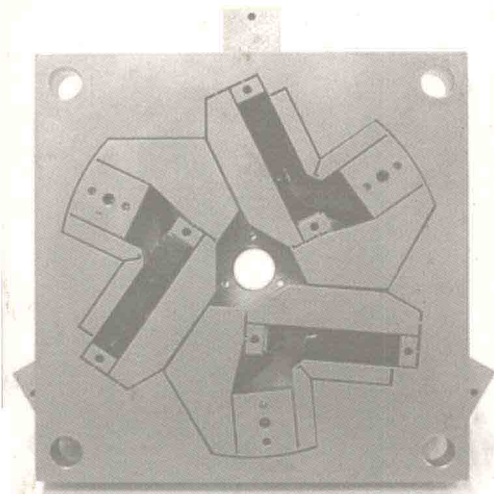
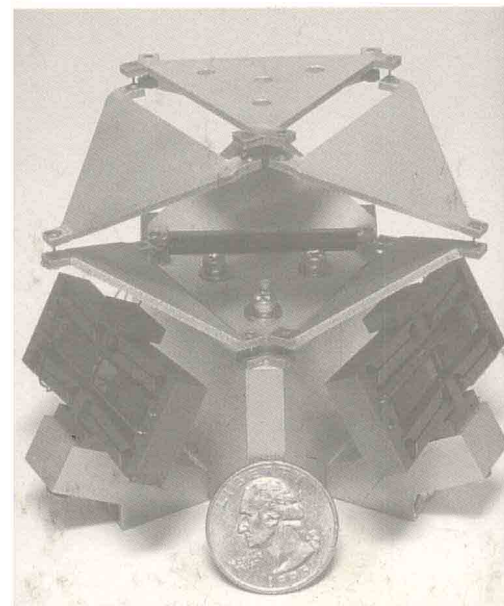


Nicolae Lobontiu



COMPLIANT MECHANISMS

Design of
Flexure Hinges



CRC PRESS

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Nicolae Lobontiu



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COMPLIANT MECHANISMS

**Design of
Flexure Hinges**

Dedication

*To Simona, Diana, and Ioana
for our loving togetherness*

To my parents, Nicolae and Ana

To Dr. Ephrahim Garcia

Preface

This book is dedicated to flexure hinges, which are the main constituents of compliant mechanisms. The flexure hinge, alternatively called flexural pivot, consists of a flexible, slender region between two rigid parts that must undergo limited relative rotation in a mechanism (which will be called *compliant*) due to the presence of at least one flexure hinge. Under the combined action of external loading and actuation, the flexure hinge bends and thus produces the relative rotation between the adjacent members. Being monolithic with the rest of the mechanism for the vast majority of applications, a flexure hinge offers several advantages over classical rotation joints, such as no friction losses, no need for lubrication, no hysteresis, compactness, capacity to be utilized in small-scale applications, ease of fabrication, virtually no assembly, and no required maintenance.

The flexure hinges are incorporated in a large number of applications, both civil and military, including translation micropositioning stages, piezoelectric actuators and motors, high-accuracy alignment devices for optical fibers, missile-control devices, displacement and force amplifiers/deamplifiers, orthotic prostheses, antennas, valves, scanning tunneling microscopes, accelerometers, gyroscopes, high-precision cameras, nanolithography, robotic microdisplacement mechanisms, nanoscale bioengineering, small-scale insect-like walking robots, actuation devices for unmanned micro aerial vehicles, or nano-imprint technology. One of the rapidly growing areas where flexure hinges are massively applied is the microelectromechanical system (MEMS) sector, where the very nature of the microscale structure of such mechanisms, together with their fabrication technology, demands almost exclusive utilization of flexure hinges as connecting joints between quasi-rigid members.

The book is primarily intended for industrial practitioners, researchers, and academics involved in designing and developing flexure-based compliant mechanisms in such areas as mechanical engineering, aerospace engineering, robotics, MEMS, and biomedical engineering. It can also serve as a supplemental text for graduate students in universities where compliant mechanisms are a curriculum subject.

The book originated from the perceived need for an information source dedicated to current problems in the design of flexure hinges and flexure-based compliant mechanisms in both macroscale and MEMS applications in a manner that would resonate with the reality of a multitude of practical engineering cases. Two main directions have been taken in this book. The first targets the design pool of flexure hinges through a systematic approach and the introduction of several new flexure configurations in the hope that the interested designer might opt for a specific flexure solution if presented

with an ample variety of choices. This aspect is all but trivial, as minor changes in the geometry of a flexure hinge might result in substantial modifications at the output port of a compliant mechanism. The second direction of the book addresses the modeling tier by recognizing that in most cases the flexure hinges will operate under small displacements. The reader acquainted with flexure modeling and design would probably agree that, except for the finite-element analysis available through commercially available software (which is predominantly the variant of choice in a large number of applications), the accepted modeling paradigm currently in operation is based on two main hypotheses: (1) the flexure hinges are flexible *constant* cross-section members, and (2) they are subject to *large deformations*. Consequently, the modeling procedure substitutes a flexure hinge with a purely rotational joint equipped with a torsional-spring stiffness. The resulting model of the flexure hinge is subsequently incorporated into a classical rigid-link mechanism model of the specific device being studied, and further static and dynamic calculations are performed according to standard procedures.

The reality is that only in a few occasions are the two fundamental premises mentioned above (constant cross-section and large deformations) concomitantly met. In the vast majority of practical situations, the flexure hinges are actually and deliberately designed to function within a small-displacement environment. The supporting rationale is twofold, as either the application itself requires this type of condition (for instance, in precision mechanisms where the output displacements are inherently small) or the physical dimensions of the flexure hinge do not permit large deformations that would automatically generate stresses over the allowable limits. Moreover, the flexure hinge geometry is seldom of constant cross-section because the fabrication technology currently in use might not allow this particular geometry to be produced in either macroscale or MEMS monolithic applications. The radius of the wire tool in electrodischarge machining, for instance, has a finite, nonzero value and, as a result, the corner of a flexure hinge fabricated by this procedure will always be filleted. Similarly, the design process itself attempts to avoid the type of geometry that would induce undesirable stress concentration in the corner areas.

Previous work supports the approach followed in this book. A very solid paper written by Paros and Weisbord in the 1960s convincingly demonstrated that a circular flexure hinge is a complex spring that not only produces the desired relative rotation between two adjacent rigid links but is also deforming axially and out of plane. The vision expressed by the authors in this fundamental paper is compelling, and present-day flexure-based applications reveal that all deformational facets of a flexure hinge have to be accounted for if an accurate assessment of the performance of a compliant mechanism is to be achieved. The path followed by Paros and Weisbord has been revisited only recently, and the present work is a modest addition to this work.

This book attempts to provide practical answers to the problems of efficiently modeling, analyzing, deciding on, and designing devices that include flexure hinges. It contains many ready-to-use plots and simple equations describing

several flexure types for professionals who need speedy solutions to their current applications. For the researcher who would prefer to find specific answers to a particular design configuration (which is probably not covered here), the book contains self-contained, easy-to-apply mathematical tools that provide guidance for real-time problem solving of further applications.

Several original features are included in this book:

- The book introduces new types of single-axis flexure hinge configurations (e.g., parabolic, hyperbolic, inverse-parabolic, secant) that supplement such classic geometries as circular, corner-filletted, and elliptical and are designed for planar compliant mechanism applications.
- The same configurations are addressed for multiple-axis (revolute) flexure hinges for spatial compliant mechanism applications.
- Newly introduced are the two-axis flexure hinges that are capable of displaying a selective response over two different compliance ranges in spatial applications.
- For all the single-axis flexure hinges, both longitudinally symmetric and nonsymmetric configurations are analyzed, and their corresponding spring rates are explicitly given.
- Short flexures and the associated shearing effects are also modeled for all flexure configurations in terms of their compliance.
- Flexure hinges are studied from a performance-oriented viewpoint in a unitary manner: flexibility, precision of rotation, stress limitations, and energy consumption are factors defined and analyzed by means of closed-form compliance equations.
- Inertia and damping properties are also derived, consistent with the compliance formulation, so that a flexure hinge can be fully characterized and included in the dynamic model of a flexure-based compliant mechanism that can be solved to evaluate its free or forced response. The inertia and damping properties of a flexure hinge are modeled by following either the long (Euler–Bernoulli) or short (Timoshenko) member hypotheses.
- An original finite-element approach is developed whereby the flexure hinges are assimilated to line elements; this approach reduces the problem dimensionality and allows us to perform static and modal/time-history analyses in a simple fashion.
- Also treated are more advanced topics related to flexure hinges, such as shape optimization, buckling, torsion of noncircular variable cross-section members, nonhomogeneous flexures, thermal effects, and large deformations.
- The book includes a list of novel industrial applications, both macro- and microscale (MEMS), for which flexure hinges are instrumental.

The book is formally organized into seven chapters. The first chapter presents the basic characteristics, scope and applications, and advantages and limitations of using flexure hinges and flexure-based compliant mechanisms instead of mechanisms based on classical rotation joints. An argument is developed regarding the importance of flexure hinges in the overall design of compliant mechanisms. Also discussed is the approach taken here of modeling and analyzing the flexure hinges by considering their variable cross-sections and small-displacement conditions.

Chapter 2 develops the generic mathematical model resulting in the compliance closed-form equations for all the flexure types that are presented in this work. The nature of the presentation in this chapter is primarily mathematical and equation based. This chapter gives an alternative to the time-consuming classical finite-element analysis (by using commercially available software) through utilizing an approach of closed-form compliance equations to evaluate the performance of flexure hinges. Specifically, a flexure hinge is characterized by quantifying its rotation capacity, sensitivity to parasitic motions, precision of rotation, level of stress (fatigue failure considerations), and energy consumption. The generic mathematical model previously formulated is applied to be specific to several flexure configurations (the majority are new types). Included are flexure hinges for such two-dimensional applications as constant rectangular cross-section and conic sections (circular, elliptical, parabolic, hyperbolic), as well as inverse parabolic and secant profiles. The validity of the closed-form compliance equations for the various flexure types is checked out by finite-element simulation, experimental measurements, and verifying that the limit case of a constant rectangular cross-section flexure can be retrieved from each individual variable cross-section flexure. The configurations mentioned previously are also considered in deriving closed-form compliance equations for multiple-axis flexure hinges in three-dimensional applications (with rotational symmetry), as well as for two-axis configurations. Conclusions and design recommendations are thus derived regarding the adequacy of employing a specific flexure in a particular application where certain performance functions are required. Graphs, plots, and tables are given for handy selection of flexures in terms of design performance criteria.

Chapter 3 is dedicated to the static modeling and analysis of flexure-based compliant mechanisms (mechanisms that incorporate only rigid links, connected by flexure hinges). A methodology is presented that enables designing serial, parallel, and hybrid (serial/parallel) flexure-based compliant mechanisms for two- and three-dimensional applications. The procedure integrates the various compliance factors into a force-displacement model for the entire mechanism. Output performance qualifiers, such as mechanical advantage, bloc load, stiffness, energy efficiency, and precision of motion, are defined and discussed here.

Chapter 4 studies the dynamic aspects of flexure-based compliant mechanisms. Based on an approach that utilizes Lagrange's equations, inertia and damping properties are derived for the different types of flexure hinges

presented so far to allow formulation of the corresponding lumped-parameter dynamic equations of flexure-based compliant mechanisms. Both the inertia and damping properties are formulated in a manner that is consistent with the modality of deriving the compliance (stiffness) characteristics in such a way that a particular flexure hinge is unitarily represented as having finite degrees of freedom. Modal and time-history analyses are thus possible by integrating the flexure properties into the ensemble of rigid components.

Chapter 5 presents an original finite-element approach to flexure hinges and flexure-based compliant mechanisms, as an alternative modeling and analysis instrument. Instead of treating the flexure hinge with its full two- or three-dimensional geometry details (by using two- or three-dimensional finite elements), which is how the commercially available finite-element software solves these cases, this approach reduces the problem of dimensionality by defining the flexure hinge as a three-node line element. Elemental stiffness and mass matrices are formulated for the various flexure types that were previously introduced. It is thus possible to study the finite-element static and modal response of two- and three-dimensional flexure hinges.

Chapter 6 presents several miscellaneous topics that are important in accurately characterizing the flexure hinges. Addressed are topics that apply to both macroscale and MEMS applications, such as shape optimization, buckling, torsion of noncircular variable cross-section members, flexures fabricated of several materials, thermal effects, and large deformations. Presented also are aspects of actuation, materials, and fabrication procedures for both macro- and MEMS-scale applications.

Chapter 7 presents several classical and up-to-date examples of flexure-based applications and illustrates both macro- and microscale (MEMS) engineering designs.

As previously mentioned, the book is dedicated to flexure hinges that are indispensable construction bricks for miniature compliant mechanisms that are highly energy efficient and are capable of providing very finely tuned output in high-end industrial applications such as those in the photonics, laser optics, or telecommunications industries. Nano-engineering and nano-biomechanical engineering, as parts of the aggressively growing MEMS domain, are areas that benefit from incorporating specifically designed flexure hinges into their miniature compliant devices and mechanisms.

The major reason for writing this book is to offer a core of modeling tools that would enable the designer to play around a bit with different flexure hinge configurations in terms of their spring, inertia, and (possibly) damping characteristics and to make a more informed choice before (inevitably, perhaps) resorting to the classical finite-element software in order to fully solve a flexure-based compliant mechanism problem. Despite all efforts, the book probably is not error free, and I would welcome feedback from the interested reader.

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My special gratitude goes to Dr. Ephraim Garcia of Cornell University who, a few years ago in the cafeteria at Vanderbilt University, sketched a flexure hinge on a piece of paper and thus opened a door for me into this world. I would like to thank Dr. Jeffrey S.N. Paine of Dynamic Structures and Materials (Franklin, TN) who offered me the opportunity of analyzing the subject of flexure hinges from so many perspectives. I would also like to thank Dr. Michael Goldfarb of Vanderbilt University and Dr. Stephen Canfield of Tennessee Technological University for their fruitful interaction over the years. My thoughts and thanks go also to my fellow colleagues in the Strength of Materials Department of the Technical University of Cluj-Napoca, Romania.

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About the Author

Nicolae Lobontiu received his B.Sc. and Ph.D. degrees in mechanical engineering from the Technical University of Cluj-Napoca, Romania, in 1985 and 1996, respectively. From 1985 to 1990, he worked as a mechanical engineer at two engineering companies in Romania and then, until 1997, as an assistant professor and junior associate professor in the Strength of Materials Department within the aforementioned university. He then worked as a postdoctoral associate at Vanderbilt University (Nashville, TN) until 1999, when he joined Dynamic Structures and Materials (Franklin, TN), where he worked as a Research Engineer until 2002. He currently holds a research position at Cornell University (Ithaca, NY) within the Sibley School of Mechanical and Aerospace Engineering. His research interests include modeling and designing flexure hinges and compliant mechanisms, finite/boundary element analysis, and rotordynamics.

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1

Introduction

This introductory chapter gives a brief presentation of flexure hinges and flexure-based compliant mechanisms for macro- and microscale applications by highlighting the main traits defining these mechanical members/devices. An outline of the treated subjects and of the associated approach in this book is also sketched here in order to identify and possibly locate the work in the context of similar dedicated information that has already been published.

A flexure hinge is a thin member that provides the relative rotation between two adjacent rigid members through flexing (bending), as shown in Figure 1.1, where a conventional rotational joint is compared to a flexure hinge. In terms of this rotary function, a flexure hinge can be considered the structural correspondent of a bearing with limited rotation capability, as illustrated in Figure 1.2.

In a classical rotary bearing, the relative rotation takes place between a shaft and its housing, these mating parts being concentrically located, and the rotation can be limited to a specific angular sector, as indicated in Figure 1.2a. A flexure hinge can provide a similar rotary output, the only difference consisting in the fact that the “centers” of the two adjacent members undergoing the relative rotation are no longer collocated, as shown in Figure 1.2b.

Physically, a flexure hinge can be realized in two different ways:

- Use an independently fabricated member (such as a strip or shim in two-dimensional applications or a cylinder-like part in three-dimensional applications) to connect two rigid members that are designed to undergo relative rotation.
- Machine a blank piece of material so that a relatively slender portion is obtained that will be the flexure hinge. In this case, the flexure is integral (or monolithic) with the parts it joins together.

As already mentioned, the flexure hinge consists of an elastically flexible, slender region between two rigid parts that must undergo relative limited rotation in a mechanism (which we will call *compliant* due to the presence of at least one flexure hinge) that is supposed to achieve a specific task. The flexure hinge is monolithic with the rest of the mechanism for the vast