Supersymmetry and Supergravity '82

Proceedings of the Trieste September 1982 School

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PREFACE

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These are the Proceedings of the 1982 school on supergravity and supersymmetry which was held at the International Centre for Theoretical Physics at Trieste, Italy from 6-15 September.

The lecturers at this school are all leading experts in the world of supersymmetry and supergravity. We would like to thank them for having accepted to lecture and to write these proceedings so soon that they can appear within six months of the school. The field is moving so fast at the present time that for students as well as experts, a regular meeting place where new developments are discussed and explained in a broader context is essential. A school of this kind has become a regular event every one or two years, and the large number of participants clearly shows an ever-growing interest in the fields of supersymmetry and supergravity. This reflects the fact that these subjects have become among the most active and popular in modern theoretical physics.

After the school was concluded, a workshop on supersymmetry and supergravity was held from Thursday, 16 September until Saturday, 18 September. The list of speakers and contributions reads:

F. ENGLERT: Spontaneous Compactification in 11-dimensional Supergravity.
P. WEST: The $N = 4$ Supersymmetric Theory and Coset Space Dimensional Reduction.
C. ORZALESI: Generalized Kaluza-Klein Theories.
B. DE WIT: On $N = 2$ Supergravity Theories.
D. NANOPOULOS: Supercosmology, Grand Unification and Supergravity.
P. TOWNSEND: Spinors and Hurwitz Algebras.
E. BERGSHOEFF: Conformal Supergravity in 10 Dimensions.
J. LUKIERSKI: Composite Gravity and Supergravity.
C. ARAGONE: The 3-Dimensional Topologically Massive Super Yang-Mills.
B. MILEWSKI: Superfield Formulation of $N = 2$ and $N = 4$ Super Yang-Mills Models with Central Charge.
E. SEZGIN: Maximally Extended Supergravity Theory in 7 Dimensions.
M. SOHNIUS: Open Gauge Algebras Revisited.
A. VAN PROEYEN: Matter coupling in $N = 1$ supergravity.
S. FERRARA: Spontaneously Broken Local Supersymmetry.

L. GIRARDELLO; Non-perturbative Aspects of Supersymmetry.
M. GRISARU: Super Higgs Effects in Superspace.
P. NATH: Locally Supersymmetric Grand Unification.
C. SAVOY: Proton Decay in Supersymmetric GUTS.
S. RAJPOOT: Mass Scale of Supersymmetry Breaking.
M. QUIROS: Spontaneously Broken SUSY Guts Free of Fine-Tuned Parameters and Δ B, Δ L Troubles.
K. STELLE: $N = 2$ Superfields and the Finiteness of $N = 4$ Yang-Mills Theory.
N. K. NIELSEN: Zeta Function Regularization of Supergravity.
K. SIBOLD: Renormalization of $N = 1$ SUSY Yang-Mills
O. PIGUET: Non-Renormalization of ABBJ anomaly in SUSY Yang-Mills.
J. DE AZCARRAGA: Super Fields from Quantization of a New Supersymmetric Particle Model.

Our deep gratitude goes out to the local organizer, Professor R. Iengo, and the secretary of the school, Louisa Sossi, for the immense amount of work which made an event of this kind possible. Finally, we thank the Director of the Centre at Trieste, Abdus Salam, for his enthusiastic support.

CONTENTS

INTRODUCTION

A. Salam
Director, International Centre for Theoretical Physics, Trieste

In 1961 R. P. Feynman was asked to summarize the Aix-en Provence Conference on Particle Physics. This is how he started his summary.

> "At each meeting it always seems to me that very little progress is made. Nevertheless, if you look over any reasonable length of time, a few years say, you find a fantastic progress and it is hard to understand how that can happen at the same time that nothing is happening in any one moment (zeno's paradox).
>
> I think that it is something like the way clouds change in the sky – they gradually fade out here and build up there and if you look later it is different. What happens in a meeting is that certain things which were brought up in the last meeting as suggestions come into focus as realities. They drag along with them other things about which a great deal is discussed and which will become realities in focus at the next meeting".

The 1982 Trieste School on Supersymmetry and Supergravity was not a meeting in the sense of Feynman. However, the above quote from him conveys the flavour of the situation in this subject. Thus even though the twin problems of giving a field theoretic formulation of off-shell supergravity as well as the problem of its (and global supersymmetry's) physical relevance still remain with us, there has been a "sharpening of focus" since the 1981 Trieste School. This is reflected in the present Proceedings where (even though, in the choice of the area covered, the pattern of the 1981 School is followed) the treatment of the subject is more extended, more relaxed and more didactic.

To be specific, the topics covered are: an introduction to supersymmetry by M. Sohnius; an introduction to supergravity by P. van Nieuwenhuizen; supergraphs by M. Grisaru, multiplet calculus by B. de Wit, gauge theories by J. Wess; extended supergravity by J. G. Taylor and hidden symmetries in extended supergravity by E. Cremmer. The new ground (relative to 1981) lies in the Kaluza-Klein domain and its compactification discussed by J. Strathdee and M. Duff, with related discussions by H. Nicolai on $N = 8$ supergravity, and a contribution from P. Frè on supergravity in 11 dimensions in the context of Cartan integrable systems. Finally there is the discussion of the crucial subjects of positive energy in supergravity by D. Freedman and of the relevance of supersymmetric grand unified theories to particle physics by S. Raby.

The School was followed by the traditional Workshop where exciting discussions centered on spontaneous compactification and breaking of higher dimensional theories, on $N = 1$ supergravity inducing breakdown of supersymmetry and superHiggs effect, on index theorems giving criteria of supersymmetry breaking, and on the finiteness of $N = 4$ supersymmetry using light cone gauge techniques. In Feynman's phrase these will no doubt become the "realities in focus" at the next School and Workshop schedules for early 1984.

Our deep appreciation for the success of the School and for the spirit which pervaded it, goes to its Directors; Sergio Ferrara, John Taylor and Peter van Nieuwenhuizen.

Abdus Salam
7 February, 1983

Supersymmetry for Beginners

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Abstract:

These lectures are an introduction to the fundamentals of supersymmetric field theories. They do not comprehensively cover the subject; rather they are supposed to provide a starting platform for the beginner who then wants to go on and learn the subject.

1. Generators of supersymmetry and their algebra

A *supersymmetry*, in a wide sense of the word, is a symmetry that relates physical properties of particles which have different spin. Thus the "non-relativistic $SU(6)$" of the 1960's which attempted to combine, for instance, spin - 3/2 resonances and spin - 1/2 baryons in single multiplets of $SU(6)$, was a supersymmetry in this wide sense. Yet longer ago, Wigner's $SU(4)$ related properties of nuclei with different spins. All these earlier attempts at "supersymmetry" had in common that they could not be incorporated into relativistic field theory: the $SU(6)$ is a low-energy symmetry which describes the particle spectrum fairly well, but it does not describe the high-energy dynamics, since it is not relativistically covariant. Every attempt to "boost" the $SU(6)$-theory failed, and these failures finally prompted a series of papers which culminated in a proof by S. Coleman and J. Mandula that under certain well-specified conditions there is no such thing as a supersymmetry which is consistent with relativistic field theory.

In 1973, however, J. Wess and B. Zumino [1] were able to write down a model field theory which was supersymmetric and satisfied all but one of the requirements in the no-go theorem: the generators of the supersymmetry did not generate a Lie group, as had been assumed before, since they did not obey commutator relations but rather *anticommutator relations*. The earliest paper in which such anticommutator relations were suggested was by Gol'fand and Likhtman [2], and the first field theory with such a symmetry (in a non-linear realization and with non-renomalizable coupling) was developed by Volkov and Akulov [3].

In this section, I shall present a framework in which to describe Coleman and Mandula's no-go theorems [4], and I then go on to discuss the additional possibilities that arise if statistics-changing symmetries are admitted [5]. These are the supersymmetries in the narrower modern sense of the word, and we shall see that they necessarily must obey anticommutator relations with each other.

1.1 Generators of symmetries

Let me first introduce the concept of a generator of a symmetry. This is an operator in Hilbert space that replaces one incoming or outgoing multiparticle state with another and furthermore "leaves the physics unchanged". The operator should act additively on direct products of states, and this implies that it can be written as the product of an annihilation operator $a_j(\vec{q})$, which picks a particle out of a state and annihilates it, and a creation operator $a_i^+(\vec{p})$, which then creates another particle in its stead with different properties (denoted by the index) and different 3-momentum. The most general operator which is bilinear in a and a^+ is

$$G = \sum_{ij} \int d^3p\, d^3q \quad a_i^+(\vec{p}) \quad K_{ij}(\vec{p},\vec{q}) \quad a_j(\vec{q}) \quad , \tag{1}$$

which is determined completely by the integral kernel $K_{ij}(\vec{p},\vec{q})$, a c-number function of the momenta $\vec{p}$ and $\vec{q}$ and the particle specifications i and j. Using the symbol $*$ for the convolutions in (1), we can write G in the suggestive form

$$G = a^+ * K * a \quad . \tag{2}$$

The operator G will replace some incoming quantum state $| in \rangle$ by another, $G | in \rangle$. We call G a *generator of a symmetry* if, in addition, it commutes with the S-matrix, i.e. if it does not matter whether we "reshuffle" the state before or after an interaction has taken place:

$$S\ G\ | in \rangle = G\ S\ | in \rangle \tag{3}$$

or

$$[S, G] = 0 \quad . \tag{4}$$

This is a formal expression for what above I have called "leaving the physics unchanged".

The sums in (1), which in (2) are included in the * symbol, run over all particle quantum numbers and thus also over all spin values. There are terms which replace bosons by bosons, and others which replace fermions by fermions, bosons by fermions and fermions by bosons. Any G can be decomposed into an *even* and an *odd part*,

$$G = B + F \quad , \tag{5}$$

where the even part B is defined to always replace bosons by bosons and fermions by fermions, while the odd part F replaces bosons by fermions and fermions by bosons. Symbolically, we can write

$$\begin{aligned} B &= b^{+} * K_{bb} * b + f^{+} * K_{ff} * f \\ F &= f^{+} * K_{fb} * b + b^{+} * K_{bf} * f \quad , \end{aligned} \tag{6}$$

where b annihilates an integer spin particle and f a half-odd spin particle*). The symbols * now of course imply summations only over the appropriate spin values. We shall continue to use a as a generic name for both b and f.

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*) throughout, we assume validity of the spin-statistics theorem

While B's may change spin only by integer amounts (or not at all), the F's must change the total spin of a state by a half-odd amount, and thus are necessarily supersymmetry generators.

Henceforth we assume that our G's are either even or odd, but use G as a generic name for both B and F.

1.2 Canonical quantisation

We assume that the particle operators obey canonical quantisation rules. These are anticommutator relations for fermionic operators with each other and commutator relations in all other cases:

$$\{ f_i(\vec{p}) , f_j^+(\vec{q}) \} = \delta_{ij} \, \delta^3(\vec{p} - \vec{q})$$
$$\{ f , f \} = \{ f^+ , f^+ \} = 0 \qquad (7)$$

$$[b_i(\vec{p}) , b_j^+(\vec{q})] = \delta_{ij} \, \delta^3(\vec{p} - \vec{q})$$
$$[b , b] = [b^+ , b^+] = 0 \qquad (8)$$

$$[b , f] = [b , f^+] = [b^+ , f] = [b^+ , f^+] = 0 \quad . \qquad (9)$$

These relations can be written more elegantly: we observe that $\delta_{ij} \, \delta^3(\vec{p}-\vec{q})$ is the unit element of the convolution product $*$, and we introduce the "graded" commutator symbol $[\; , \; \}$ which denotes the anticommutator if both operators are fermionic and the commutator in all other cases. The canonical quantisation rules then read

$$[a , a^+ \} = 1 \quad ; \quad [a , a \} = [a^+ , a^+ \} = 0 \quad . \qquad (10)$$

1.3 Algebra of generators

Let us now try to concoct a third symmetry generator G^3 from two known ones G^1 and G^2. If both commute with the S-matrix, then so does their product $G^1 G^2$. This product, however, is not a generator of a symmetry in the sense defined above since it is quadrilinear in particle operators a and a^+. The canonical quantisation rules suggest that we try the commutator $[G^1, G^2]$.

We first do this for two bosonic generators B^1 and B^2. We can use the quantisation rules together with the identities

$$[\,ab\,,\,c\,] = a\,[b,c] + [a,c]\,b = a\,\{b,c\} - \{a,c\}\,b \quad , \tag{11}$$

which hold for any three associative objects abc, and we get after a bit of algebra

$$[\,B^1\,,\,B^2\,] = B^3 \quad , \tag{12}$$

where the kernels of B^3, which define it, are given by

$$\begin{aligned} K^3_{bb} &= K^1_{bb} * K^2_{bb} - K^2_{bb} * K^1_{bb} \\ K^3_{ff} &= K^1_{ff} * K^2_{ff} - K^2_{ff} * K^1_{ff} \quad . \end{aligned} \tag{13}$$

The commutator has turned out to be bilinear in particle operators, and is thus another generator of a symmetry. It was the quantisation rules that got rid of two of the four particle operators originally present in the product.

In a very similar way, we can show that the commutator of an F with a B gives another F:

$$[\,F^1\,,\,B^2\,] = F^3 \tag{14}$$

with

$$\begin{aligned} K^3_{fb} &= K^1_{fb} * K^2_{bb} - K^2_{ff} * K^1_{fb} \\ K^3_{bf} &= K^1_{bf} * K^2_{ff} - K^2_{bb} * K^1_{bf} \quad . \end{aligned} \tag{15}$$

There is, however, no way to decompose a commutator $[\,b^+f\,,\,f^+b\,]$ in such a way that only the proper combinations $\{\;,\;\}$ appear and that therefore the canonical quantisation rules can be used to eliminate two of the four particle operators. Such terms appear in $[F^1,F^2]$, and we conclude that the commutator of two odd generators is not bilinear in particle operators and hence not a symmetry generator.

On the other hand, we find that we can use the identities

$$\{ ab , c \} = a \{b,c\} - [a,c] b = a [b,c] + \{a,c\} b \tag{16}$$

as well as the ones of eq. (11), and then evaluate the *anti*commutator of two F's, which turns out to be the generator of an even symmetry:

$$\{ F^1 , F^2 \} = B^3 \tag{17}$$

with

$$\begin{aligned} K^3_{bb} &= K^1_{bf} * K^2_{fb} + K^2_{bf} * K^1_{fb} \\ K^3_{ff} &= K^1_{fb} * K^2_{bf} + K^2_{fb} * K^1_{bf} \quad . \end{aligned} \tag{18}$$

The B's and the F's thus satisfy the following relations characteristic of a *graded Lie algebra*:

$$\begin{aligned} [B^1 , B^2] &= B^3 \\ [F^1 , B^2] &= F^3 \\ \{ F^1 , F^2 \} &= B^3 \quad . \end{aligned} \tag{19}$$

In this subsection we have seen how the grading of the canonical quantisation rules, i.e. that the fermions obey anticommutation relations, induces a similar grading for the bilinear combinations, the symmetry generators. Those generators which so far we have called "odd" behave like fermionic objects, the even ones like bosonic objects. We can, of course, now summarily write the algebra (19) as

$$[G^1 , G^2 \} = G^3 \quad , \tag{20}$$

if we augment our definition of [, } by defining the grading property of the result to be as in (19).

1.4 Graded Lie algebras

Let us explore some fundamental properties of graded Lie algebras. These are defined by algebraic relations between a number of B_i and a number of F_α:

$$[B_i, B_j] = i\, c_{ij}{}^{k}\, B_k$$
$$[F_\alpha, B_i] = s_{\alpha i}{}^{\beta}\, F_\beta \tag{20}$$
$$\{F_\alpha, F_\beta\} = \gamma_{\alpha\beta}{}^{i}\, B_i \quad .$$

The structure constants $c_{ij}{}^{k}$ and $\gamma_{\alpha\beta}{}^{i}$ have symmetry properties,

$$c_{ij}{}^{k} = -c_{ji}{}^{k} \quad ; \qquad \gamma_{\alpha\beta}{}^{i} = \gamma_{\beta\alpha}{}^{i} \quad , \tag{21}$$

and all structure constants are subject to consistency conditions which follow from the *graded Jacobi identities*

$$\{[G^1, G^2\}, G^3] + \text{graded cyclic} = 0 \quad . \tag{22}$$

The *graded cyclic sum* which appears here is defined just as the cyclic sum,

$$G^1 G^2 G^3 + \text{gr.cycl.} = G^1 G^2 G^3 + G^3 G^1 G^2 + G^2 G^3 G^1 \quad ,$$

except if two of the operators are fermionic and one is bosonic:

$$F^1 F^2 B^3 + \text{gr.cycl.} = F^1 F^2 B^3 + B^3 F^1 F^2 - F^2 B^3 F^1 \quad .$$

That (22) is an identity for any three associative objects G^1, G^2 and G^3 is easily checked.

The requirement that the Jacobi identities be fulfilled turns out to be equivalent to demanding that certain matrices constructed from the structure constants should form a representation of the algebra, the *adjoint representation*. These matrices are

$$B_l = \begin{bmatrix} C_l & 0 \\ 0 & S_l \end{bmatrix} \quad ; \quad F_\alpha = \begin{bmatrix} 0 & \Sigma_\alpha \\ \Gamma_\alpha & 0 \end{bmatrix} \tag{23}$$

with

$$(C_l)_j{}^k = i\, c_{jl}{}^k \quad ; \quad (S_l)_\alpha{}^\beta = s_{\alpha l}{}^\beta$$
$$(\Gamma_\alpha)_\beta{}^l = \gamma_{\beta\alpha}{}^l \quad ; \quad (\Sigma_\alpha)_l{}^\beta = s_{l\alpha}{}^\beta \quad . \tag{24}$$

The requirement that these matrices should form a representation is equivalent to the following four conditions:

(a) the matrices C_l represent the *Lie subalgebra* of the B's (this is the subalgebra's adjoint representation);

(b) the matrices S_l also represent the Lie subalgebra of the B's (this is the representation of the subalgebra under which the F's transform and which need not be irreducible);

(c) the $\gamma_{\alpha\beta}{}^l$ are a numerically invariant tensor under the Lie subalgebra of the B's:

$$(S_l)_\alpha{}^\delta\, \gamma_{\delta\beta}{}^j + (S_l)_\beta{}^\delta\, \gamma_{\alpha\delta}{}^j - \gamma_{\alpha\beta}{}^k\, (C_l)_k{}^j = 0 \quad ; \tag{25}$$

(d) there is a cyclic identity involving s and γ:

$$\gamma_{\alpha\beta}{}^l\, s_{\gamma l}{}^\delta + \text{cyclic } (\alpha\to\beta\to\gamma) = 0 \quad . \tag{26}$$

1.5 The Coleman-Mandula theorem

S. Coleman and J. Mandula [4] have studied the properties of all bosonic generators of symmetries in the mathematical framework of relativistic field theory. The assumptions they made (non-trivial S-matrix, etc.) can hardly be questioned from the point of view of physics, except perhaps for the "strong spectral assumption" that there should be only one zero-mass state, the unique vacuum. That assumption, however, is introduced mainly to avoid infrared problems, and relaxing it - with the hope that infrared problems will somehow take care of themselves - alters their result only in that the conformal group may be admitted as symmetry group in place of the Poincaré group if all one-particle states are massless [5].

The fact that their investigation was limited to Lie groups of symmetries excluded fermionic generators, and thus supersymmetry as we know it, from the beginning, since these do not generate Lie groups. Nevertheless, their results still hold for the bosonic subset of all our generators and, through the conditions of the previous subsection, severely restrict the fermionic subset as well.

They found that any group of bosonic symmetries of the *S*-matrix in relativistic field theory is the *direct product* of the Poincaré group with an internal symmetry group. The latter must furthermore be compact and itself be the direct product of a semisimple group with $U(1)$ factors.

The bosonic generators are thus the four momenta P_μ and the six Lorentz generators $M_{\mu\nu}$, plus a certain number of internal symmetry generators, which I now want to call B_r. The algebra is that of the Poincaré group

$$[P_\mu, M_{\rho\sigma}] = i(\eta_{\mu\rho} P_\sigma - \eta_{\mu\sigma} P_\rho) \tag{27}$$

$$[M_{\mu\nu}, M_{\rho\sigma}] = i(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\mu\sigma} M_{\nu\rho}) ,$$

plus that of the internal symmetry group

$$[B_r, B_s] = i c_{rs}{}^t B_t , \tag{28}$$

and the direct-product structure manifests itself in the vanishing of the commutators

$$[P_\mu, B_r] = [M_{\mu\nu}, B_r] = 0 . \tag{29}$$

In other words, the B_r must be translationally invariant Lorentz scalars.

The Casimir operators of the Poincaré group are the mass-square*) operator $P^2 = P_\mu P^\mu$, and the generalised spin operator $W^2 = W_\mu W^\mu$, where W^μ is the Pauli-Lubanski vector**)

$$W^\mu = -\frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}\,P_\nu\,M_{\rho\sigma} \quad . \tag{30}$$

In the rest frame of a massive state we have $P_\mu = (m,0,0,0)$ and $W^2 = -m^2\vec{M}^2$ with $\vec{M} = (M_{23}, M_{31}, M_{12})$.

These Casimir operators commute with the entire Poincaré group and also with all internal symmetry generators:

$$[\,B_r\,,\,W^2\,] = 0 \tag{31}$$

$$[\,B_r\,,\,P^2\,] = 0 \quad . \tag{33}$$

The first of these equations means that all members of an irreducible multiplet of the internal symmetry group must have the same spin, i.e. that there are *no bosonic generators of supersymmetries*. The second equation says that they all must have the same mass. This latter result is somewhat older and known as *O'Raifeartaigh's theorem* [6].

In the case that all states are massless and have discrete spin, we have $W^\mu = \lambda P^\mu$ (λ half-integer) and $P^2 = W^2 = 0$. Again, no B_r can change the helicity λ since $[\,B_r\,,\,P_\mu\,] = [\,B_r\,,\,W_\mu\,] = 0$, and the above theorem about the absence of bosonic supersymmetries still holds. Let me state the Coleman-Mandula no-go theorem in a positive way:

> *All generators of supersymmetries must be fermionic, i.e. they must change the spin by a half-odd amount and change the statistics of the state.*

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*) my metric is $\eta_{00} = -\eta_{ii} = 1$.

**) the totally antisymmetric tensor is normalised to $\epsilon_{0123} = 1$