Irving H. Shames

INTRODUCTION TO SOLID MECHANICS

Second Edition

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PREFACE

It is now 14 years since the initial publication of *Introduction to Solid Mechanics*. It has received steady use at an undiminished level that continues to this day. The users, who have been exceptionally loyal over the years, consist of a variety of schools, from prestigious engineering schools to small two-year community colleges. Here at Buffalo during this period, I have been teaching sophomore classes of 200 to 300 students using this book. These students consist of mechanical, aerospace, civil, and industrial engineers.

One reviewer of the first edition, although he liked the book, felt strongly that it was suitable only for majors in mechanics, engineering science, engineering physics, or possibly for very strong, analytically oriented versions of the more conventional programs. Over the years, I have given this opinion much thought. So when some years ago, I was afforded the opportunity at Buffalo to teach junior fluid mechanics to all aerospace and mechanical engineers (200 students), I tried to investigate this premise. This was possible since half the class consisted of transfer students from a wide variety of schools. Seven years of observation, together with countless individual in-depth interviews with students, generally showed me that there was distinctly greater ease of learning and deeper insight into the difficult aspect of fluids on the part of those students who had had a more basic, analytic approach than for those (many transfers) who had a more elementary, more practical exposure to mechanics and solids as sophomores. This tendency extended even to the practical aspects of my fluids course but was decidedly more pronounced for the theoretical aspects of the course. Furthermore, I have found an even more pronounced disparity in my advanced senior–graduate courses in solid mechanics, where a good understanding of fundamentals is essential. This alone would have been sufficient to propel my present undertaking along the same direction as that taken in the first edition. But now other experiences during recent years have given me additional impetus to continue in this direction.

In recent years, I have been teaching senior–graduate courses in *finite elements.* This culminated in the publication of the book *Energy and Finite Element Methods in Structural Mechanics* with C. Dym (Hemisphere Publishing Corporation), New York, 1985. Many of our seniors in civil, mechanical, and aerospace engineering take this course as an elective. Parenthetically, the recruitors from industry are now asking for competence in this area from prospective hirees. To become proficient in this growing area of expertise, I feel that the student needs now *more than ever* a sound, basic, in-depth treatment of solid mechanics starting as a sophomore and building carefully from that point onward. I fear that students who play with software and "canned" programs without this sound, carefully developed background are headed for careers as technicians rather than as engineers.

These then are some of the overall sentiments I brought to the task of developing the second edition. The following list of items will detail further how I have tried to implement these sentiments.

1. The treatment continues to be basic and careful so that follow-on courses will not have to undo or redo the concepts of this course even as to notation. For instance, I use for normal stresses the notation τ_{xx} and τ_{xy} than σ_x and τ_{xy}. Moments and products of inertia are denoted as I_{xx}, I_{xy}, etc., rather than I_x, P_{xy}, etc. This anticipates tensor notation and the concept of rank of expressions in an equation. The concept of the Cartesian tensor is not skirted or waved at but instead explained carefully in a manner I have found even in large classes to be comprehensible to the bulk of sophomores. This results in the dual benefits of great saving of time and an increase of unity and clarity in the course. Also, this permits students to know precisely why second moments and products of area, plane stress, and plane strain share a variety of vital properties despite their outward physical disparities, as well as how they relate to three-dimensional stress, strain, and inertia.

2. The first edition knowingly encompassed more material than could possibly be covered in a three-credit course. I must admit that as a writer, I have always found it difficult to be impeded by the artificial constraints of credit hours from presenting additional important concepts and topics at a time and place when and where sufficient, well-cultivated background is already in place and where and when these additional concepts and topics follow naturally. Furthermore, consistent with this, as a teacher it is my practice to give a thumbnail sketch of this additional material for orientation purposes — a practice I consider important. Parenthetically, I use as a benchmark of success or lack thereof in my course the degree of interest and curiosity developed for the subject matter of the course. In no small measure I gauge this degree of success by how extensive the casual reading or the careful study is of the material in the text which is outlined but not covered formally in class. Hence, you will appreciate why, even knowing that the additional material may in some cases mitigate against the adoption

of the book, I have not retreated from this extended approach in the second edition. But now I have an additional factor to impel me to continue as I have earlier. Over the years the students at Buffalo have consistently told me that they found the first edition almost as useful after finishing the course as during the course for use in their upper-class courses, such as structures, machine design, finite elements, etc., for providing bridges from their earlier studies to the new endeavors.

With these considerations in mind, I have tried to improve the treatment in the following ways: I have reformulated a number of large chapters into smaller ones so as to entail more homogeneous treatments in each and to delineate certain differences in viewpoint and approach between certain topics. This is designed to make the material more relevant for new areas appearing in the curriculum, such as the already mentioned finite elements and, to mention another, inelastic solid mechanics.† Also, I might add, new topics have been introduced while other topics that I feel have lost their importance have been deleted. (For those not familiar with the first edition, I have included a tentative course outline on the page following this preface.)

The above items will be further addressed in this preface.

3. For instance, my treatment of energy methods is now quite different as a result of the extensive introduction of finite elements into many undergraduate curricula. I now have a separate chapter on *displacement methods* and a separate chapter on *force methods*. As you can see on inspecting these chapters, this material is quite basic and is based on the kind of depth of understanding needed for a good grasp of finite elements in particular, and modern structural mechanics in general. I do not expect these chapters to be covered by everybody in a first three-credit course. However, I do cover the chapter on displacement methods in my course (about $3\frac{1}{2}$ credits). My experience is that "mickey mouse" treatments of this topic are virtually useless in follow-on studies of structures. The energy chapters will bring the students to a point where they will be able to anticipate the important concept of the *functional* and be on the threshold of *variational methods* in general.

4. At Buffalo, as a result of strong insistence by ABET, we endeavor to maintain use of the computer in virtually all our engineering courses while trying not to distort or compromise the basic thrust of each course. In the second edition, I have addressed this goal. There are 50 computer-oriented projects in Appendix I. They are designed so that the student will essentially need only a knowledge of Fortran learned in standard freshman courses. Anything beyond this knowledge in terms of algorithms or numerical methods is fully explained in the project statements. It is my practice *not to take any time out of my solids course* for programming or other ancillary computer instruction. Two

†We have in recent years introduced this kind of course as a high-level elective course for competent seniors and for graduate students. This course covers the basics of linear elasticity, linear and nonlinear viscoelasticity, plasticity, and viscoplasticity. The impetus for this course is the growing need for engineers to deal with inelastic materials (such as composites) and the ability to use finite and boundary element methods to solve problems in these areas. My colleague F. Cozzarelli and I have coauthored a new text, *Elastic and Inelastic Stress Analysis*, published by Prentice Hall. Again the growing need to work with such materials signals to me the need for a basic in-depth treatment of solid mechanics even at the outset of a program.

or three projects are assigned a semester over and above the usual load of problem solving and reading. These projects give a *design* dimension to the course beyond what was possible before. In what has turned out to be a voluminous instructor's manual, there is to be found all the computer programs in *Fortran* and many in *Basic* plus printouts of results. The manual is spiral bound so that instructors can photocopy whatever they desire for their students. The Fortran programs also are on a floppy disk inserted in the manual for the instructor's use so that he or she can easily edit them for use on an IBM-compatible machine.

5. Last, but not least in my mind, is the input and encouragement I have received from instructors who used the first edition. Many have indicated that except for particularly weak students, they found this book actually easier to teach from than more elementary treatments. They credited this to the increase in logic and the decrease in "hand-waving" needed to proceed through the course as a result of the greater depth and rigor. I don't think the second edition will alter these reactions.

I would like to call your attention to a number of other more specific changes I have made in the second edition.

1. I have simplified the treatment in several respects. Instead of developing the transformation of stress and strain first in three dimensions and then simplifying to plane stress and plane strain, I have formulated the transformation equations for plane stress and plane strain directly in separate chapters, Chapters 7 and 8, without recourse to the three-dimensional approach. Of course, this is simpler and allows the use of such things as Mohr's circle earlier in the text. Those instructors wishing to derive the benefits of seeing the whole picture first before getting into special cases can still do so in the second edition by going to the general case first (in Chapters 15 and 16) and then coming back to the chapters on plane stress and plane strain. Long-time users of the first edition may be glad to know that even in the two-dimensional introduction described above, the concept of the tensor is still set forth, albeit in a simple, natural way, with the resulting saving of time and increase in clarity.

2. Over the years, I have become less than satisfied with the chapter on shear and bending-moment, and so in the new edition I have redone this chapter. I believe that it is now clearer and easier to use. Furthermore, the instructors who have used this material in later courses tell me that there is better retention from the new exposition.

3. As a further change, I have added an introductory chapter on *finite elements* applied to plane trusses. This will, at the same time, introduce the student to matrix structural mechanics.

4. Of lesser note I would like to mention that I have eliminated the use of singularity functions for shear and bending-moment equations and now have a separate starred chapter for singularity functions used for the deflection of beams. I continue to use the step, ramp, delta, and doublet functions rather than the Clebsch brackets. The singularity functions I use are widely used in circuit theory, applied physics, and in the study of viscoelasticity and creep, not to speak of many areas of applied mathematics such as the important *boundary element* method.

5. Lecturing to a very large class has made me seek clear, straightforward delineations of the subject matter. As a result of this constant and continuing impetus, I

have added explanations and examples at various places in the first edition to augment and to drive home a concept or an idea more effectively. What has not been changed or rearranged has survived the exacting test of 14 years of lectures to large classes of diverse backgrounds.

6. I have overhauled the best of the problems from the first edition and have added many new problems. About 60% of the problems are now in SI units.

Finally, I wish to thank my esteemed colleague, Professor Shahid Ahmad, who has been teaching the odd-semester class in solid mechanics. He has been most encouraging in supporting efforts at achieving greater depth and rigour. Professor J. Pitarresi of the State University of New York at Binghampton took solid mechanics in my class as a sophomore while using the first edition. He has since taught from this book and has spent much time discussing the first edition with me from the dual viewpoints of student and instructor. These conversations were very helpful and encouraging to me as I worked on the second edition. Next I wish to acknowledge the constant stimulation and encouragement I have enjoyed from senior faculty at Buffalo, namely Professors Frank Cozzarelli, Richard Shaw, and Raj Kaul. My good friend Professor M. Grasso of nearby State University of New York at Fredonia has been most helpful in pointing out corrections. I wish most heartily to thank my excellent graduate student Mr. Anoop Dhingra for doing, under my direction, the programming of the computer projects. Another graduate student Mr. Paohong Tong checked the entire manuscript and the solutions manual with much skill and patience. I thank him most sincerely. I want to thank my excellent secretary, Mrs. Debra Kinda, for her fine typing efforts and for her patience and ever-present good cheer. My thanks extend to my editor, Doug Humphrey, for his support of the aims and philosophy of this book and for his direct help. Last, but certainly not least, I wish to express my gratitude to Dean George Lee and Associate Dean Ken Kiser. As Faculty Professor, I report directly to them. Their support, with the able and tireless assistance of Bill Wachob, has at all times been constant, generous, and unwavering.

Irving H. Shames

COURSE OUTLINE FOR FIRST 3-CREDIT COURSE

	Hours (50 min.)
Chapter 2: 2.1–2.6 Stress	3
Chapter 3: Strain	4
Chapter 4: 4.1–4.4, 4.7 Tensile Test Discussion	2
Chapter 5: 5.1–5.5, 5.7 One-Dimensional Problems	4
Chapter 6: 6.1–6.2 Three-Dimensional Hooke's Law	2
Chapter 7: 7.1–7.5 Plane Stress	4
Chapter 8: 8.1–8.5 Plane Strain	3
Chapter 10: Shear and Bending Moment	4
Chapter 11: 11.1–11.5, 11.13 Stresses in Beams	5
Chapter 12: 12.1–12.5 Bending Deflections	3
Chapter 14: 14.1–14.4 Torsion	3
Chapter 15: 15.1–15.4 Three-Dimensional Stress Tensor	3
Chapter 16: Three-Dimensional Strain Tensor	2
Chapter 17: 17.1–17.6, 17.8 Elastic Stability	3
	45

This outline reflects the pace maintained at Buffalo involving students in all fields.

A second course may be fashioned from material not covered above. This might include such topics as:

1-D Viscoelasticity Problems	Composite Beams
Non-Isotropic Hooke's Law	Singularity Functions
1-D Thermal Stress Problems	Index Notation
Failure Criteria	Fatigue
Non-Symmetric Bending	Force Energy Methods
Shear Centers	Displacement Energy Methods
Inelastic Bending and Torsion	Introduction to Finite Elements

CONTENTS

FUNDAMENTAL NOTIONS

chapter 1

1.1 INTRODUCTION

Mechanics is the physical science concerned with the movement and deformation of media acted on by mechanical or thermal disturbances. Since such considerations are involved in most situations confronting the engineer, mechanics lies at the core of much engineering analysis. In fact, it is the oldest of all the physical sciences. The writings of Archimedes covering buoyancy and the lever were recorded before 200 B.C. Our modern knowledge of gravity and motion was established by Isaac Newton (1642–1727), whose laws founded Newtonian mechanics. In this book we shall be concerned with the statics of deformable bodies. We shall now discuss certain general considerations which will underlie our future efforts.

To be able to represent an action using the known laws of mechanics and also to be able to form equations simple enough to be susceptible to mathematical computational techniques, invariably in our deliberations we must replace the actual physical action, and the participating bodies, with hypothetical, highly simplified substitutes. We must be sure, of course, that the results of our substitutions have some reasonable correlation with reality. All analytical physical sciences must resort to this technique, and, consequently, their computations are not cut and dried but involve a considerable amount of imagination, ingenuity, and insight into physical behavior. We shall at this time set forth the most fundamental idealizations that we shall rely on in this book.

The continuum. Even the simplification of matter into molecules, atoms, electrons, etc., is too complex a picture for many problems of engineering mechanics. In most problems we are interested only in the average measurable manifestations of these elementary bodies. Pressure, density, and temperature are actually the gross effects of the actions of the many molecules and atoms, and they can be conveniently assumed to arise from a hypothetically continuous distribution of matter, which we shall call the *continuum*, instead of from a conglomeration of discrete bodies. Without such an artifice, we would have to consider the action of each of these elementary bodies—a virtual impossibility for most problems.

The rigid body. In many cases involving the action on a body by a force, we simplify the continuum concept even further. The most elemental case is that of a rigid body, which is a continuum that undergoes theoretically no deformation whatever. Actually, every body must deform a certain amount when acted on by forces, but in many cases the deformation is too small to affect the desired analysis. It is then preferable to consider the body as rigid and proceed with the simplified computations. For example, assume that we are to determine the forces transmitted by a beam to the earth as the result of a load P (Fig. 1.1). If P is reasonably small, the beam will undergo little deflection and we can carry out a straightforward simple analysis as if the body were indeed rigid; that is, we use the *undeformed geometry*. If we were to attempt a more accurate analysis—even though a slight increase in accuracy is not required—we would then need to know the exact position that the load assumes relative to the earth *after* the beam has ceased to deform, as shown in an exaggerated manner in Fig. 1.2. To do this accurately is a hopelessly difficult task, especially when we consider that the supports must also "give" in a certain way. Although the alternative to a rigid-body analysis here leads us to a virtually impossible calculation, situations do arise in which more realistic models must be employed to yield the accuracy required. *The guiding principle is to make such simplifications as are consistent with the required accuracy of the results.*

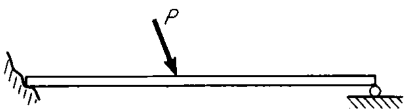

Figure 1.1

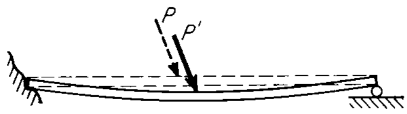

Figure 1.2

We must generally abandon the rigid-body model when the applied loads cause the body to deform to such an extent that the final orientation of these applied loads is not known with sufficient accuracy for the problem at hand. At other times we find that the deformation of a body, however small, must be taken into account to solve the problem. We shall encounter the latter problems often; they are called *statically indeterminate* problems, for reasons that will soon be clear.