Introduction to DYNAMIC MACROECONOMIC THEORY

An Overlapping Generations Approach

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with Neil Wallace

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Preface

This book grew out of a series of lecture notes that we used at the University of Minnesota, University of Chicago, Universidad Central de Venezuela, and Centro de Estudios Macroeconomicos de Argentina. Our objective here is to provide relatively easy access to a wide range of basic issues in dynamic (or intertemporal) macroeconomics, yet to maintain the firm microeconomic foundation that is at the heart of almost all serious modern macroeconomic work. We use a single basic model throughout, the overlapping generations model under perfect foresight. Although this choice imposes some restrictions (for example, we do not consider economies with uncertainty), it enables us to use simple mathematics (nothing more advanced than calculus is required) and to concentrate on economic issues. In addition, we are able to illustrate each of our results with analytical derivations, graphically, and by working out examples. The exercises, an integral part of the book, are strategically placed and should be worked when encountered. Those marked with an asterisk are more difficult or more open ended than the rest.

Chapters 1 through 3, 5, and 6 construct the basic model and illustrate the methods for finding equilibria with different kinds of assets. The agents in all of the economies are selfish except in those economies described in Chapter 4, where some individuals care about their children. Chapter 7 proposes variants on the basic model that allow for equilibria with fluctuations. Chapter 8 adds a simple storage or linear production technology, and Chapter 9 modifies the basic model to generate the standard neoclassical growth results. Chapters 10 through 12 introduce money as an asset and explore restrictions that generate equilibria in which individuals hold both money and assets with higher rates of return.

The basic structure of the book was developed jointly. McCandless is responsible for the exposition and, at Wallace's request, this is reflected in the attribution. There is no portion of the book, however, that did not benefit from Wallace's detailed ideas and recommendations.

At Harvard University Press, Michael Aronson has been extremely supportive. We are grateful for the improvements in consistency and style that have come from Jodi Simpson's careful editing. Patricia Pitzele and Mustafa Koluman read and critiqued the entire manuscript. Barbara Belisle provided invaluable administrative support.

Contents

PART ONE

REAL ECONOMIES

Most questions in what has traditionally been called macroeconomics involve consideration of events at more than one date. For example, a decision to save is a decision to sacrifice current consumption for (at least the hope of) future consumption. There are two traditions in economics for dealing with intertemporal questions. One tradition attempts to study, simultaneously and explicitly, the entire time interval or time horizon that is relevant for the question at hand. This view was brought into prominence by Irving Fisher at the turn of the century (see, in particular, his *Theory of Interest*). The other tradition attempts to deal with such questions by focusing only on the present. It attempts to embed the future—or more precisely, present views of the future—by having them be one of the givens in the analysis. This tradition received its main impetus from John Maynard Keynes's *General Theory of Employment, Interest, and Money*.

We try to take Fisher's view of the world seriously, not only in terms of how we deal with time, but also by building our models entirely from microeconomic, general equilibrium fundamentals. We are explicit about the tastes or preferences of individuals, the resources available to them, and the technologies they can use. One advantage of this approach is that we can use the tools of standard welfare analysis for evaluating different policies. However, models that use this type of analysis can quickly become very complicated, so we are forced to make certain simplifications.

One major decision we make is how long people in our models will live. Suppose we are interested in studying an economy from now to some date T units of time into the future. One way to model individuals is to assume that they all live for the entire time period. Another is to assume that some of them die and some new individuals are born during the period. We choose the second approach for several reasons. First, it is more interesting in the sense that it allows more kinds of outcomes. Second, it forces people to think about the future because they may be engaging in transactions with individuals who are not yet alive. Third, it allows us to deal with very long lived economies, ones that outlive any individual. The kind of model we use is called overlapping generations—new generations of people appear before all of the people from the earlier period die off.

We make other simplifying assumptions. Time is discrete; we label time periods by the integers. Each generation lives only two periods, so at each date there are some young and some old. In the next period the young have become old, the old have passed on, and a new generation of young have arrived. There is just one consumption good each period, and we will only consider very simple technologies for transferring this good from one period to another. All of the equilibria we examine are competitive, and individuals, except in the earliest chapters, have perfect foresight.

For the first nine chapters of this book, we deal exclusively with real economies; economies in which there is no money and all exchanges are barter exchanges, even exchanges that involve what we normally think of as assets. The kinds of exchanges we concentrate on are intertemporal exchanges. Individuals have claims to goods in a consumption pattern over time that is not necessarily their preferred consumption pattern, and we consider alternative objects that they might use to help them move closer to their preferred consumption pattern. We study how tastes, technologies, and time paths of income interact to determine interest rates, asset prices, and either growth or the price level.

We study a number of possible government policies: tax-transfer schemes that resemble social security programs, government borrowing and the effects of different maturity government debt, and different kinds of taxes (consumption versus income taxes). The economies we study will be able to exhibit fluctuations, from either real or endogenous sources. For most of Part One, we limit ourselves to endowment economies, where we do not allow storage or production, but eventually we relax these restrictions and get simple versions of neoclassical growth models.

The theory we present in Part One is not especially controversial. It comes from standard neoclassical economics. In Part Two we add a particular asset we call money. The use of overlapping generation models to model money is more controversial, so we have excluded that discussion from our examination of real economies in Part One and have placed it in Part Two.

CHAPTER 1

Describing the Environment

There are at least two steps in constructing an economic model. First, one describes the environment in which the economic agents exist; and, second, one describes the kinds of social organizations that can be imposed on that environment and the equilibria that these organizations generate. Any given economic environment can permit a number of different social organizations for producing goods and allocating the goods that it produces.

For a group of individuals on some isolated island, the physical structure of the economic environment is simple to describe: it is the island, the quality of its land and the plants and animals that currently exist on it, the coral reef and the stock of fish that live there, the individuals themselves, their skills and ages, and the passage of time. There are many ways this tribe of individuals could organize an economy in their environment. They could choose one member to dictate work assignments and to allocate the produced goods. They could decide collectively how to assign work and goods. They could allow individuals to work independently and make trades with one another. Whatever organization they form, it must make work assignments and goods allocations and it must be able to do this beyond the life of any one member of the tribe.

In this chapter, we describe the economic environment that we will be using throughout the book. This environment is explicitly dynamic, so we begin with a description of time, how much of it we have, and how we keep track of it. We describe the lifetimes of the individuals who inhabit our economy. The seven ages of man get reduced to two:

youth and old age. Individuals have preferences for consumption over both of these periods of their lives. We define consumption allocations for this economy and Pareto conditions for comparing these consumption allocations.

In most of our models there is no production. The actions of interest are the exchanges, mainly intertemporal exchanges, that individuals choose to make with one another. Therefore, we endow individuals with quantities of goods and observe the exchanges they choose to make. We can safely study these *pure exchange models* because most of the results will carry over to economic environments with production.

Time

In a description of a dynamic economy one needs to be very specific about time. There are two possible ways of modeling time: time can be continuous, or it can be discrete. For the physicist this dichotomy is a serious problem, and there are physical theories of the world in which time is continuous and theories in which it is discrete (although the increments are very small).[1] We have chosen to make time discrete and index time periods by integers. We may be giving up some generality by doing so (and we may be losing some important characteristics of the real world), but we gain in expositional and mathematical simplicity.

All actions, decisions, and events occur at points of time, called periods or dates, which we denote by an index, t. The index can have values from minus infinity ($-\infty$, the infinite past) to plus infinity ($+\infty$, the infinite future). We use the convention of calling the current date (or equivalently, the initial period of the model) period $t = 1$, and we are interested in how the economy evolves into the infinite future. The *history* of the economy, that is, events corresponding to the t's from $-\infty$ to 0 (which is the most recent period already completed), is given and determines the initial conditions of our economy.

The fact that we are allowing time to go into the infinite future is important for some of the equilibria of our models. Even if no one believes that time will go infinitely into the future (the physicists say the universe will end, at least operationally, in finite time), the fact that this

1. John Barth in *Giles Goat Boy* pokes fun at this issue with a character who is trying to define ever more precisely when tick becomes tock.

date is far into the future and is unknown may be sufficient to allow infinity to be a good approximation of that future, unknown end point.

Starting at time 1 and taking history as given, we will be asking the following type of questions: How would this economy evolve under different types of policy rules? The question is carefully phrased. Note that it speaks of policy *rules*. We are interested in more than what the possible government policies will be in time $t = 1$. In fact, in some of our economies, we are not able to say what happens at time 1 without specifying what the policies will be in later periods. The notion of a *policy rule* that determines the policies at each period from now into the future is crucial to our discussions of policy.

The Population

At each time period t, a new generation appears (is born, if you wish). This generation, called generation t, is denoted by its date of birth.[2] There are $N(t)$ members of generation t. These individuals live for two periods. The members of generation t are alive in period t (when they are called the young) and in period $t + 1$ (when they are called the old). No member of generation t makes it into period $t + 2$; at the end of period $t + 1$ they all die.[3]

This construction of the evolution of the population is called overlapping generations, because there are two generations alive in any one period, the period in which they overlap. Figure 1.1 shows the life spans of several generations. This figure also makes it clear how the generations overlap. In time 0, for example, only the old who were born in period $t = -1$ and the young who were born in period 0 are alive. As time progresses to period 1, the old from generation -1 die and the young from generation 0 become old. A new generation (generation 1) is born and is young. This pattern repeats. Each generation overlaps

2. This practice is different from that used in universities, which denote each generation by the date of their "death" as students (or, as it is more commonly called, their graduation).

3. Kenneth Arrow has pointed out that one of the great economic problems individuals face is that they do not know the exact date of their death. This lack of knowledge prevents them from consuming the last bit of their income just before they die (suppose they consume the last bit of their income, thinking they will die during the upcoming night, but wake up the next morning alive and impoverished). We ignore that problem here.

	Time period					
generation	0	1	2	3	4	5
-1	old					
0	young	old				
1		young	old			
2			young	old		
3				young	old	
4					young	old

Figure 1.1

for one period with the previous generation and then overlaps for one period with the next generation. It overlaps with no other generations.

Figure 1.1 also allows us to find the number of individuals who are alive in any period. In period 2, $N(1)$ old are around and so are $N(2)$ young. [$N(t)$ is the population of generation t.] In any time period t, there are $N(t - 1)$ old and $N(t)$ young. The total live population at period t is $N(t - 1) + N(t)$. Population size can change, so $N(t)$ need not equal $N(t + j)$ for any $j \neq 0$. Both population size and length of life are exogenous and are not affected by any actions of any individuals. (The mystery of life shall remain a mystery here.) We are interested in studying economies in which the population grows, some in which it shrinks, and others in which it remains constant.

Total Resources

There is only one good in each period. This good is something that the members of the economy consume. It is the same good at different dates (although that is not crucial); but the date at which the good exists is important, so we denote the good by its date. The good that is around in time period 7 is called *time 7 good.* (You can think of this as time t bread, time $t + 1$ bread, time $t + 2$ bread, and so on.) In an important sense there are an infinite number of goods: one for each of the infinite