EARLY DEVELOPMENTS IN MATHEMATICAL ECONOMICS

REGHINOS D THEOCHARIS

FOREWORD BY
LORD ROBBINS

SECOND EDITION

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First edition 1961
Second edition 1983

Published by
THE MACMILLAN PRESS LTD
London and Basingstoke
Companies and representatives throughout the world

ISBN 0 333 27344 3

Printed in Hong Kong

Preface to the Second Edition

The present edition includes a major addition. This is the section on Cournot and his contribution to the development of mathematical economics. Since the appearance of the first edition, I have been feeling that the book was incomplete. A process in the development of mathematical economics was described which was shown to be gathering momentum and which was bound to lead to a major development but the book just stopped at the threshold of this. This is why a section on Cournot is now included. He marks the end of the era of the 'protohistory' of mathematical economics and the beginning of its new era of rapid developments.

What I have just said does not mean, however, that I would now wish to belittle the importance of the contributions to mathematical economics of the authors before Cournot. For, as Professor G.L.S. Shackle has said in his review in the *Economica* of the first edition of this book, one of its major purposes was, apart from describing the essentials of the early contributions to mathematical economics, to point out:

> their often charming directness and incisive power, their frequently astonishing anticipations of what we think of as the discoveries of our own generation, the tale they so plainly tell, in many cases, of a mind of true genius struggling, alone, in an utterly untrodden country, enjoying, of course, by that very fact the exquisite freedom of the first explorer:
>
> Libera per vacuum posui vestigia princeps

The present work springs from a thesis approved by the University of London in 1958 for a Ph.D. under the title 'Augustin Cournot and his Contribution to Economic Analysis with special reference to the General Theory of Monopoly'.

The work was originally undertaken at the London School of

Economics under the guidance of Lord Robbins and, partly, of Dr G. Morton and Professor M. Peston.

I wish to express my sincere thanks to Dr Morton and Professor Peston for the help so generously given, especially in the mathematical sections. The responsibility for any mistakes is, of course, entirely mine.

My major debt is, however, to Lord Robbins who, with his unequalled knowledge of the literature of economics, supervised the work at all its stages and guided my steps towards discovering sources which would have remained unknown to me. Lord Robbins's keen interest in seeing that the work is published as well as his kindness and generous encouragement will never be forgotten.

There is another debt that I wish to acknowledge towards Lord Robbins. Throughout these years his warm-hearted friendship has been one of the main-stays of my work and life.

I also wish to thank Professor K. Okuguchi, who has done so much to revitalise Cournot's thinking in a dynamic setting, for his keen interest in my work and his appreciation.

Lastly, but not least, I wish to thank my wife; without her devotion, love and help this work and its revision would have never been completed.

Athens School of Economics
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Athens, December 1978 REGHINOS D. THEOCHARIS

Foreword

The history of mathematical economics before Cournot must in some respects be regarded as consisting of antiquarian *curiosa*. For it cannot be claimed that anything that was done during this period has survived in immediately recognisable form to the present day. It was not until Cournot, that great original mind, presented, with the simplicity of genius, the idea of demand as a function of price, that a contribution which has proved to be of permanent value and influence was forthcoming from this quarter.

Nevertheless, all such beginnings must have a profound significance for those who are interested in the struggles of the mind to understand the universe about it: and these early attempts to enlist the most powerful method of logical reasoning in the service of economic thought have an interest and a fascination of their own. Nor should it be imagined that their authors belonged chiefly to an underworld of the neglected and obscure. There were indeed such to whom perhaps for the first time this book does full justice. But many were outstanding in the general history of thought. The names of Bernoulli, Hutcheson, Beccaria, Du Pont de Nemours, for instance, have an importance far transcending their place in this history. And this perhaps is an added justification for this kind of study.

I must confess, however, that when Dr Theocharis, who has since had the great distinction to become the first Finance Minister of the Republic of Cyprus, first told me that he was planning the earlier chapters of his doctor's thesis on Cournot so as to cover this period, I had certain qualms and hesitations, although I did not communicate them to him. I knew my Jevons and Irving Fisher, I had read the copious opening chapters of the *epigoni* of the School of Lausanne, I knew at first hand a few of the authors whose work he proposed to investigate; and I had doubts whether the field would repay further intensive study on this scale. But it is one of the privileges of being a supervisor in a graduate school that if you are at all lucky with your pupils, you learn much more than you teach; and I am sure that all readers of this book will agree that Dr Theocharis has triumphantly proved the baselessness of my initial reserves and has produced in these chapters a monograph of lasting value and interest. Not only has he succeeded in providing for the first time a systematic survey of the literature; he has also made discoveries of authors not previously recognised and compelled the revaluation of contributions already known. His discovery of Joseph Lang and his re-examination of Canard are outstanding cases in point. I am confident that historians of economic thought will join with me in gratitude and admiration for the results of Dr Theocharis' researches.

There is another circumstance connected with this book which I hope Dr Theocharis will not mind my mentioning. These chapters, and indeed the whole thesis of which they are part, were written at the London School of Economics when relations between his country and ours were at their worst. I like to think of them as a testimony to the fact that, in these disturbed times, there is at least one spot on earth where, as in the Temple of Isis and Osiris in the *Magic Flute*, 'Kennt man die Rache nicht', and where the pursuit of knowledge and truth can proceed, as it proceeded in less troubled days, unimpeded by political divisions.

ROBBINS

London School of Economics
10 October 1960

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1 Introduction

The aim of the present work is to examine the early development of mathematical economics. No attempt has so far been made to examine systematically the literature of mathematical economics before 1838. Occasional attempts[1] have been made, however, to evaluate the work of certain of the authors of that period and the interesting fact about these attempts is that they practically all agree that the authors of that period 'stand now as more or less isolated figures, who cannot be said to have contributed to a current of thought because there is no discernible flow'.[2]

We hope to show that, on the contrary, certain currents of thought may be detected, in the sense that not only do we find contributions from people, not in direct connection with one another, who use the same method, but beyond this it will be shown that authors, knowing the work and ideas of their predecessors, have been able to develop further these ideas. So we shall find Ceva leading to Spinelli, Beccaria to Silio, Frisi through Ortes, Ferroni, Venturi and others to Valeriani and Fuoco, Canard to Fuoco, Gioja and Cournot, Kröncke to Thünen, Isnard to Lang. The fact is, however, that having followed the thread up to a point, we suddenly come to an end in most cases. In Italy, especially, mathematical economics flourished during the eighteenth and early nineteenth centuries and it would be very interesting to know what influence these early authors exercised on authors after Cournot, but such a task is outside the scope of this study.

In our treatment we have preferred to divide the authors according to the language they used. We have, however, made some exceptions.

We have preferred to examine the very early contributors to mathematical economics, with the exception of Daniel Bernoulli, separately because they all use the geometric method; they start with certain axioms and the proof of their theorems is made in terms of logic rather than formal mathematical treatment. This is the reason why we have called them 'the logicians'.

The second exception is that we have called the Italian contribution, with the exception of Fuoco who has been considerably influenced by the French, the 'Milanese School'. This has been done in order to stress the importance of the Lombard contribution, with its centre at Milan, to mathematical economics. It is made up of an illustrious conglomeration of men, mostly nobles, as G. Pecchio[3] has observed, who had the widest possible interests from economics to criminology, from poetry to astronomy. As G. Rossi remarks, 'having been educated in a period when mathematical analysis was fashionable and poets wrote poems about science and geometric treatises, they often used eagerly and advantageously reasonings and demonstrations, which, if not always taking the form of, had all the force and character of mathematics'.[4]

NOTES

1. Chiefly (a). A. Montanari, *La matematica applicata all'economia politica* (1892), who deals with Beccaria, Silio, Valeriani and Scialoja. (b) Notes by M. Fasiani to F. Fuoco, *Applicazione dell'algebra all economia politica* (1937). (c) Helmut Reichardt, *A.A. Cournot* (Tübingen, 1954) pp. 67–91. (d) Ross M. Robertson, 'Mathematical Economics before Cournot' in *Journal of Political Economy* (1949) pp. 523–36, who deals with only some of the mathematical economists of the period. (e) G.H. Bousquet, 'Histoire de l' économie mathématique jusqu' à Cournot', *Metro-economica* (1958) pp. 121–35; G.H. Bousquet, 'Le Systéme Mathématique de l' équilibre économique selon Léon Walras et ses origines', *Revue d' Economie Politique* (1963) pp. 948–76.
   A very useful anthology of works in mathematical economics, including early ones, has appeared since the first edition of this Book. This is W.J. Baumol and S.M. Goldfeld, *Precursors in Mathematical Economics: An Anthology* (London School of Economics, 1968).
2. Robertson, op. cit., p. 535; see also Reichardt, op. cit., p. 67
3. G. Pecchio, *Histoire de l'économie politique en Italie* (1830) pp. 395 and 379.
4. G. Rossi, *La matematica applicata alla teoria della ricchezza sociale* (Reggio Emilia, 1889) p. ix.

2 The Logicians

2.1 ARISTOTLE

The earliest author to use mathematics in an economic argument was Aristotle. In the fifth book of the *Nicomachean Ethics*, in his general examination of Justice, Aristotle deals with two kinds of 'particular justice', justice in the distribution of wealth and justice in the various transactions between men.

In the distribution of wealth the shares of two persons must be proportional to the persons themselves. If the persons are not equal, they must not have equal shares. This he illustrates by assuming that A and B are the persons and C and D the things. 'The ratio between the two pairs of terms is the same, because the persons and the things are divided similarly. It will stand then thus, $A : B :: C : D$, and then permutando $A : C :: B :: D$ and then (supposing C and D to represent the things)

$$A + C : B + D :: A : B$$

The distribution in fact consisting in putting together these terms thus: and if they are put together so as to preserve this ratio, the distribution puts them together justly.'[1]

One kind of transaction, the principles of justice of which Aristotle examines, is exchange. If this is to be just there must be what the Pythagoreans called 'Reciprocation'. He visualises an agriculturist and a shoemaker exchanging their products and explains that 'there will be Reciprocation when the terms have been equalised so as to stand in this proportion; Agriculturist: Shoemaker :: wares of Shoemaker: wares of Agriculturist'.[2] 'Let A represent an agriculturist, C food, B a shoemaker, D his wares equalised with A's. 'Then the proportion will be correct, $A : B :: C : D$; now Reciprocation will be practicable, if it were not, *there would have been no dealing*.'[3] The last phrase seems to be of special interest, as Aristotle appears to be no longer interested in merely defining a just exchange but is attempting to lay down some form of equilibrium conditions of exchange. The value of each product is determined by the quality of labour spent upon it[4] and, in equilibrium, quantities of products exchanged must be proportional to the quality of labour which produced them.

Aristotle was much inspired in his treatment of this subject by geometrical concepts. In his presentation of exchange there occurs a passage[5] which is rendered thus by his translator:[6]

> Now the acts of mutual giving in due proportion may be represented by the diameters of a parallelogram, at the four angles of which the parties and their wares are so placed that the side connecting the parties be opposite to that connecting the wares and each party be connected by one side with his own ware (as in Figure 2.1.)

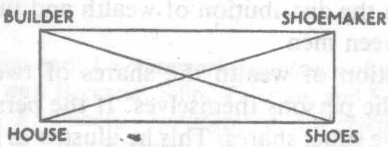

FIG. 2.1

2.2 GIOVANNI CEVA

The period intervening between Aristotle and the early eighteenth century produced no use of mathematics in the economic writings of the scholastics of the Middle Ages or the laical intellectuals.

The first clear attempt after Aristotle to use mathematics in an economic problem was made by Ceva in an Essay published in Mantua in 1711 under the title 'De re numaria, quoad fieri potuit geometrice tractata, ad illustrissimos et excellentissimos dominos praesidem quaestoresque huius arciducalis Caesarei magistratus Mantuae'.[7]

His views on the use of the mathematical method are clear and decided. In the dedication, at the beginning of his book, he points out that commerce is so great and complex that the exploration of its nature cannot be done in any other way except through the use of geometry.[8] This he does in his treatment of money by building up a model through the use of definitions and postulates.

After defining 'monetary material' to include every kind of money circulating in a country, he distinguishes between its intrinsic value (internus valor) which is the quantity of pure metal it contains, and its 'external value', which he defines as the purchasing power of money, adding, however, that it corresponds to the value of the metal plus minting expenses.[9]

Ceva's first postulate is that, other things being equal, the external value of a currency is inversely related to its quantity. The second postulate links changes in the purchasing power of money with changes in the population of a country; other things being equal, the external value of money is directly proportional to the population of a country.[10]

Given these, he proceeds to prove his first theorem that the 'external values' of two currencies are in compound proportion, made up of the direct proportion of their respective populations and inverse proportion of their quantities.

He uses the following illustration which I reproduce.[11]

$$\begin{array}{cccc} I & a & b & c \\ L & a & e & h \\ K & d & e & f \end{array}$$

Ceva assumes that at time I the population of a country is a, the quantity of money b and the external value of money c. At time K the population is d, the quantity of money e and the external value f. To prove his theorem he makes use of an intermediate step, assuming that at time L the population is equal to that at time I, while the quantity of money is equal to that at time K; the external value of money is equal to h.

Now he compares time I with time L. By the first postulate, when other things are equal (and here a is unchanged) value is inversely related to the quantity of money:

$$c : h :: e : b$$

Ceva then proceeds to compare values at times L and K. As the quantities are now the same, by the second postulate, the value of money is directly related to population:

$$h : f :: a : d$$

As, however, $\frac{c}{f} = \frac{c}{h} \times \frac{h}{f}$, we have $\frac{c}{f} = \frac{e}{b} \times \frac{a}{d}$, which demonstrates that the ratio of the two values is equal to a compound ratio made up of the inverse ratio of their respective quantities and the direct ratio of their populations.

Through appropriate postulates, Ceva now proceeds to introduce a situation where gold, silver and copper coins circulate and attempts to prove a second theorem[12] according to which, when the quantity of gold coins circulating remains unchanged, their value is directly related to the quantity of copper money and inversely related to the quantity of silver money. He uses an illustration which we reproduce and argues in the following way:

$$\begin{array}{c} G\\ I\\ H\\ \text{Time}\end{array} \quad \underbrace{\begin{array}{ccc} a & b & c\\ d & b & c\\ d & e & c\\ \text{copper} & \text{silver} & \text{gold}\end{array}}_{\text{coins}}$$

Let at time G the quantity of copper, silver and gold money respectively be a, b and c; and at time H, d, e and c; and further assume that at an intermediate time I, the quantity of copper coins is d, silver coins b and gold coins c. Ceva starts by comparing times G and I where the quantities of gold and silver are the same; as, however, the quantity of copper coins changes, the value of gold coins will change proportionally. An increase in the quantity of copper coins leads to an increase in the value of gold coins, presumably because people trust gold more than copper, although Ceva does not point this out and had not included it in his postulates.[13] He then compares times I and H where, with unchanged quantities of copper and gold coins, the value of gold is inversely proportional to changes in the quantity of silver coins, i.e. $e : b$. An increase therefore of silver leads to a decrease in the value of gold. Gold and silver are put on the same plane.

But[14] $$\frac{\text{value of gold coins at time } G}{\text{value of gold coins at time } H}$$

$$= \frac{\text{value of gold coins at } G}{\text{value of gold coins at } I} \times \frac{\text{value of gold coins at } I}{\text{value of gold coins at } H} = \frac{a}{d} \times \frac{e}{b}$$

In the second part of his essay[15] he solves various problems, where there are coins of the same metal but of different intrinsic value or of different metals, and demonstrates that their intrinsic values are proportional to the product of their respective 'titles'[16] by their respective weights;[17] he also discusses various other cases of minting coins when their weights, minting expenses, titles, etc are given. In all these problems he uses extensively algebraic notation, which he stresses can be translated by anyone into numbers.[18]