THE QUANTUM
AND ITS INTERPRETATION

THE QUANTUM
AND ITS INTERPRETATION

BY

H. STANLEY ALLEN, M.A., D.Sc.

PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF ST. ANDREWS

WITH THIRTY DIAGRAMS

METHUEN & CO. LTD.
36 ESSEX STREET W.C.
LONDON

PREFACE

MANY books have been written on the quantum theory and on its applications to special branches of physics, the work of Arnold Sommerfeld on *Atomic Structure and Spectral Lines*, in particular, being indispensable to all concerned with these subjects. The present volume is not a treatise on quantum theory, but an attempt to deal with the baffling problem of the nature of the quantum. Let it be said at once that no final solution of this problem has yet been reached; indeed some investigators would maintain that in the last analysis of all physical problems we must rest content merely with mathematical formulation. It is not at present possible to bridge the gap between the undulatory theory of light, which pictures the disturbance as a spreading wave, and the quantum theory of radiation which supposes the energy to be concentrated in bundles or "light units." It is still true, as Sir William Bragg said at the Robert Boyle lecture in 1921, that "we are obliged to use each theory as occasion demands." There seems hope, however, that some type of electromagnetic model, probably subject to suitable restrictions expressing quantum conditions, may serve to correlate the known facts of electricity, magnetism, and radiation. This anticipation receives support from the new undulatory or wave mechanics associated with the name of Schrödinger, of which an account is given in this volume.

It is becoming more and more evident that the electron must be regarded as the seat of a periodic process, and that particles of matter may themselves be spoken of as "waves." While this volume was passing through the press this view has been confirmed by the experiments of Davisson, who examined the reflection of electrons from a crystal, and of G. P. Thomson, who studied the scattering of a beam of electrons in passing through a thin crystalline plate. C. G. Darwin has discussed the theoretical aspects of the electron as a "vector wave."

The quantum theory has revealed an atomicity in nature of a kind previously unsuspected. This aspect of the theory, emphasized by Dr. J. H. Jeans in his first *Report on Radiation and the Quantum Theory* (1914), retains its mystery. But the atomicity of electricity, in its dual aspect of positive and negative electrification, has also to be explained; and it may prove to be the case that these three types of atomicity represent only different aspects of some fundamental property of the electromagnetic field.

In this book emphasis has been laid on the magnetic aspect of the interpretation of the quantum. It is, of course, recognized that a complete theory must take into account both electric and magnetic forces; indeed, in the theory of relativity these become most closely linked together. The genius of Faraday led to the development of a mode of representing electric and magnetic fields by physical lines of force, and these may still serve a useful purpose in forming mental pictures of certain atomic phenomena, even though we may be driven eventually to employ the four-dimensional tubes of electro-magnetic force introduced by Prof. E. T. Whittaker. There is still much for the mathematician to accomplish in treating these "calamoids" as quanta.

The greater part of this work is based upon lectures delivered to advanced students in the University of St. Andrews. This accounts for the mode of presentation, for the somewhat detailed description of fundamental principles, and for occasional repetition. I have endeavoured to take a middle course between a mathematical and a purely descriptive treatment.

I wish to express my thanks to all those who have assisted me, and in particular to Mr. R. S. Maxwell, Mr. W. G. Robson and Dr. Ian Sandeman for help in the preparation of the manuscript or in reading the proofs. For permission to reproduce diagrams, or for the loan of blocks, my thanks are due to the Royal Society of Edinburgh, the Editor of *Nature*, the Editors of the *Philosophical Magazine*, Mr. C. H. Douglas Clark, Sir Alfred Ewing, Mr. H. Moore, Prof. Arnold Sommerfeld, and Drs. Goudsmit and Uhlenbeck. I am indebted to Prof. A. Fowler for tables showing the arrangement of electrons in atoms.

H. S. A.

THE UNIVERSITY,
ST. ANDREWS,
January, 1928

CONTENTS

PART I

FUNDAMENTAL FACTS AND PRINCIPLES

PART II

SPECIAL INVESTIGATIONS

LIST OF DIAGRAMS

* *Reproduced from "The Basis of Modern Atomic Theory" (Douglas Clark), Methuen & Co., Ltd.*

† *Reproduced from "Atomic Structure and Spectral Lines" (Sommerfeld), Methuen & Co., Ltd.*

‡ *Reproduced from "Proceedings of Royal Society of Edinburgh," by permission of the Secretary.*

§ *Reproduced from "Philosophical Magazine," by permission of the Editors.*

PART I

FUNDAMENTAL FACTS AND PRINCIPLES

CHAPTER I

THE QUANTUM THEORY: PRELIMINARY SURVEY

The quantum theory . . . represents a complete departure from the old Newtonian system of mechanics.

We, as physical machines, are built on a scale which is large compared with the scale of light waves and electrons, from which it has resulted that our first physical experiments, as a race, have been concerned with matter also on a scale very large in comparison with its ultimate structure. The Newtonian laws have undoubtedly been found adequate to explain the whole series of what we may call large-scale phenomena, but no adequate reasons have, so far, been given for asserting that they must also be the laws which govern small-scale phenomena. The fact seems to be that the old laws are not, so to speak, fine-grained enough to supply the whole truth with regard to small-scale phenomena.

J. H. Jeans

To be living in a period which faces such a complete reconstruction of our notions as to the way in which aether waves are absorbed and emitted by matter is an inspiring prospect. The atomic and electronic worlds have revealed themselves with beautiful definiteness and wonderful consistency to the eye of the modern physicist, but their relation to the world of aether waves is still to him a profound mystery for which the coming generation has the incomparable opportunity of finding a solution.

R. A. Millikan

1. Origin of the Quantum Theory

THE quantum theory, which has proved of such great importance in the study of Physics in the present century, originated in an attempt to account for the characteristic properties of the heat radiated from a hot body. We shall suppose that in the interior of such a body is a cavity, and that the body is maintained throughout its substance and for an indefinite time at a constant temperature. Inside the cavity (Hohlraum) there will be a continuous stream of radiation in all directions. The problem of radiation is that of determining the way in

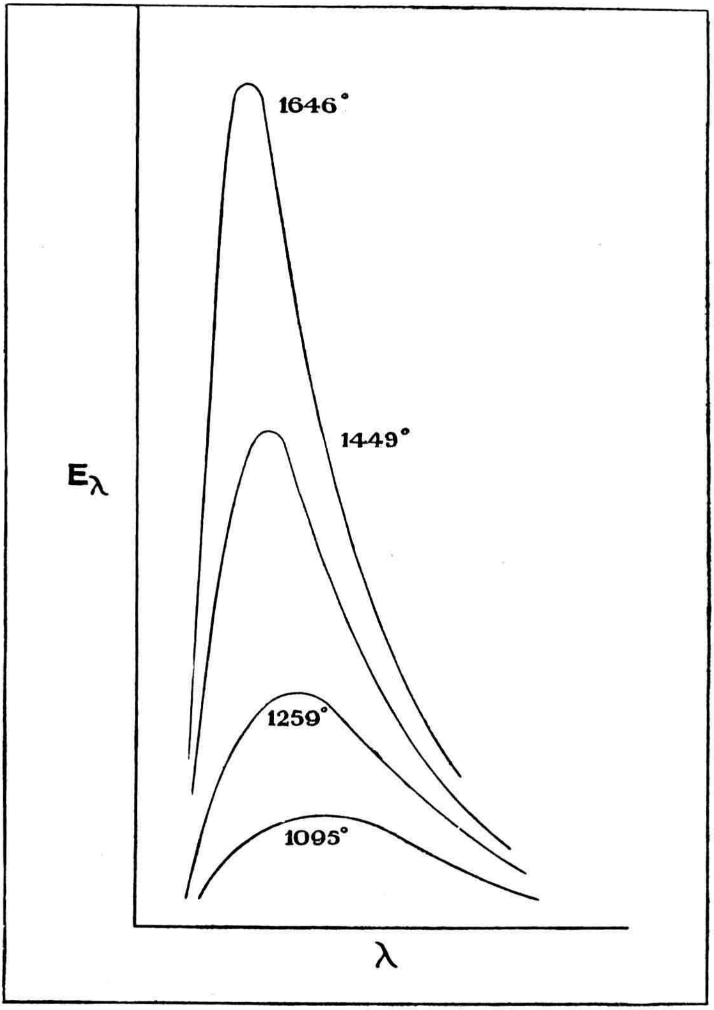

FIG. 1.—Energy and Wave-length of Radiation.
Temperatures are in degrees on the absolute scale.

which the energy is divided between the various wave-lengths or frequencies which go to make up the complete continuous spectrum of the full or "black body" radiation.

On the experimental side the problem has been solved with considerable accuracy by the concordant experiments of a number of independent investigators. By heating the body to incandescence and allowing a beam of the radiation to issue through a *small* opening, the character of the radiation may be examined after it has been analysed in a spectroscope. It is found that there is a particular wave-length, λ, usually in or beyond the red part of the spectrum, corresponding to maximum energy of emission (Fig. 1).

On the theoretical side the problem presented great difficulties. If the laws of classical mechanics are true, there appears to be no escape from the conclusion that the whole of the energy ought to be found at the extreme ultra-violet end of the spectrum, no matter what may be the temperature of the enclosure. It was to meet this difficulty that Dr. Max Planck,* of Berlin, put forward his new theory in a communication to the German Physical Society on December 14, 1900. In this paper "On the distribution of energy in the normal spectrum," he described a new method of obtaining the formula, now known as Planck's formula,† which he had announced a few weeks earlier. In order to secure agreement with the experimental results he was compelled to introduce a new hypothesis which was quite inconsistent with the classical theory. It was not unreasonable to suppose that the hot body contained certain "vibrators" or "resonators" which could emit radiation, a simple harmonic oscillator of frequency ν being concerned in the emission of radiation of corresponding frequency. But the new and strange assumption was that such a vibrator could only possess energies $h\nu$, $2h\nu$, $3h\nu$ . . . , where h is a constant, now known as Planck's constant.

This assumption is practically equivalent to saying that radiant energy E of any assigned frequency ν can be emitted and absorbed only as an integral multiple of an element of energy, that is

$$E = nh\nu \qquad . \quad . \quad . \quad . \quad . \quad . \quad 1:1‡$$

where n is always a positive integer.

This may be called the hypothesis of "energy quanta," as it seems to imply the existence of a unit of energy $h\nu$; but it is hardly correct to say that the quantity $h\nu$ represents a universal "atom" of energy, since the amount depends on the frequency of the vibration considered. The numerical value of the quantum constant h deduced by Planck from his formula as applied to

* Max Planck, *Verh. d. deutsch. phys. Ges.*, vol. 2, p. 237, 1900; *Ann. d. Physik*, vol. 4, p. 553, 1901.

† Planck's radiation formula is given in Appendix III, p. 258.

‡ Mathematical equations are referred to by means of two numbers, of which the first is the number of the chapter, the second the number of the equation in the chapter.

the measurements of Kurlbaum and of Lummer and Pringsheim was

$$h = 6{\cdot}55 \times 10^{-27} \text{ erg sec.},$$

a value which is in remarkably good agreement with later and more accurate determinations by various methods. The value adopted in this book is

$$\mathbf{h = 6{\cdot}558 \times 10^{-27} \text{ erg sec.}}$$

It will be noticed that h is a quantity of the dimensions of energy multiplied by time, i.e. of *action*, as the term is used in applied mathematics in connection with the Principle of Least Action.

The fundamental relation of Planck's theory may obviously be written in the form

$$\frac{E}{\nu} = nh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1:2$$

ν being the frequency. The equation may also be written

$$E\tau = nh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad 1:3$$

where τ is the period of the vibration. This may be interpreted by regarding E/ν or $E\tau$ as the action, and h as an element of action. From this point of view, which agrees closely with that adopted in the later developments of the subject, the quantum of action, h, is a universal constant, a true *atom* of action. We may gain some modicum of satisfaction from the idea that action is atomic, though as Jeans has remarked " an attempt to imagine a universe in which action is atomic leads the mind into a state of hopeless confusion ! "

An alternative mode of regarding the quantum constant may be mentioned here. J. W. Nicholson,* in an investigation of the nebular and coronal spectrum, suggested that the angular momentum of an atomic system might be equated to $nh/2\pi$. Here $h/2\pi$ appears as a natural unit of angular momentum, the physical " dimensions " of this quantity being the same as those of action.

2. Photo-electricity

In 1905 Einstein† made a notable suggestion which has proved fruitful in predicting experimental results although it raises many grave difficulties. This was the hypothesis of the existence of " light quanta," an idea which has also been developed by J. J. Thomson in his unitary theory of light. According to

* J. W. Nicholson, *Monthly Notices, R.A.S.*, vol. 72, p. 677, 1912.

† A. Einstein, *Ann. d. Physik*, vol. 17, p. 132, 1905 ; vol. 20, p. 199, 1906.

this hypothesis, the energy of radiation is to be treated as though it were done up in bundles, the energy of the bundle or light quantum being given by Planck's expression $h\nu$. Further, only a single quantum is involved in any single physical change. This is sometimes known as Einstein's law, which may be stated as follows: "In every case where there is a mutual transformation between moving electrons and electromagnetic radiation, a single light quantum is utilized or is liberated."

One aspect of this reciprocal relation is seen in the photoelectric effect. When light of sufficiently short wave-length is allowed to fall upon a polished metal surface, negative electrons are set free with a velocity which depends upon the wave-length of the exciting light. The maximum kinetic energy of the electrons increases with frequency in accordance with a formula first suggested by Einstein in the paper referred to. Einstein's formula is:

$$\tfrac{1}{2}\, mv^2 = h\nu - \mathrm{P} \qquad \text{I:4}$$

Here $\frac{1}{2}mv^2$ corresponds to the maximum kinetic energy of the emission; $h\nu$ corresponds to the energy of the light quantum, and P measures the work done when an electron escapes from the atom to which it is attached. It is found that this quantity P, which is called the "electron affinity," can be expressed as $h\nu_0$ where ν_0 is a definite frequency characteristic of the metal on which the radiation falls. The equation, which has been called "the fundamental law of photo-electric activity," now takes the form

$$\tfrac{1}{2}\, mv^2 = h(\nu - \nu_0) \qquad \text{I:5}$$

The kinetic energy of the electron is proportional to the difference between the frequency of light and the frequency characteristic of the particular metal.

The significance of the relation may best be appreciated by considering a particular case. For sodium the characteristic or "threshold" frequency is about $5{\cdot}15 \times 10^{14}$ vibrations per second, corresponding to green light. If the light is redder than this, it may fall on sodium for centuries without liberating any electrons, but if the light is bluer than the specified green light, it will at once bring about the separation of electrons and the maximum kinetic energy of emission will increase as the frequency increases. Einstein's equation has been verified by numerous experimenters—in the first instance by Hughes * and by Richardson and Compton,† then by Millikan ‡ and his pupils—indeed Millikan's experiments based on this relation,

* A. Ll. Hughes, *Phil. Trans.*, vol. 212, p. 205, 1912.

† O. W. Richardson and K. T. Compton, *Phil. Mag.*, vol. 24, p. 575, 1912.

‡ R. A. Millikan, *Phys. Rev.*, vol. 7, p. 355, 1916.

furnish us with one of the most accurate methods of determining Planck's constant *h*. The equation possesses a very high degree of generality: it applies to ordinary light and to X-rays, and appears to be valid not only in the emission of electrons under the influence of light, but also when emission of radiation is brought about by the impact of electrons. Consider for example the production of X-rays in a Coolidge bulb. "A plentiful supply of electrons is provided at the cathode by heating a fine spiral of tungsten wire to a high temperature. A high potential difference between cathode and target is provided by some appropriate means, and the electrons are hurled at the target, each possessing an amount of energy equal to the product of the electron charge and the applied potential. Where the electrons strike, some of their energy is converted into electro-magnetic waves of very high frequency, the so-called X-rays. Suppose that we measure the energy supplied to each electron—not an easy matter with the usual arrangements, but very easily done if, as in certain experiments of Duane and Hunt at Harvard University, the potential is derived from a great storage battery of 40,000 volts. Suppose, further, that we analyse by the X-ray spectrometer the X-radiation that issues from the target. We find that the frequencies of the emitted rays may have a wide range of values, but that the upper limit of the frequencies is always proportional to the energy of the electron, and, therefore, to the potential imposed on the tube. This ratio remains the same no matter what the intensity of the electron discharge, and no matter what the nature of the target." The ratio is, in fact, Planck's constant *h*. Sir William Bragg, from whose Kelvin lecture * the quotation is taken, points out the reciprocal character of the relation in the case of X-rays, and emphasizes the extraordinary and, at present, insoluble problem involved. "It is not known how the energy of the electron in the X-ray bulb is transferred by a wave motion to an electron in the photographic plate or in any other substance on which the X-rays fall. It is as if one dropped a plank into the sea from a height of 100 feet, and found that the spreading ripple was able, after travelling 1,000 miles and becoming infinitesimal in comparison with the original amount, to act upon a wooden ship in such a way that a plank of that ship flew out of its place to a height of 100 feet. How does the energy get from one place to another?" "In many ways the transference of energy suggests the return to Newton's corpuscular theory. But the wave theory is too firmly established to be displaced from the ground that it occupies. We are obliged to use each theory as occasion demands and wait for

* *Nature*, vol. 107, p. 79, 1921.

further knowledge as to how it may be possible that both should be true at the same time."

3. The Photo-chemical Equivalent

Another illustration of Einstein's hypothesis is provided by his photo-chemical law: when light of given frequency is incident on a system sensitive to such light, for each single light quantum absorbed one molecule of the absorbing substance is decomposed. In other words, the number of molecules affected is equal to the number of light quanta absorbed. As Jeans points out: "The law not only prohibits the killing of two birds with one stone, but also the killing of one bird with two stones. Since the energy of a quantum $h\nu$ is proportional to its frequency ν we understand how a small amount of violet light can accomplish what no amount of red light would suffice to do, a fact familiar to every photographer—we can admit quite a lot of light if only it is red, but the smallest amount of violet light spoils our plates."

It must, however, be said that most photochemical processes are complicated by secondary reactions which make it extremely difficult to test the direct applicability of the law. It is now generally admitted that the absorbing molecule absorbs energy in quanta; but the number of molecules *decomposed* may differ considerably from the number of *absorbing* molecules. In a few reactions, and within certain narrow spectral limits, these numbers have been found the same. It is, then, extremely probable, if not certain, that the primary process consists in the absorption of an energy quantum; but the subsequent changes are frequently so complicated that it is not at present possible to express them in terms of the quantum theory.

Suggestions have been made by Trautz, Lewis, and Perrin that radiation is an important factor in all chemical action, Perrin in particular having developed the view that "ordinary" chemical reactions may be regarded as due to radiation, that is as photo-reactions. But the observed velocities of reaction are much in excess of those required (apparently) by this hypothesis. Although the radiation hypothesis is in some respects attractive, its supporters have not yet been able to present it in such a form as to make it rank as a theory capable of giving quantitative expression of the results of experiment. Indeed it would seem that far more attention than it has yet received should be paid to the effect of electron collisions in promoting chemical change.*

* "Photochemical Reaction in Liquids and Gases," Faraday Society Discussion, *Trans.*, vol. 21, p. 438, 1926.

4. The Quantum Theory of Spectral Series

The application of the quantum theory to the study of spectra continues to excite very great interest. This work, which will always be associated with the name of the Danish physicist Niels Bohr,* is based on two fundamental ideas. The first is a natural extension of the principle involved in the photo-electric effect. Bohr argued that when an atom emits monochromatic radiation of frequency ν, it must be because the atomic system has lost energy of amount $h\nu$. Thus

$$h\nu = H_a - H_e \qquad \text{I:6}$$

where H_a and H_e are the values of the energy in the two states † of the atomic system under consideration. This relation, generally known as Bohr's Frequency Condition, at once explains the Combination Law in connection with spectral series. This law states that the frequencies of the lines of a spectrum are given by the difference between pairs of terms of a sequence.

But a second application of the quantum principle is required to fix the "stationary states" of the atomic system, that is to determine the *permissible* orbits by "quantizing the orbits." In the earlier work this was effected by employing the suggestion of J. W. Nicholson that h may be regarded as a natural unit of angular momentum. The determination of the orbits is simple when a single electron is moving in the field of a positively charged nucleus.

By the application of these hypotheses Bohr was successful in deducing Balmer's and certain similar series emitted by hydrogen, and the enhanced spectrum of helium, i.e. the spectrum given by helium which has lost one electron so that there is a surplus positive charge. The same principles have also been employed in the explanation of the spectra of other elements.

In particular we must notice the remarkable agreement between the value of the fundamental Rydberg constant of spectroscopy deduced by Bohr and that found as the result of observation. Bohr's value for this fundamental frequency is

$$\nu_\infty = \frac{2\pi^2 me^4}{h^3} \qquad \text{I:7}$$

where e is the charge in ordinary electrostatic units, m is the

* Bohr, *Phil. Mag.*, vol. 26, pp. 1, 476, 857, 1913.

† It is convenient to use the subscript letters a and e to distinguish between the initial (antecedent) state and the final (end) state of the system.