRODUCTION

For the analysis of hypersurface singularities and their deformations the Milnor lattice and the invariants associated with it play an important rôle. The middle homology group of a Milnor fibre of the singularity is a free abelian group, which is endowed with a bilinear form, the intersection form. This bilinear form is symmetric or skew-symmetric, if the dimension is even or odd respectively. This group with this additional structure is called the Milnor lattice H. The monodromy group Γ of the singularity is a subgroup of the automorphism group of this lattice. It is generated by reflections, respectively symplectic transvections, corresponding to certain elements of the Milnor lattice, the vanishing cycles, which are defined in a geometric way. These form the set $\Delta \subset H$ of vanishing cycles. The monodromy group is already generated by the respective automorphisms corresponding to the elements of certain geometrically distinguished bases of vanishing cycles. The set of Dynkin diagrams (or intersection diagrams) corresponding to such bases yields another invariant. A survey of the relations between these invariants and their importance for the deformation theory of the singularities in the hypersurface case is given by E. Brieskorn in his expository article [Brieskorn$_3$].

By stabilizing we can restrict ourselves to the symmetric case in the hypersurface situation. For the simple hypersurface singularities the monodromy groups and the sets of vanishing cycles are finite, and it is well-known that they coincide with the Weyl groups and root systems respectively corresponding to the classical Dynkin diagrams [1] of type A_m, D_m, E_6, E_7, or E_8. In the general (symmetric) case there arises the question about the nature of these infinite reflection groups and these infinite subsets of the Milnor lattice and about a characterization of these invariants. The work on this question was stimulated by a result of H. Pinkham [Pinkham$_1$] of the year 1977. Pinkham showed that the monodromy groups of the fourteen exceptional unimodal hypersurface singularities are arithmetic. In the sequel we were able to prove that the result of Pinkham is true for large classes of hypersurface

---

1) The notation "Dynkin diagrams" for these graphs was introduced by N. Bourbaki [Bourbaki$_2$], and is commonly used for these graphs since. Usually one also denotes the more general intersection diagrams of arbitrary hypersurface singularities by this name. We shall also use this notation in this monograph. Unfortunately, this notation does not seem to be historically justified. More accurately, these graphs should be attributed to H.S.M. Coxeter. However, Bourbaki calls the closely related graphs associated with Coxeter systems Coxeter graphs. We follow Bourbaki and reserve this notation for these graphs, too (cf. Remark 4.1.4, Example 5.2.4, and Chapter 5.5).

singularities, using a theorem of M. Kneser. On the other hand it was clear from the beginning that this result could not be true for all hypersurface singularities. One can find exceptions among the other unimodal hypersurface singularities, which appear at the beginning of Arnol'd's classification, namely among the hyperbolic singularities forming the $T_{p,q,r}$-series.

The Milnor lattices, monodromy groups, and vanishing cycles are more generally defined for isolated singularities of complete intersections. Therefore one can also consider the above question in this larger context. The aim of this monograph is to explore these invariants for this general class of singularities, which embraces the hypersurface singularities. This investigation is centred upon the above question.

In order to be able to study these invariants, we first elaborate suitable procedures to compute these invariants. Here we are guided by the hypersurface case. In this case one uses Dynkin diagrams corresponding to geometric bases of vanishing cycles for the calculation of these invariants. However, the notion of a geometric basis of vanishing cycles cannot be readily transferred to the general case. Though one can also consider the corresponding systems of vanishing cycles in the complete intersection case, they are no longer linearly independent, unless we are in the hypersurface case, and one also has to take the linear relations among these cycles into account. We introduce an appropriate notion of a Dynkin diagram for an isolated complete intersection singularity. We present methods to compute such Dynkin diagrams.

We apply these methods to determine Dynkin diagrams and the invariants derived from these graphs for the singularities at the lowest levels of the hierarchy of isolated complete intersection singularities. Using these results, we are in particular able to classify the isolated complete intersection singularities with a definite, parabolic, or hyperbolic intersection form. This extends the corresponding classification of Arnol'd in the hypersurface case [Arnol'd$_1$] to the case of complete intersections. Moreover, one discovers many interesting relations in this way.

As the main result of this monograph we can answer the question about a characterization of the monodromy groups and of the vanishing cycles in the symmetric case completely. We can show that the monodromy groups and vanishing cycles of all even-dimensional isolated complete intersection singularities except the hyperbolic singularities can be described in a purely arithmetic way, i.e. only in terms of the Milnor

lattice. In particular the simple parts of the monodromy groups are arithmetic in these cases. For the hyperbolic singularities one can give another, though non-arithmetic, description of these invariants. These results can also be applied to global monodromy groups. So for example one can prove that the monodromy group of the universal family of projective complete intersections of a fixed (even) dimension and multidegree is arithmetic. The proof of these results is reduced to algebraic results on reflection groups corresponding to vanishing lattices by means of our calculation of Dynkin diagrams.

We now give a survey of the contents of this work in detail. The monograph is divided into five chapters.

In the first chapter we introduce the invariants of isolated singularities of complete intersections to be considered later. Here we alter the point of view compared to the hypersurface case slightly. In the general case we do not regard merely the Milnor lattice, but a short exact sequence of lattices

$$0 \longrightarrow H' \longrightarrow \hat{H} \longrightarrow H \longrightarrow 0,$$

including the Milnor lattice H, as the basic invariant. This sequence is defined as follows (cf. Section 1.1). Let (X,x) be an isolated singularity of an n-dimensional complete intersection, and let $F = (F_1,\ldots,F_p) : (\mathbb{C}^{n+p},0) \longrightarrow (\mathbb{C}^p,0)$ be the semi-universal deformation of (X,x). We choose a line ℓ in the base space $\mathbb{C}^p$ through the origin which intersects the discriminant of F at the origin transversally. Without loss of generality we assume that the coordinates of $\mathbb{C}^p$ are chosen in such a way that this line coincides with the last coordinate axis. Then $F' = (F_1,\ldots,F_{p-1}):(\mathbb{C}^{n+p},0) \longrightarrow (\mathbb{C}^{p-1},0)$ defines an isolated singularity (X',x). Let X_s and X'_t be the Milnor fibres of (X,x) and (X',x) respectively. Then the above sequence is part of the long exact (reduced) homology sequence of the pair (X'_t,X_s). This means that $H = \tilde{H}_n(X_s,\mathbb{Z})$, $H' = H_{n+1}(X'_t,\mathbb{Z})$, and $\hat{H}$ is the relative homology group $H_{n+1}(X'_t,X_s)$. On these modules we consider the bilinear forms induced by the intersection form on H.

For each invariant associated with the Milnor lattice H we can then consider a corresponding relative invariant which is related to the lattice $\hat{H}$. In this way the vanishing cycles are associated with the thimbles which were already considered by Lefschetz (Section 1.2). These form the set $\hat{\Delta}$. To the monodromy group Γ corresponds the relative monodromy group $\hat{\Gamma}$ (Section 1.3). The natural generalization of the notion of a weakly or strongly distinguished basis of vanishing

cycles in the hypersurface case is considered in Section 1.4: This is a weakly or strongly distinguished basis of $\hat{H}$ consisting of thimbles. So the Dynkin diagrams which are introduced in Section 1.5 are Dynkin diagrams corresponding to weakly or strongly distinguished bases of thimbles of $\hat{H}$. As in the hypersurface case they are not uniquely determined, and we also study the possible transformations of these diagrams in this section. At the end of Section 1.5 we show the invariance of the introduced objects. A Dynkin diagram corresponding to a strongly distinguished basis of thimbles determines the remaining relative invariants, in particular also a special element of the relative monodromy group $\hat{\Gamma}$, namely the relative monodromy operator $\hat{c}$. We discuss in Section 1.6 to what extent one can get informations about the module H', and hence about the whole fundamental exact sequence above, from the knowledge of $\hat{H}$ and of the relative monodromy operator $\hat{c}$.

In Chapter 2 we describe our methods for the computation of Dynkin diagrams corresponding to strongly distinguished bases of thimbles. We derive a generalization of a procedure of Gabrielov in the hypersurface case [Gabrielov$_3$] (Section 2.2). Our calculations are essentially based on this method. This procedure allows us to reduce the calculation of Dynkin diagrams to the calculation of Dynkin diagrams for simpler singularities. Here the polar curve of the singularity plays an important rôle. The necessary definitions and facts about polar curves and polar invariants in the case of complete intersections are collected in Section 2.1. The simplest singularities of complete intersections which are not hypersurfaces are the isolated singularities of intersections of two quadrics. For such a singularity H. Hamm has given a basis of the Milnor lattice H [Hamm$_2$]. In Section 2.3 we show that the basis of Hamm consists of vanishing cycles and that these cycles bound the thimbles of a strongly distinguished basis of $\hat{H}$. We compute the Dynkin diagram corresponding to this basis and analyze the invariants of this special singularity by means of suitable transformations of this Dynkin diagram. It turns out that there exists a close relation to K. Saito's theory of extended affine root systems [Saito$_1$], which we discuss in Section 2.4. Finally Section 2.5 deals with another method to compute Dynkin diagrams. This is a generalization of a method of F. Lazzeri in the hypersurface case to determine the intersection matrix using the relations of the fundamental group of the complement of the discriminant. We explain this method by means of an example which is essential for the later applications, but where the application of the procedure of Section 2.2 already leads to considerable difficulties.

In Chapter 3 we apply the method of Chapter 2.2 to calculate Dynkin diagrams for some special singularities. Here we only consider singularities which are given by map-germs $f : (\mathbb{C}^{n+2},0) \to (\mathbb{C}^2,0)$ with $df(0) = 0$ and with regular 2-jet. This means in particular that H' has rank 1 in this case. In Section 1.1 we consider the classification of these singularities of any dimension, and we introduce the classes of singularities which will be considered later. Part of them will play a rôle in Chapter 4. These classes are characterized by the Segre symbol of the 2-jet. At the beginning of the classification one finds after the intersections of two quadrics the n-dimensional singularities $T^n_{2,q,2,s}$ (Segre symbols $\{1,\ldots,1,2\}$ and $\{1,\ldots,1,(1,1)\}$). In Section 3.2 we explain in detail how one can compute Dynkin diagrams for these singularities using the method of Chapter 2.2. It turns out that these singularities are hyperbolic in the even-dimensional case. Section 3.3 is devoted to two other classes of n-dimenional singularities, namely the singularities of the $J^{(n-1)}$- and $K^{(n-1)}$-series (Segre symbols $\{1,\ldots,1,3\}$ and $\{1,\ldots,1,(1,2)\}$ respectively). Here the calculation of Dynkin diagrams follows the pattern of Section 3.2. The remaining sections of Chapter 3 are not needed for Chapter 4. In Section 3.4 we focus our attention on the case of curves and consider in particular Dynkin diagrams for the simple space curve singularities. The surface case is considered more fully in Sections 3.5 and 3.6. Section 3.6 deals in particular with the triangle singularities and the extension of Arnol'd's strange duality observed in [Ebeling-Wall]. For the space curve and surface singularities of Sections 3.4 and 3.6 we obtain Dynkin diagrams which are closely related to the Dynkin diagrams of Gabrielov for the unimodal hypersurface singularities. We study the Coxeter elements of these graphs and show how our results fit together with new results of K. Saito about Coxeter elements of a certain class of graphs. Section 3.6 provides supplementary information on the extension of the strange duality exceeding and completing the paper [Ebeling-Wall] in certain aspects. Finally we mention a particular result of our calculations: One finds topologically non-equivalent singularities, for example already among the triangle singularities, whose monodromy operators are conjugate over $\mathbb{Q}$ (Corollary 3.6.4).

The main results of this work are described in Chapter 4. These results are stated in Section 4.1. We first classify the isolated complete intersection singularities with a definite, parabolic or hyperbolic intersection form (Theorem 4.1.1). Then we give a description of the monodromy groups, of the relative monodromy groups, and of the sets of vanishing cycles and thimbles for almost all isolated singularities

of even-dimensional complete intersections (Theorems 4.1.2 and 4.1.3). The only exceptions for which these characterizations are not true belong to the hyperbolic singularities. In Remark 4.1.4 we show that in this case one can find a description of these invariants in the framework of Kac-Moody-Lie algebras. For the hypersurface case and partially for the case of two-dimensional complete intersections in $\mathbb{C}^4$ these results are already published in [Ebeling$_5$] and generalize earlier results in [Pinkham$_1$], [Ebeling$_1$],[Ebeling$_2$], and [Ebeling$_3$]. At the end of Section 4.1 we quote the most important results corresponding to these results in the odd-dimensional case. Here we refer to [Janssen$_1$] for details. The proof of our central results consists in a reduction to algebraic theorems, which are proven in Chapter 5. For this reduction, which is described in Section 4.2, we need the results of the first three chapters. In Section 4.3 we discuss applications to global monodromy groups and Lefschetz pencils. In this way we return to the context in which the vanishing cycles and thimbles were originally introduced by S. Lefschetz.

In Chapter 5 we have collected the algebraic results on which the proof of the theorems in Chapter 4 is based. Here we consider subgroups Γ_Δ of the group of units of an integral symmetric lattice L which are generated by reflections corresponding to the vectors of a subset $\Delta \subset L$. Here the pair (L,Δ) has to satisfy the following conditions: (i) Δ consists only of minimal vectors of square length 2ε, $\varepsilon \in \{+1,-1\}$ fixed, (ii) Δ generates L, (iii) Δ is a Γ_Δ-orbit, (iv) unless $\operatorname{rk} L = 1$, there exist $\delta_1,\delta_2 \in \Delta$ with $\langle \delta_1,\delta_2 \rangle = 1$. Such a pair is called a vanishing lattice, following a terminology of W.A.M. Janssen and E. Looijenga (Section 5.2). Typical examples of such vanishing lattices are the pairs (H,Δ) and $(\hat{H},\hat{\Delta})$. We show that a vanishing lattice which contains a certain small vanishing sublattice of Witt index 2 (and which is called complete in this case) is already the maximum possible vanishing lattice (Sections 5.3 and 5.4). This means that the subset Δ is maximum, hence contains all minimal vectors v of square length 2ε with $\langle v,L \rangle = \mathbb{Z}$, and Γ_Δ contains all reflections corresponding to minimal vectors of square length 2ε. It then follows from a theorem of M. Kneser [Kneser$_1$] that the elements of Γ_Δ are characterized by the properties that they have spinor norm 1 and act trivially on the quotient $L^\#/j(L)$ of the dual lattice $L^\#$ by the image of the lattice L. In Section 5.5 we show that these statements also hold true for some vanishing lattices defined by Coxeter systems, whereas in general these statements are not true for such vanishing lattices. Up to some supplements, Chapter 5 is largely identical with §§ 1 - 3 and part of § 5 of the paper [Ebeling$_5$].

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This monograph is a translation of the author's "Habilitationsschrift" (Bonn 1986) with some minor modifications and corrections. A summary of the main results of the first four chapters is contained in the author's preprint "Vanishing lattices and monodromy groups of isolated complete intersection singularities" (to appear in Invent. math.).

This work was supported by a research grant of the Deutsche Forschungsgemeinschaft. I wish to express my thanks to this institution for this support. This grant also enabled me to visit the University of North Carolina at Chapel Hill/U.S.A. for two months in spring 1986 and to participate in the Special Year in Singularities and Algebraic Geometry. Part of this work was done during this period. I am especially grateful to the organizers of the Special Year, J. Damon and J. Wahl, for the hospitality and the pleasant working atmosphere.

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Bonn, March 1987

Wolfgang Ebeling

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1. INVARIANTS OF COMPLETE INTERSECTIONS

1.1. The fundamental exact sequence

Let (X,x) be an isolated complete intersection singularity of dimension n. This means that (X,x) is a germ of a complex analytic space of dimension n, which is isomorphic to the fibre $(f^{-1}(0),0)$ of an analytic map-germ

$$f:(\mathbb{C}^{n+k},0) \longrightarrow (\mathbb{C}^k,0),$$

and $x \in X$ is an isolated singular point of X. For $k=1$ (X,x) is an isolated hypersurface singularity.

An isolated complete intersection singularity has a semi-universal deformation

$$F:(\mathbb{C}^{n+p},0) \longrightarrow (\mathbb{C}^p,0)$$

with smooth base space. Let B_ε be an open ball of radius ε around the origin in $\mathbb{C}^{n+p}$. We consider a representative of F of the form

$$F:X = F^{-1}(S) \cap B_\varepsilon \longrightarrow S$$

for a neighborhood S of 0 in $\mathbb{C}^p$. The germ of the set

$$C_F = \{y \in X \mid y \text{ is a critical point of } F\}$$

in 0 is the critical locus of F. The germ of its image $D_F = F(C_F)$ in 0 is the discriminant (locus) of F. The discriminant $(D_F,0)$ is a reduced irreducible hypersurface in $(\mathbb{C}^p,0)$.

For a sufficiently small ε and a sufficiently small neighborhood S of 0 in $\mathbb{C}^p$, the mapping

$$F|_{X-F^{-1}(D_F)} : X - F^{-1}(D_F) \longrightarrow S - D_F$$

is the projection of a differentiable fibre bundle. The typical fibre X_s of this bundle over a base point $s \in S - D_F$ has by [Milnor$_1$], [Hamm$_1$] the homotopy type of a bouquet of μ spheres S^n of real dimension n. The fibre X_s is called the Milnor fibre, the number μ the Milnor number of (X,x). The only non-trivial reduced homology group of X_s is the group $\tilde{H}_n(X_s,\mathbb{Z})$. This is a free $\mathbb{Z}$-module of rank μ.

The intersection number of cycles defines a bilinear form $<,>$ on this module, which is symmetric for n even and skew-symmetric for n odd. We call

$$H = (\tilde{H}_m(X_s,\mathbb{Z}), <,>)$$

the Milnor lattice of (X,x). If the dimension n is even, then one has in addition $<v,v> \in 2\mathbb{Z}$ for all $v \in H$, i.e. the Milnor lattice is an even lattice in this case.

We now consider a construction, which can be traced back to Lê Dũng Tráng (cf. [Lê$_2$], [Looijenga$_3$]). We regard a generic complex line ℓ through the origin $0 \in \mathbb{C}^p$, i.e. a line which is not contained in the tangent cone of the discriminant $(D_F,0)$ in 0. We assume that the coordinates of $\mathbb{C}^p$ are chosen in such a way that this line coincides with the last coordinate axis. This means in particular that this line meets the discriminant only in 0. But this is equivalent to the fact that

$$F' = (F_1,\ldots,F_{p-1}):(\mathbb{C}^{n+p},0) \longrightarrow (\mathbb{C}^{p-1},0)$$

defines an isolated complete intersection singularity of dimension $n+1$.

We now choose the neighbourhood S of the form $S = T \times \bar{\mathbb{D}}$ for a disc

$$\bar{\mathbb{D}} = \{z \in \mathbb{C} \mid |z| \leq \eta\}.$$

Let ε, η, and T be chosen so small and suitable that the following conditions are satisfied:

(i) For the representative

$$F':X' = F'^{-1}(T) \cap B_\varepsilon \longrightarrow T,$$

the following is valid: The mapping

$$F'\Big|_{X'-F'^{-1}(D_{F'})}:X' - F'^{-1}(D_{F'}) \longrightarrow T - D_{F'}$$

is the projection of a differentiable fibre bundle. The typical fibre of this bundle is a Milnor fibre of $(X',0)$.

(ii) There exists a homeomorphism

$$h:\overline{X} = F^{-1}(S) \cap \overline{B}_\varepsilon \longrightarrow \overline{X}' = F'^{-1}(T) \cap \overline{B}_\varepsilon ,$$

such that the following diagram commutes:

$$\begin{array}{ccc} \overline{X} & \xrightarrow{F} & S = T \times \overline{\mathbb{D}} \\ \downarrow h & & \downarrow \pi \\ \overline{X}' & \xrightarrow{F'} & T \end{array} .$$

Here π denotes the projection onto the first factor, and h is the identity on $F^{-1}(T \times \mathbb{D}_1)$ with $\overline{\mathbb{D}}_1 \subset \mathbb{D}$. This is possible by [Looijenga$_3$, Proposition (5.4)].

(iii) $S - D_F$ represents the homotopy type of $S - D_F$ at 0 [Looijenga$_3$, (7.3)].

The restriction of the projection $\pi:S \longrightarrow T$ to the discriminant $D = D_F$ is finite. Let t be a point of T which is not contained in the image of the ramification locus of $\pi|_D$.

We now consider the mapping

$$F|_{X'_t}:X'_t = F'^{-1}(t) \cap B_\varepsilon \longrightarrow \{t\} \times \overline{\mathbb{D}} .$$

This mapping corresponds to the function

$$F_p:X'_t \longrightarrow \overline{\mathbb{D}} .$$

By the choice of t there are exactly m intersection points of $\{t\} \times \overline{\mathbb{D}}$ with the discriminant D, where m is the multiplicity of the discriminant at the origin. We denote these points by $s_1,\ldots,s_m$. A fibre X_{s_i} over such a point has exactly one singularity, and this is an ordinary double point. Off these singular fibres, the mapping $F|_{X'_t} = F_p$ is the projection of a differentiable fibre bundle. We choose $s = (t,\eta) \in \{t\} \times \overline{\mathbb{D}}$ as base point. The fibre X_s over the point s is a Milnor fibre of (X,x). The situation is illustrated in Figure 1.1.1. Here the manifold X'_t appears as a ball, which is, however, in fact only true in the hypersurface case. The manifold X'_t has the homotopy type of a bouquet of μ' spheres S^{n+1}, where μ'

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