

Steven C. Chapra
Raymond P. Canale



Numerical Methods for Engineers

With Software and Programming Applications • Fourth Edition

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With Software and Programming Applications

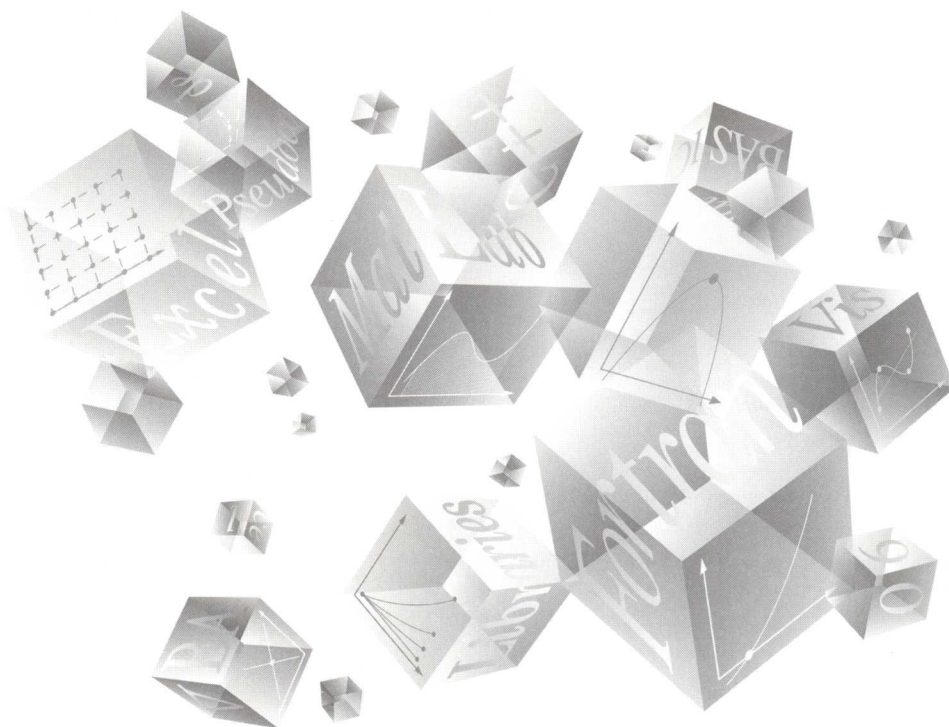
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Numerical Methods for Engineers

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Fourth Edition



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To

Margaret and Gabriel Chapra

Helen and Chester Canale

PREFACE

Back in the Stone Age (actually the 1970s and early 1980s), numerical methods were primarily taught at the graduate level. About 15 years ago, we began concurrently teaching the subject to sophomore and junior students at Texas A&M and Michigan. We used newly developed “microcomputers” along with a strong focus on engineering problem solving to make the subject accessible and exciting to our students. And it seemed to work.

Based on our experiences, we wrote a thin silver book complete with a 5-1/4" floppy disk in the back (Chapra and Canale, 1985). Over time that book has evolved into the edition you are now reading. Although we’ve added new material and changed with the times, the present book is still founded on the same principles as the original:

- **Problem Orientation.** Engineering students learn best when they are motivated by problems. This is particularly true for mathematics and computing. Consequently, we have approached numerical methods from a problem-solving perspective.
- **Student-Oriented Pedagogy.** We have developed a number of features to make this book as student-friendly as possible. These include the overall organization, the use of introductions and epilogues to consolidate major topics and the extensive use of worked examples and case studies from all areas of engineering. We have also endeavored to keep our explanations straightforward and oriented practically.
- **“Clear Box” Approach.** Although we emphasize problem solving, we believe that it is self-limiting for engineers to approach numerical algorithms as “black boxes.” Thus, we include sufficient theory to allow users to understand the basic concepts behind the methods. In particular, we stress theory related to error analysis, the limitations of the methods, and trade-offs among methods.
- **Personal Computing Orientation.** When we first wrote this book, there was a great chasm between the batch world of mainframe computers and the interactive world of PCs. Today, as PC performance grows, the distinctions are falling away. That said, this book still emphasizes visualization and interactive calculations, which are the hallmarks of personal computing.
- **Empowering Students.** We obviously introduce students to the standard “point-and-shoot” problem-solving capabilities of packages like Excel and MATLAB. However, students are also shown how to develop simple, well-structured programs to extend the base capabilities of those environments. This knowledge carries over to standard

programming languages such as Fortran 90 and C/C++. We believe that the current flight from computer programming represents something of a “dumbing down” of the engineering curriculum. The bottom line is that as long as engineers are not content to be tool-limited, they will have to write code. Only now they may be called “macros” or “M-files.” This book is designed to empower them to do that.

Beyond these five original principles, the new edition has a number of new features:

- **Software Orientation.** Just as the silver book capitalized on early hardware developments, the present book takes advantage of software development. In particular, material and problems throughout the text are devoted to implementing numerical methods with Microsoft's Excel and Mathworks' MATLAB. These tools place an incredible array of numerical capability at the ready disposal of any student engineer willing to climb their mild and sunny learning curves.
- **New and More Challenging Problem Sets.** The primary enhancement in the fourth edition is the revision of the problems sets, including over 125 new problems. Many of the new problems are challenging and relate to exciting growth areas such as bioengineering.
- **New Material.** New sections have been added. These include a modified false position method and multidimensional integration.

As always, our primary intent is to provide students with a sound introduction to numerical methods. We believe that motivated students who enjoy numerical methods, computers, and mathematics will, in the end, make better engineers. If our book fosters an enthusiasm for these subjects, we will consider our efforts a success.

Acknowledgments. Special thanks to Prof. Wally Grant, Olga Pierrakos, Amber Phillips, Justin Griffiee, and Kevin Mace of Virginia Tech for developing many of the new problems in this edition. In addition, Prof. Theresa Good of Texas A&M contributed a number of excellent new problems. As in past editions, David Clough of the University of Colorado and Jerry Stedinger of Cornell University generously shared their insights and suggestions. Useful suggestions were also made by Alon Barlevy (Cal State Fullerton), Bill Philpot (Cornell), David Hefer (The Technion, Israel), Dong-Il Seo (Chungnam National University, Korea), Anton Krueger (Iowa), and Tim Ameel and Tahar Loulou (Utah). This edition has also benefited from comments and suggestions made by Tony Cahill (Texas A&M), Lizette Chevalier (Southern Illinois University), Tim D'Orazio (San Francisco State University), Charles K. Roby (Clemson), Steve Salley (Wayne State University), and Jude T. Sommerfeld (Georgia Tech). In addition, David V. Chase (The University of Dayton), Raymundo Cordero and Karim Muci (ITESM, Mexico), James W. Hiestand (University of Tennessee at Chattanooga), Steve Klegka (U.S. Military Academy), James L. Kuester and Robert L. Rankin (Arizona State University), Elisa D. Sotelino (Purdue University), and Hewlon Zimmer (U.S. Merchant Marine Academy) provided useful feedback for the prior edition.

Finally, it should be stressed that although we received useful advice from the aforementioned individuals, we are responsible for any inaccuracies or mistakes you may detect in this edition.

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ABOUT THE AUTHORS

Steve Chapra teaches in the Civil and Environmental Engineering Department at Tufts University. His other books include *Surface Water-Quality Modeling* and *Introduction to Computing for Engineers*.

Dr. Chapra received engineering degrees from Manhattan College and the University of Michigan. Before joining the faculty at Tufts, he worked for the Environmental Protection Agency and the National Oceanic and Atmospheric Administration, and taught at Texas A&M University and the University of Colorado. His general research interests focus on surface water-quality modeling and advanced computer applications in environmental engineering.

He has received a number of awards for his scholarly contributions, including the 1993 Rudolph Hering Medal (ASCE) and the 1987 Meriam-Wiley Distinguished Author Award (American Society for Engineering Education). He has also been recognized as the outstanding teacher among the engineering faculties at both Texas A&M University (1986 Tenneco Award) and the University of Colorado (1992 Hutchinson Award).

Raymond P. Canale is an emeritus professor at the University of Michigan. During his over 20-year career at the university, he taught numerous courses in the area of computers, numerical methods, and environmental engineering. He also directed extensive research programs in the area of mathematical and computer modeling of aquatic ecosystems. He has authored or coauthored several books and has published over 100 scientific papers and reports. He has also designed and developed personal computer software to facilitate engineering education and the solution of engineering problems. He has been given the Meriam-Wiley Distinguished Author Award by the American Society for Engineering Education for his books and software and several awards for his technical publications.

Professor Canale is now devoting his energies to applied problems, where he works with engineering firms and industry and governmental agencies as a consultant and expert witness.

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