THE FAST FOURIER TRANSFORM AND ITS APPLICATIONS

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PREFACE

The fast Fourier transform (FFT) is a widely used signal-processing and analysis concept. Availability of special-purpose hardware in both the commercial and military sectors has led to sophisticated signal-processing systems based on the features of the FFT. The implementation of FFT algorithms on large mainframe computers has made unprecedented solution techniques readily achievable. Personal computers have generated yet a further proliferation of FFT applications. To the student, the professional at home, engineers, computer scientists, and research analysts, the FFT has become an invaluable problem-solving tool.

Popularity of the FFT is evidenced by the wide variety of application areas. In addition to conventional radar, communications, sonar, and speech signal-processing applications, current fields of FFT usage include biomedical engineering, imaging, analysis of stock market data, spectroscopy, metallurgical analysis, nonlinear systems analysis, mechanical analysis, geophysical analysis, simulation, music synthesis, and the determination of weight variation in the production of paper from pulp. Clearly, an applications text cannot address in depth such a breadth of technology. The objective of this book is to provide the foundation from which one can acquire the fundamental knowledge to apply the FFT to problems of interest.

The book is designed to be *user friendly*. We stress a pictorial, intuitive approach supported by mathematics, rather than an elegant exposition that is difficult to read. Every major concept is developed by a three-stage sequential process. First, the concept is introduced by an intuitive graphical development. Second, a nonsophisticated (but theoretically sound) mathematical treatment is developed to support the intuitive arguments. The third stage consists of practical examples designed to review and expand the concept. This three-step procedure, with an emphasis on graphical techniques, gives *meaning* as well as mathematical substance to the basic properties and applications of the FFT. Readers should expect a high efficiency in transferring the development of the text into practical applications.

This book is a sequel to *The Fast Fourier Transform*. The focus of the original volume was on the Fourier transform, the discrete Fourier transform, and the FFT. Only a cursory examination of FFT applications was presented. This text extends the original volume with the incorporation of extensive developments of fundamental FFT applications. *Applications* of the FFT are based on its unique property to rapidly compute the Fourier, inverse Fourier, or Laplace transforms. For this reason, we develop in detail the methods for applying the FFT to transform analysis and interpreting results. We then extend the development and apply the FFT to the computation of convolution and correlation integrals. All developments employ a rich use of graphical techniques and examples to insure clarity of the presentation. We then build on these fundamentals and expand the basic FFT uses to a higher level of application topics. Topical areas include two-dimensional FFT analysis, FFT digital filter design, FFT multichannel band-pass filtering, FFT signal processing, and FFT systems applications.

The text should provide an excellent basis for a senior level or introductory graduate course on digital signal processing. Course outlines emphasizing a thorough examination of the Fourier Transform will find the text particularly appealing. The added applications material allows students to develop the experience necessary to apply the FFT to problems spanning a wide variety of disciplines. Students are expected to have access to a digital computer. The text should serve equally well as a supplementary text for a course with broad systems analysis and signal-processing objectives. The book should also be very attractive as a reference to the practicing signal-processing community because it offers not only a readable introduction to the FFT, but a thorough and unified reference for applying the FFT to any field of interest. Readers should also find that the material provides an excellent self-study text.

The text is divided into five major subject areas:

1. *The Fourier Transform.* In Chapters 2 through 6, we lay the foundation for the entire book. We investigate the Fourier transform, its inversion formula, and its basic properties; graphical explanation of each discussion lends physical insight to the concept. The transform properties of the convolution and correlation integrals are explored in detail. Numerous examples are presented to facilitate understanding. For reference in later chapters, Fourier series and waveform sampling of baseband signals are developed in terms of Fourier transform theory.

2. *The Discrete Fourier Transform.* Chapters 6 and 7 develop the discrete Fourier transform. A graphical presentation develops the discrete transform from the continuous Fourier transform. This graphical presentation is substantiated by a theoretical development. Discrete transform properties are derived. The relationship between the discrete and continuous Fourier transform is explored in detail; numerous waveform classes are considered by illustrative examples. Discrete convolution and correlation are defined and compared with continuous equivalents by illustrative examples.

3. *The Fast Fourier Transform.* In Chapter 8, we develop the FFT algorithm. A simplified explanation of why the FFT is efficient is presented. We follow with the development of a signal flow graph, a graphical procedure for examining the FFT. Based on this flow graph, we describe sufficient generalities to develop a computer flowchart and computer programs. Theoretical developments of the various forms of the FFT are presented.

4. *Basic Applications of the FFT.* Chapters 9 through 11 focus on an investigation of the basic applications of the FFT. Application of the FFT to the computation of discrete and inverse discrete Fourier transforms is presented with emphasis on a graphical examination of resolution and common FFT user mistakes (aliasing, time-domain truncation, noncausal time functions, and periodic functions). FFT data-weighting functions are examined in depth. Laplace transform computation using the FFT is presented with graphical examples. FFT implementation of discrete convolution and correlation is developed by extensive graphical presentations. Computational procedures are carefully defined and a computer program is provided. Two-dimensional Fourier transforms, convolution, and correlation are developed (graphically and by example), as in the one-dimensional case. Application of the FFT to two-dimensional Fourier transform and convolution computation are described and computer programs are provided.

5. *Signal Processing and System FFT Applications.* The design and application of digital filters using the FFT is explored from a practical usage perspective. A novel application of the FFT to multichannel band-pass filtering is developed in a manner from which the reader can readily expand the results.

Because waveform sampling is fundamental to FFT signal-processing applications, band-pass and quadrature waveform sampling is addressed in detail. The philosophy underlying the remaining discussions is to address a range of FFT techniques that are applicable to sonar, seismic, radar, communications, medical, optical, system analysis, and antenna applications. Specific FFT application areas addressed include signal-to-noise enhancement, matched filtering, deconvolution filtering, time-difference-of-arrival measurements, phase interferometry measurements, antenna analysis, system simulation, power spectrum analysis, and array beamforming.

I would like to take this opportunity to express my sincere appreciation to the many individuals who have contributed to the contents of this book. A special note of thanks goes to Dr. Patty Patterson, who contributed significantly in correcting and improving the manuscript. Charlene Rushing and Neil Ishman contributed to the computer subroutines.

To my wife Vangee, I am indebted for her patience and understanding for the many hours I have stolen from her life while preparing the manuscript. To my daughter Cami, I thank you for your efforts, dedication, and enthusiasm towards a *commitment to excellence*; I hope some of your ideals are incorporated in this book.

CONTENTS

1

INTRODUCTION

The fast Fourier transform (FFT) is a fundamental problem-solving tool in the educational, industrial, and military sectors. Since 1965 [1], FFT usage has rapidly expanded and personal computers fuel an explosion of additional FFT applications. **The single focus of this book is the FFT and its applications.**

In this chapter, we survey briefly the broad application areas of the FFT to give the reader a perspective for its seemingly universal appeal. We will establish the FFT as one of the major developments in signal-processing technology. The diverse applications of the FFT follow from the roots of the FFT: the discrete Fourier transform and hence the Fourier transform. Our overview of the Fourier transform and its interpretation with respect to the time and frequency domains is presented.

1.1 THE UBIQUITOUS FFT

Ubiquitous is defined as being everywhere at the same time. The FFT is certainly ubiquitous because of the great variety of *apparent* unrelated fields of application. However, we know that the proliferation of applications across broad and diverse areas is because they are united by a common entity, the Fourier transform. For years only the *elitist* theoretical mathematician was capable of staying abreast of such a broad spectrum of technologies. However, with the FFT, Fourier analysis has been reduced to a readily available and practical procedure that can be applied effectively with-

Applied Mechanics

- structural dynamics
- aircraft wing-flutter suppression
- machinery dynamics diagnostics
- nuclear power plant modeling
- vibration analysis

Sonics and Acoustics

- acoustic imaging
- passive sonar
- ultrasonic transducers
- array processing
- architecture acoustic measurement
- music synthesis

Biomedical Engineering

- diagnosis of airways obstruction
- muscle fatigue monitoring
- assessing heart valve damage.
- tissue structure characterization
- gastric disturbances investigation
- cardiac patients diagnosis
- ECG data compression
- artery dynamics investigation

Numerical Methods

- high-speed interpolation
- conjugate gradient method
- boundary value problems
- Riccati and Dirichlet equations
- Rayleigh's integral
- Wiener-Hopf integral equation
- diffusion equation
- numerical integration
- Karhunen-Loeve transform
- elliptic differential equations

Signal Processing

- matched filters
- deconvolution
- real-time spectral analysis
- cepstrum analysis
- coherence function estimation
- speech synthesis and recognition
- random process generation
- transfer function estimation
- echo/reverbation removal

Instrumentation

- chromatography
- microscopy
- spectroscopy
- x-ray diffraction
- electrochronography

Radar

- cross-section measurement
- moving target indicator
- synthetic aperture
- doppler processor
- pulse compression
- clutter rejection

Electromagnetics

- microstrip line propagation
- conducting bodies scattering

Figure 1.1 Summary of FFT Applications.

- antenna radiation patterns
- dielectric substrate capacitance
- phased-array antenna analysis
- time-domain reflectometry
- waveguide analysis
- network analysis

Communications

- systems analysis
- transmultiplexers
- demodulators
- speech scrambler system
- multichannel filtering
- M-ary signaling
- signal detection
- high-speed digital filters
- voice coding systems
- video bandwidth compression

Miscellaneous

- magnetotellurics
- metallurgy
- electrical power systems
- image restoration
- nonlinear system analysis
- geophysics
- GaAs FET transient response
- integrated circuit modeling
- quality control

Figure 1.1 (cont.)

out sophisticated training or years of experience. The FFT has become a *standard analysis module* because of its usefulness and availability.

The FFT is no longer a textbook novelty. In Fig. 1.1, we show an abbreviated listing of typical application areas of the FFT. Key reference materials in the FFT application fields shown are included in the bibliography. The FFT, once the province of engineers and scientists, has become a technique used in areas ranging from the analysis of stock market trends to the determination of weight variations in the production of paper from pulp. Computer technology evolution, particularly that of the personal computer, has positioned the FFT as a handy and powerful analysis tool whose availability is no longer limited only to the signal-processing specialist. As shown in Fig. 1.1, the application fields of the FFT are extremely diverse. In an age where it is virtually impossible to stay abreast of technology, it is stimulating to find an analysis concept that enables one to approach an unfamiliar field with familiar tools. Certainly, the FFT has become one of the major developments in digital signal-processing technology.

As stated previously, the common bond throughout the varied application of the FFT is the Fourier transform. A key property of the Fourier transform is its ability to allow one to examine a function or waveform from the perspective of both the time and frequency domains. The Fourier transform is the cornerstone of this text.