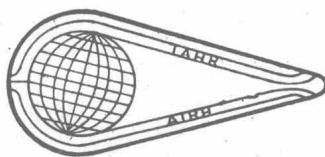


ASSOCIATION INTERNATIONALE  
DE RECHERCHES HYDRAULIQUES

INTERNATIONAL ASSOCIATION  
FOR HYDRAULIC RESEARCH

NEUVIEME ASSEMBLEE GENERALE  
NINTH CONVENTION

Dubrovnik 1961  
Yougoslavie



Publié avec le concours financier de l'UNESCO  
Published with the financial assistance of UNESCO

# INTRODUCTION

La Neuvième Assemblée Générale de l'Association Internationale de Recherches Hydrauliques a eu lieu à Dubrovnik (Yougoslavie) du 4 au 7 Septembre 1961. L'organisation du Congrès a été confiée à l'Association Yougoslave de Recherches Hydrauliques, membre de l'Union Yougoslave des Ingénieurs et Techniciens. Au Congrès assistaient 241 participants et 56 dames de 33 pays. Le programme du Congrès comprenait:

## A. THEMES PRINCIPAUX

1. Effets de turbulence sur des constructions hydrau-hydrauliques (27 mémoires)
2. Mécanique des courants d'eaux souterraines (37 mémoires)
3. Problèmes hydrauliques pour les machines à calculer (43 mémoires)
4. Modifications de courants naturels causées par des ouvrages (27 mémoires)

## B. CONFERENCES GENERALES

1. Problèmes hydrauliques du Karst
2. Ecoulement en trois dimensions à travers la couche de limite
3. Transformation de l'énergie dans les zones de séparation

## C. SEMINAIRES

1. Hydraulique de petites constructions dans l'irrigation et le drainage (10 abrégés)
2. Hydrodynamique fondamentale des écoulement à surface libre et des écoulements non uniformes (38 abrégés)

## D. SYMPOSIUM

- Machines hydrauliques

## E. SEANCE ADMINISTRATIVE

- Rapports concernant l'activité de l'AIRH entre les deux dernières Assemblées et l'élection des nouveaux conseils d'administration et d'exécution

Le Congrès a été ouvert le 4 Septembre à 10h 00 par le Président du Comité d'organisation Prof. Bogić Knežević. A la séance d'ouverture les congressistes étaient salués par le délégué du Conseil Exécutif Fédéral Yougoslav, M. Vinko Hafner et le maire de la ville de Dubrovnik, M. Vinko Betica. Les discours d'ouverture ont été prononcés par le président de l'AIRH, Dr. Arthur Ippen et le secrétaire Dr. J. H. Schoemaker.

Les séances du Congrès ont été tenues dans deux salles: les conférences générales, les mémoires relatifs aux thèmes 1 à 4 avec leur discussions et la Séance administrative ont eu lieu dans la grande salle, tandis que les deux séminaires et le Symposium se déroulaient dans la salle de séminaire.

Après la clôture du Congrès deux excursions — «A» et «B» — ont été organisées. La première, excursion «A», comprenait la visite de l'aménagement hydroélectrique «Trebišnjica», du port maritime de Bar, des aménagements hydroélectriques «Perućica» et «Bistrica», du barrage «Kokin Brod» et du Laboratoire d'Hydraulique situé au pied du mont Avala, près de Beograd. La seconde, excursion «B», consistait en visites des aménagements hydroélectriques «Split» et «Peruća», de l'Institut de l'Hydrodynamique Maritime de Zagreb et du Laboratoire Hydraulique de la Faculté Technique de Ljubljana.

Ces Comptes-rendus contiennent les mémoires selon les thèmes principaux avec leur discussions, conférences générales, rapport généraux sur les abrégés de deux séminaires ainsi que leur registres, et le rapport sur le Symposium.

La publication de ces Comptes-rendus a été possible grâce à la subvention du Conseil Exécutif Fédéral Yougoslave, de l'AIRH, de l'UNESCO et de diverses institutions et entreprises hydrauliques yougaslaves.

Le Comité d'organisation tient à exprimer sa profonde gratitude à tous ceux qui ont contribué au succès de la Neuvième Assemblée Générale de l'AIRH.

# INTRODUCTION

The Ninth Convention of the International Association for Hydraulic Research was held from September 4 to 7, 1961, in Dubrovnik (Yugoslavia). The Yugoslav Association for Hydraulic Research, a member of the Yugoslav Union of Engineers and Technicians, was in charge of its organisation. The Congress was attended by 241 participants and 56 ladies from 33 countries. The Program of the Congress was as follows:

## A. MAIN SUBJECTS

1. Effect of turbulence on hydraulic structures (27 papers)
2. Mechanics of groundwater flow (37 papers)
3. Hydraulic problems for computers (43 papers)
4. Modifications of natural streams by engineering structures (27 papers)

## B. GENERAL LECTURES

1. Hydraulic problems of the Karst
2. Three-dimensional boundary layer flow
3. Energy transformation in the zones of separation

## C. SEMINARIES

1. Hydraulics of small structures in irrigation and drainage systems (10 concise communications)
2. Fundamental hydrodynamics of free surface flow and unsteady flow (38 concise communications)

## D. SYMPOSIUM

- Hydraulic machinery

## E. BUSINESS MEETING

- Reports concerning the activity of the IAHR between the last two congresses and the election of the new executive and administrative boards.

The Ninth Convention was opened by the president of the Organizing Committee, Prof. Bogić Knežević, on September 4th at 10.00 a. m. At the opening session the Congress was addressed and welcomed by the delegate of the Federal Executive Council, Mr. Vinko Hafner and the mayor of the city of Dubrovnik, Mr. Vinko Betica. The introductory speeches were delivered by the president of the IAHR, Dr. Arthur Ippen and the secretary, Dr. H. J. Schoemaker.

The Congress sessions were held in two halls. The general lectures and the papers involving subjects 1 to 4 were presented and discussed in the main hall where the business meeting was also held, while the seminars and the symposium took place in the seminar room.

After the close of the Congress two excursions — »A« and »B« — were organized. The first one, excursion »A«, included visits to the hydroelectric power plant »Trebišnjica«, the seaport of Bar, hydroelectric power plants »Perućica« and »Bistrica«, the barrage »Kokin Brod« and the Laboratory of Hydraulics, situated at the foot of Avala Hill, near Beograd. The second one, excursion »B«, included visits to the hydroelectric power plants »Split« and »Peruća«, the Institute of ship hydrodynamics near Zagreb and the Hydraulic Laboratory in Ljubljana.

This Congress publication contains papers on the main subjects together with their discussions, general lectures, general reports of the chairmen of the two seminars, registers of the concise seminary communications and the report on the symposium.

The publication of the Proceedings of the Ninth Convention was financially supported by the Yugoslav Federal Executive Council, IAHR und UNESCO, as well as by several Yugoslav hydraulic institutes and enterprises.

The Organizing Committee gratefully acknowledges the services of all those who contributed to the success of the Ninth Convention.

## Conseil de l'AI RH

Le Conseil pour la période 1<sup>er</sup> janvier 1962 — 31 décembre 1963 a été élu conformément aux articles 17—21 des Statuts précédents et l'article 15 des Statuts approuvés par l'assemblée général à Dubrovnik au 7 septembre 1961.

Les membres du Conseil élu sont:

|                         |                       |                   |
|-------------------------|-----------------------|-------------------|
| <b>Président:</b>       | A. T. Ippen           | (Etats-Unis)      |
| <b>Vice-présidents:</b> | D. V. Joglekar        | (Indes)           |
|                         | F. H. Allen           | (Grande-Bretagne) |
|                         | L. Escande            | (France)          |
| <b>Secrétaire:</b>      | H. J. Schoemaker      | (Pays-Bas)        |
| <b>Membres:</b>         | R. Boucher            | (Canada)          |
|                         | J. P. Duport          | (France)          |
|                         | P. Canisius           | (Allemagne)       |
|                         | B. V. Proskouriakov   | (U.R.S.S.)        |
|                         | M. Hom-ma             | (Japon)           |
|                         | A. Schlag             | (Belgique)        |
|                         | O. J. Maggiolo Campus | (Uruguay)         |

---

## Council of AIRH

The Council for the period January 1st, 1962 — December 31st, 1963 has been elected, according to the articles 17—21 of the former Constitution and the article 15 of the Constitution adopted by the general assembly at Dubrovnik on September 7 th, 1961.

The members of the elected Council are:

|                        |                       |               |
|------------------------|-----------------------|---------------|
| <b>President:</b>      | A. T. Ippen           | (U.S.A.)      |
| <b>Vice-president:</b> | D. V. Joglekar        | (India)       |
|                        | F. H. Allen           | (U. K.)       |
|                        | L. Escande            | (France)      |
| <b>Secretary:</b>      | H. J. Schoemaker      | (Netherlands) |
| <b>Members:</b>        | R. Boucher            | (Canada)      |
|                        | J. P. Duport          | (France)      |
|                        | P. Canisius           | (Germany)     |
|                        | B. V. Proskouriakov   | (U.S.S.R.)    |
|                        | M. Hom-ma             | (Japan)       |
|                        | A. Schlag             | (Belgium)     |
|                        | O. J. Maggiolo Campus | (Uruguay)     |

## TABLE DE MATIERE — TABLE OF CONTENTS

### THEME 1

#### EFFECTS DE TURBULENCE SUR DES CONSTRUCTIONS HYDRAULIQUES

#### THE EFFECTS OF TURBULENCE ON HYDRAULIC STRUCTURES

N. V. Chandrasekhara Swamy

TURBULENT FLOW IN OPEN CHANNELS . . . . . 1

N. S. Govindra Rao

N. Rajaratnam

ON THE INCEPTION OF AIR-ENTRAINMENT IN OPEN CHANNEL FLOWS 9

E. Razvan

L'ETUDE DE L'ECOULEMENT MACROTURBULENT EN AVAL DES  
EVACUATEURS DES CRUES . . . . . 13

Rex A. Elder

MODEL — PROTOTYPE TURBULENCE SCALING . . . . . 24

T. E. Unny

THE SPATIAL HYDRAULIC JUMP . . . . . 32

Jiří Kališ

DIMINUTION DE LA TURBULENCE DERRIERE LE RESSAUT . . . . . 43

Petr Hořeni

PRESSURE FLUCTUATIONS IN THE MIXING CHAMBER OF AN EJECTOR 50

K. Haindl, L. Doležal

TURBULENT PRESSURE FLUCTUATIONS IN A COMPLEX HYDRAULIC  
STRUCTURE . . . . . 56

Pavel Novák

INFLUENCE OF BED LOAD PASSAGE ON SCOUR AND TURBULENCE  
DOWNSTREAM OF A STILLING BASIN . . . . . 66

Anton Grzywiński

THE EFFECT OF TURBULENT FLOW ON MULTI-SECTION VERTICAL  
LIFT GATES . . . . . 76

|                                                                                                                                  |     |
|----------------------------------------------------------------------------------------------------------------------------------|-----|
| P. S. Eagleson, F. E. Perkins                                                                                                    |     |
| A TOTAL HEAD TUBE FOR THE BROAD-BAND MEASUREMENT OF TURBULENT VELOCITY FLUCTUATIONS IN WATER . . . . .                           | 86  |
| Jean Pantelopulos                                                                                                                |     |
| INFLUENCE DE LA TURBULENCE SUR LA REPARTITION DE LA FORCE TRACTRICE ENTRE LES MATERIAUX D'UN FOND MOBILE . . . . .               | 98  |
| J. W. Daily, J. D. Lin, R. S. Broughton                                                                                          |     |
| TURBULENCE AND STATIC PRESSURE IN RELATION TO INCEPTION OF CAVITATION . . . . .                                                  | 110 |
| Mikio Hino                                                                                                                       |     |
| ON THE MECHANISM OF SELF-AERATED FLOW ON STEEP SLOPE CHANNELS. APPLICATIONS OF THE STATISTICAL THEORY OF TURBULENCE . . . . .    | 123 |
| J. J. Lévy                                                                                                                       |     |
| EFFET DYNAMIQUE D'UN COURANT A HAUTE TURBULENCE SUR DES OUVRAGES HYDRAULIQUES ET SUR LE LIT DES RIVIERES . . . . .               | 133 |
| S. Angelin, K. Flagestad                                                                                                         |     |
| VIBRATION OF ARCHED STOP-LOGS IMMERSED IN STREAMING WATER . . . . .                                                              | 141 |
| Harold M. Martin, William E. Wagner                                                                                              |     |
| EXPERIENCE IN TURBULENCE IN HYDRAULIC STRUCTURES . . . . .                                                                       | 153 |
| Praxitelis A. Argyropoulos                                                                                                       |     |
| THE HYDRAULIC JUMP AND THE EFFECT OF TURBULENT ON HYDRAULIC STRUCTURES. CONTRIBUTION TO THE RESEARCH OF THE PHENOMENON . . . . . | 173 |
| O. Györke                                                                                                                        |     |
| ENERGY DISSIPATION IN PROTECTED BEDS DOWNSTREAM OF RIVER BARRAGES IN THE CASE OF SHALLOW STILLING POOLS . . . . .                | 184 |
| I. V. Nagy                                                                                                                       |     |
| INVESTIGATIONS INTO THE DISTRIBUTION OF SUSPENDED SEDIMENT . . . . .                                                             | 197 |
| G. H. Toebes                                                                                                                     |     |
| HYDROELASTIC FORCES ON HYDRAULIC STRUCTURES DUE TO TURBULENT WAKE FLOWS . . . . .                                                | 205 |
| Jozef Procházka                                                                                                                  |     |
| THE EFFECT OF TURBULENCE ON CURRENT METER . . . . .                                                                              | 222 |
| Z. Szgyártó                                                                                                                      |     |
| EFFECT OF TURBULENCE ON THE CALIBRATION ACCURACY OF HYDRAULIC STRUCTURES . . . . .                                               | 227 |
| Zivko Vladislavljević-Medak                                                                                                      |     |
| QUELQUES ASPECTS QUANTITATIFS DU MOUVEMENT TURBULENT . . . . .                                                                   | 239 |

|                                                                                                                                                                                                                    |     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| J. Čabelka, T. Horsky                                                                                                                                                                                              |     |
| CONTRIBUTION A L'ETUDE DE L'AMORTISSEMENT DE L'ENERGIE<br>CINETIQUE D'UN COURANT D'EAU SUR LES DEVERSOIRS DE HAUTS<br>BARRAGES ET DE LA CHARGE DU RADIER . . . . .                                                 | 244 |
| F. R. Brown                                                                                                                                                                                                        |     |
| FLUCTUATION OF CONTROL GATES . . . . .                                                                                                                                                                             | 258 |
| R. Curtet, F. Ricou                                                                                                                                                                                                |     |
| INFLUENCE DES CONDITIONS A L'ENTREE DE LA CHAMBRE SUR LES<br>CARACTERISTIQUES MOYENNES ET DE TURBULENCE D'UN JET DE<br>REVOLUTION EN PRESENCE D'UNE VITESSE AMBIANTE DANS UNE<br>CHAMBRE CYLINDRIQUE . . . . .     | 270 |
| <br><b>T H E M E 2</b>                                                                                                                                                                                             |     |
| <br><b>MECANIQUE DES COURANTS D'EAUX SOUTERRAINES</b><br><b>MECHANICS OF GROUNDWATER FLOW</b>                                                                                                                      |     |
| S. D. L. Luthra, D. V. Joglekar                                                                                                                                                                                    |     |
| UPLIFT PRESSURE BELOW HYDRAULIC STRUCTURES ON STRATI-<br>FIED, PERMEABLE FOUNDATIONS . . . . .                                                                                                                     | 279 |
| R. O. Bullen                                                                                                                                                                                                       |     |
| AN ELECTRICAL ANALOGUE INVESTIGATION OF THE STANDPIPE<br>FIELD PERMEABILITY TEST . . . . .                                                                                                                         | 287 |
| J. Zeller                                                                                                                                                                                                          |     |
| THE SIGNIFICANCE OF AQUIFER POROSITY IN NON-STEADY SEEPAGE<br>FLOW WITH FREE SURFACE . . . . .                                                                                                                     | 299 |
| J. Zaoui                                                                                                                                                                                                           |     |
| DETERMINATION DES CARACTERISTIQUES HYDRODYNAMIQUES DES<br>MILIEUX POREUX . . . . .                                                                                                                                 | 308 |
| A. Preissman, R. Brepson, M. L. Clement                                                                                                                                                                            |     |
| ECOULEMENT EN MILIEUX POREUX DE PROFONDEUR FINIE SOUS UN<br>RADIER PROTEGE A L'AMONT ET A L'AVANT PAR DES ECRANS D'ETAN-<br>CHEITE-PROCEDE NUMERIQUE DE DETERMINATION DE LA REPRESEN-<br>TATION CONFORME . . . . . | 317 |
| Selim Yalin, Lehn Franke                                                                                                                                                                                           |     |
| EXPERIMENTAL INVESTIGATION OF THE LAWS OF FILTER FLOW . . . . .                                                                                                                                                    | 324 |
| Vitalie Pietraru                                                                                                                                                                                                   |     |
| CONTRIBUTIONS A L'ETUDE DES INFILTRATIONS NON PERMANENTES<br>A NIVEAU LIBRE . . . . .                                                                                                                              | 332 |
| Harold A. Weggel                                                                                                                                                                                                   |     |
| MECHANICS OF GROUND-WATER FLOW THROUGH LAVA BED AREA<br>ADJACENT TO A RESERVOIR . . . . .                                                                                                                          | 341 |

|                                                                                                                                                   |     |
|---------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| V. I. Aravin                                                                                                                                      |     |
| EXPERIMENTAL STUDIES OF PERCOLATION THROUGH REGIONS INFLUENCED BY RAISING OF GROUND-WATER TABLE DUE TO HYDRAULIC STRUCTURES . . . . .             | 350 |
| M. Szalay                                                                                                                                         |     |
| EFFECT OF INCOMPLETE SHEET PILINGS AND IMPERMEABLE APRONS ON FILTRATION FLOW UNDER PRESSURE . . . . .                                             | 360 |
| L. Bernell                                                                                                                                        |     |
| NON-STATIONARY GROUNDWATER FLOW THROUGH EMBANKMENTS                                                                                               | 364 |
| Aasmund Tveiten                                                                                                                                   |     |
| WATER FLOW THROUGH PEAT . . . . .                                                                                                                 | 371 |
| G. Öllös                                                                                                                                          |     |
| INVESTIGATION INTO BOUNDARY CONDITIONS OF SEEPING SYSTEMS AND THEIR SMALLEST FLOW CROSS-SECTION . . . . .                                         | 377 |
| František Slepíčka                                                                                                                                |     |
| THE LAWS OF FILTRATION AND LIMITS OF THEIR VALIDITY . . . .                                                                                       | 383 |
| František Slepíčka                                                                                                                                |     |
| HYDRAULIC FUNCTION OF A CYLINDRICAL WELL IN AN ARTESIAN AQUIFER WITH REGARD TO NEW RESEARCH ON FLOW THROUGH POROUS MEDIA . . . . .                | 395 |
| Václav Zajiček                                                                                                                                    |     |
| APPLICATION OF THE HYDROLOGICAL METHOD FOR THE EVALUATION OF NON-PERMANENT FLOW OF ALLUVIAL GROUND WATER .                                        | 400 |
| Miroslav Knežek                                                                                                                                   |     |
| DETERMINATION OF THE SEEPAGE CAPACITY OF INFILTRATION RESERVOIRS AT FINITE DEPTH OF AN IMPERMEABLE BASE . . .                                     | 408 |
| S. Irmay                                                                                                                                          |     |
| UNSTEADY FLOW THROUGH POROUS MATERIALS . . . . .                                                                                                  | 414 |
| L. Šuklje                                                                                                                                         |     |
| FACTEURS GOUVERNANT LES GRADIENTS CRITIQUES DANS LA BASE D'EXCAVATION . . . . .                                                                   | 428 |
| Giorgio Nosedà                                                                                                                                    |     |
| ÉCOULEMENT D'INFILTRATION À TRAVERS UN BARRAGE EN TERRE SUR ALLUVIONS PERMEABLES . . . . .                                                        | 437 |
| Borivoje Mijatović                                                                                                                                |     |
| SOME CHARACTERISTICS OF THE HYDRAULIC MECHANISM OF THE UNDERGROUND WATERS IN DISSOLUTION FRACTURES AND CHANNELS OF THE CALCAREOUS ROCKS . . . . . | 446 |



|                                                                                                                                                                    |     |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| W. C. Bischoff Van Heemskerck                                                                                                                                      |     |
| INVESTIGATION OF EXCESS WATER-PRESSURE UNDER THE ASPHALT<br>FACING OF SAND-CORED DYKES . . . . .                                                                   | 455 |
| Kâzim Çeçen                                                                                                                                                        |     |
| SEEPAGE ONTO THE SLOPES IN ISOTROPIC AND ANISOTROPIC SOILS                                                                                                         | 468 |
| G. Öllös                                                                                                                                                           |     |
| POSSIBILITIES OF MODEL INVESTIGATIONS INTO WATER MOVEMENTS<br>OCCURRING IN FISSURED ROCKS . . . . .                                                                | 485 |
| Charles-Sébastien Gerber, Daniel Manry                                                                                                                             |     |
| ETUDE EXPERIMENTALE DES PHENOMENES SUPERFICIELS EN MILIEU<br>POREUX . . . . .                                                                                      | 495 |
| Mladen Boreli, B. Batinić                                                                                                                                          |     |
| CONSIDERATION SUR LA LOI NON-LINEAIRE DE FILTRATION REPAR-<br>TITION DE VITESSES DE FILTRATION AU VOISINAGE DU POINT DE<br>SORTIE . . . . .                        | 506 |
| M. Boreli, D. Jovašević                                                                                                                                            |     |
| CLOGGING OF THE POROUS MEDIA . . . . .                                                                                                                             | 516 |
| Milan Vuković                                                                                                                                                      |     |
| METHODES DE TRAITEMENT DES ECOULEMENTS DANS UN MILIEU<br>STRATIFIE . . . . .                                                                                       | 522 |
| Václav Hálek                                                                                                                                                       |     |
| LA SIMULATION DES PROCESSUS NATURELS ET LEUR UTILISATION<br>DANS LA SOLUTION DES REGIMES DES COURANTS D'EAUX SOUTER-<br>RAINES DANS DES REGIONS ETENDUES . . . . . | 529 |
| Vladimirescu I., Lates M.                                                                                                                                          |     |
| RECHERCHES EXPERIMENTALES SUR LA FILTRATION EN REGIME<br>NONPERMANENT, AVEC APPLICATION AUX DIGUES ET BARRAGES<br>EN TERE . . . . .                                | 539 |
| Sukeyuki Shima                                                                                                                                                     |     |
| ON THE SO-CALLED DIFFUSION PHENOMENA OF SEEPAGE FLOW . .                                                                                                           | 550 |
| G. Karádi, I. V. Nagy                                                                                                                                              |     |
| INVESTIGATIONS INTO THE VALIDITY OF THE LINEAR SEEPAGE LAW                                                                                                         | 556 |
| Ján Benetin, Otto Mišút                                                                                                                                            |     |
| DEFORMATION OF THE PERMEABILITY OF ANISOTROPIC SOIL . .                                                                                                            | 567 |
| K. Ubell                                                                                                                                                           |     |
| VERTICAL-VELOCITY CURVE FOR GROUND-WATER FLOW BY SMALL<br>GRADIENT . . . . .                                                                                       | 574 |
| Boleslaw Kordas                                                                                                                                                    |     |
| CONTRIBUTION A L'ETUDE DES PUIITS A DRAINS RAYONNANTS . .                                                                                                          | 581 |

|                                                                                                                                         |     |
|-----------------------------------------------------------------------------------------------------------------------------------------|-----|
| Josip A. Grčić                                                                                                                          |     |
| MODEL TEST OF STEADY FLOW TOWARD A WELL . . . . .                                                                                       | 591 |
| J. Riarez, R. Gallea                                                                                                                    |     |
| REMARQUE SUR LES ECOULEMENTS NON PERMANENTS DANS LES<br>SOTS: VIDANGE RAPIDE DE BARRAGE — INJECTION DANS UN SOT<br>NON SATURE . . . . . | 601 |
| T H E M E 3                                                                                                                             |     |
| PROBLEMES HYDRAULIQUES POUR LES MACHINES A CALCULER<br>HYDRAULIC PROBLEMS FOR COMPUTORS                                                 |     |
| Armando Balloffet, Arnold Kupferman                                                                                                     |     |
| SURGES DUE TO PARTIAL BLOCKAGE OF A TAILRACE TUNNEL . . . . .                                                                           | 613 |
| R. M. Advani                                                                                                                            |     |
| IMPORTANCE OF ELECTRONIC COMPUTERS AND SIMILAR OTHER<br>DEVICES IN THE FIELD OF HYDRAULIC ENGINEERING . . . . .                         | 622 |
| N. J. Kendall                                                                                                                           |     |
| ANALISYS OF PIPE NETWORKS BY ELECTRONIC DIGITAL COMPUTERS                                                                               | 626 |
| R. Sabljak, W. H. Doran                                                                                                                 |     |
| HYDRAULIC ANALYSIS OF A SURGE SHAFT SYSTEM BY DIGITAL<br>COMPUTER . . . . .                                                             | 639 |
| A. Preissmann, J. Cunge                                                                                                                 |     |
| CALCUL DES INTUMESCENCES SUR MACHINES ELECTRONIQUES . . . . .                                                                           | 656 |
| M. Normand                                                                                                                              |     |
| L'ELECTRONIQUE AU SERVICE DE L'HYDROLOGIE . . . . .                                                                                     | 665 |
| E. Biro                                                                                                                                 |     |
| ETUDE DU COUP DE BELIER DANS DES CONDUITES A CARACTERI-<br>STIQUES MULTIPLES. . . . .                                                   | 673 |
| M. F. Gohin                                                                                                                             |     |
| ETUDE DES DENIVELATIONS ET DES COURANTS DUS A LA MAREE<br>EMPLOI DE MACHINES A CALCULER . . . . .                                       | 680 |
| F. Biesel, B. Ranson                                                                                                                    |     |
| CALCULS DE DIFFRACTION DE LA HOULE . . . . .                                                                                            | 688 |
| J. Zaoui .                                                                                                                              |     |
| QUELQUES APLICATIONS DES ORDINATEURS AUX PROBLEMES<br>D'ECOULEMENTS SOUTERRAINS . . . . .                                               | 700 |
| B. Ranson                                                                                                                               |     |
| CALCUL DES ECOULEMENTS IRROTATIONNELS A SYMETRIE<br>CILINDRIQUE . . . . .                                                               | 707 |

|                                                                                                                                    |     |
|------------------------------------------------------------------------------------------------------------------------------------|-----|
| George Bugliarello                                                                                                                 |     |
| TOWARD A COMPUTER LANGUAGE FOR HYDRAULIC ENGINEERING                                                                               | 714 |
| J. E. Nash                                                                                                                         |     |
| A LINEAR TRANSFORMATION OF A DISCHARGE RECORD                                                                                      | 730 |
| G. N. Gvazava                                                                                                                      |     |
| APPLICATION OF THE ELECTRONIC ANALOG COMPUTER FOR CALCULATION OF UNSTEADY FLOW IN CONDUITS AND SURGE TANKS OF HYDROELECTRIC PLANTS | 732 |
| Taizo Hayashi                                                                                                                      |     |
| WATER-HAMMER IN BIFURCATING PIPE-LINES                                                                                             | 740 |
| Tojiro Ishihara, Yasuo Ishihara                                                                                                    |     |
| RUNOFF ANALYSIS BY ANALOG COMPUTER                                                                                                 | 749 |
| C. W. Thomas, P. F. Enger                                                                                                          |     |
| USE OF AN ELECTRONIC COMPUTER TO ANALYZE DATA FROM STUDIES OF CRITICAL TRACTIVE FORCES FOR COHESIVE SOILS                          | 760 |
| J. Van Den Berg                                                                                                                    |     |
| CALCULATION OF RUN OFF FROM RAINFALL FIGURES                                                                                       | 772 |
| G. Pistilli, G. Savastano                                                                                                          |     |
| THE STUDY OF LEVEL'S OSCILLATIONS IN CYLINDRICAL SURGE TANKS WITH THE DIGITAL DIFFERENTIAL ANALIZER OF THE UNIVERSITY OF NAPLES    | 775 |
| P. Canisius, F. K. Rubbert                                                                                                         |     |
| THE APPLICATION OF ELECTRONIC COMPUTERS TO THE PREDICTIVE CALCULATION OF RIVER TIDES                                               | 790 |
| C. Thirriot                                                                                                                        |     |
| ETUDE DES INTUMESCENCES EN VALEURS ADIMENSIONNELLES                                                                                | 793 |
| C. Thirriot                                                                                                                        |     |
| ETUDE THEORIQUE DES PHENOMENES PNEUMATIQUES DANS UNE GALERIE DE FUITE D'USINE SOUTERRAINE                                          | 803 |
| Jean Piquemal                                                                                                                      |     |
| APPLICATION DE L'ANALOGIE A L'ETUDE DES SURPRESSION DANS LES CONDUITES FORCES A CARACTERISTIQUES MULTIPLES                         | 813 |
| M. T. Guyot, J. Nougaro, Cl. Thirriot                                                                                              |     |
| ETUDE NUMERIQUE DES REGIMES TRANSITOIRES DANS LES CANAUX                                                                           | 820 |
| R. Bonnefille, F. Voyer                                                                                                            |     |
| CALCUL DE LA PROPAGATION DE LA MAREE DANS UNE ZONE COTIERE                                                                         | 832 |

|                                                                                                                                                 |     |
|-------------------------------------------------------------------------------------------------------------------------------------------------|-----|
| J. Faure, N. Nahas                                                                                                                              |     |
| DEUX PROBLEMES DE MOUVEMENTS NON PERMANENTS A SURFACE<br>LIBRE RESOLUT SUR ORDINATEURS ELECTRONIQUES . . . . .                                  | 854 |
| R. P. Sijbesma, M. de Vries                                                                                                                     |     |
| COMPUTATIONS OF BOUNDARY CONDITIONS OF A RIVER MODEL<br>WITH MOVABLE BED . . . . .                                                              | 870 |
| M. de Vries                                                                                                                                     |     |
| COMPUTATIONS ON GRAIN — SORTING IN RIVERS AND RIVER MODELS                                                                                      | 876 |
| Guy Thibessard                                                                                                                                  |     |
| LA SIMULATION DU COUP DE BELIER SUR CALCULATEUR NUMERIQUE                                                                                       | 881 |
| M. F. Biesel                                                                                                                                    |     |
| OPTIMISATION DE LA RECETTE D'UNE USINE MAREMOTRICE . . . . .                                                                                    | 891 |
| M. F. Biesel, M. P. Arnaud                                                                                                                      |     |
| RECHERCHE DE LA REPARTITION ECONOMIQUE DES DIAMETRES<br>DANS UN RESEAU DE DISTRIBUTION RAMIFIE . . . . .                                        | 899 |
| G. Ransford, B. Ranson                                                                                                                          |     |
| FREQUENCY REGULATION BY A HYDROELECTRIC PLANT SUPPLIED<br>FROM A FREE-SURFACE CHANNEL . . . . .                                                 | 906 |
| M. Bouvard, J. Zaoui                                                                                                                            |     |
| LA DETERMINATION DU MOUVEMENT DES MATERIAUX EN ECOULE-<br>MENT TURBULENT (SUSPENSION ET DECONTANTATION) SUR MACHINES<br>ELECTRONIQUES . . . . . | 913 |
| J. Murillo                                                                                                                                      |     |
| APPLICATION D'UN ORDINATEUR A UN PROBLEME DE CHAMBRE<br>D'EQUILIBRE . . . . .                                                                   | 926 |
| H. Van Wijngaarden, R. P. Sijbesma                                                                                                              |     |
| COMPUTATIONS OF WATER LEVELS AT HIGH DISCHARGES ALONG<br>THE MAIN DIKES OF THE RIVER RHINE AND ITS BRANCHES IN THE<br>NETHERLANDS . . . . .     | 931 |
| Romolo Pasquali, Giorgio Rossi                                                                                                                  |     |
| RESOLUTION DE PROBLEMES HYDROLOGIQUES PAR LE CALCULA-<br>TEUR ELECTRONIQUE . . . . .                                                            | 947 |
| Hans Mogens Jensen                                                                                                                              |     |
| STATEMENT OF SOME METHODS FOR CALCULATION OF HEAD LOSSES<br>AT CHANNELS, BACKWATERS AND CANALS USED AT THE SWEDISH<br>STATE BOARD . . . . .     | 956 |
| L. Borel                                                                                                                                        |     |
| ETUDE GENERALE DE L'ECOLEMENT DE L'EAU EN REGIME VARIABLE<br>DANS UNE CONDUITE DE SECTION VARIABLE . . . . .                                    | 965 |

|                                                                                                                                                  |      |
|--------------------------------------------------------------------------------------------------------------------------------------------------|------|
| L. Bonny                                                                                                                                         |      |
| DETERMINATION PRATIQUE DES PARAMETRES HYDRAULIQUES FONDAMENTAUX D'UNE TURBINE . . . . .                                                          | 979  |
| E. O. Macagno, Matilde Macango                                                                                                                   |      |
| PRESSURE WAVE ANALYSIS FOR VARIABLE LENGTH OF A FLUID COLUMN . . . . .                                                                           | 993  |
| Torben Sørensen, Ian Larsen                                                                                                                      |      |
| HYDRAULIC CALCULATION OF STORM FLOOD LEVELS IN CONNECTED SHALLOW BASIN WITH SPECIAL REFERENCE TO THE LIMFJORD, DENMARK . . . . .                 | 1003 |
| Geza L. Bata, Petar B. Madich                                                                                                                    |      |
| SOLUTION FOR MULTIPLE SURGE-TANK SYSTEMS WORKED OUT ON REPETITIVE DIFFERENTIAL ANALYZER . . . . .                                                | 1011 |
| D. Rose                                                                                                                                          |      |
| THE INFLUENCE OF THE CLOSED SEA-ARMS ON THE WATER LEVEL AND CURRENT BEFORE THE COAST OF THE NETHERLANDS AND IN THE NEW WATERWAY . . . . .        | 1024 |
| <br><b>T H E M E 4</b>                                                                                                                           |      |
| MODIFICATIONS DE COURANTS NATURELS CAUSES PAR DES OUVRAGES<br>MODIFICATIONS OF NATURAL STREAMS BY ENGINEERING STRUCTURES                         |      |
| Z. Hankó                                                                                                                                         |      |
| INVESTIGATIONS TO DETERMINE THE NECESSARY LENGTH OF THE LINING FOLLOWING THE STILLING BASIN OF THE BOTTOM DROPS ON SMALL WATER COURSES . . . . . | 1029 |
| L. Muszkalay                                                                                                                                     |      |
| EFFECT OF PUMPING PLANT ON THE FLOW CONDITIONS OF THE RIVER . . . . .                                                                            | 1043 |
| A. H. Blyskowski                                                                                                                                 |      |
| MODIFICATION OF NATURAL STREAMS BY ENGINEERING STRUCTURES                                                                                        | 1051 |
| W. Kresser                                                                                                                                       |      |
| INFLUENCE D'INJECTION DE LIQUIDES SUR LE COURANT D'EAU AUTOUR D'UNE PILE . . . . .                                                               | 1055 |
| Enzo Levi, Humberto Luna                                                                                                                         |      |
| DISPOSITIFS POUR REDUIRE L'AFFOUILLEMENT AU PIED DES PILES DE PONTS . . . . .                                                                    | 1061 |
| D. R. P. Farleigh                                                                                                                                |      |
| CHANNEL — MOVEMENTS RESULTING FROM CONSTRUCTION OF NEW BRIDGE ABUTMENTS AT CONWAY, N. WALES . . . . .                                            | 1070 |

|                                                                                                                                                             |      |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| Al. Maruta                                                                                                                                                  |      |
| L'EFFET DES RETENUES A SEUIL SUR LES MODIFICATIONS DES LITS<br>DES RIVIERES . . . . .                                                                       | 1080 |
| S. Hincu, I. Lupu                                                                                                                                           |      |
| SUR LA SIMILITUDE DES AFFOUILLEMENTS LOCAUX DANS LA ZONE<br>DES CONSTRUCTIONS HYDROTECHNIQUES . . . . .                                                     | 1087 |
| O. J. Maggiolo                                                                                                                                              |      |
| ETUDE SUR LE MODELE REDUIT DE L'EROSION DU LIT ROCHEUX EN<br>AVAL DU DEVERSOIR DU BARRAGE DE „RINCON DEL BONETE“ —<br>(URUGUAY) . . . . .                   | 1094 |
| A. K. Ananian                                                                                                                                               |      |
| DETERMINATION DE LA FORMATION DU LIT RIVIERES, CREE PAR<br>SUITE DE L'ABAISSMENT DE LA COTE LEURS BASES D'EROSION . . . . .                                 | 1102 |
| S. V. Isbash, I. V. Lebedev                                                                                                                                 |      |
| CHANGE OF NATURAL STREAMS DURING CONSTRUCTION OF HIDRAU-<br>LIC STRUCTURES . . . . .                                                                        | 1114 |
| T. Horsky                                                                                                                                                   |      |
| L'AMORTISSEMENT DE L'ENERGIE MECHANIQUE PAR COLLISION DES<br>LAMES DEVERSANTES . . . . .                                                                    | 1122 |
| Jean Pantelopulos                                                                                                                                           |      |
| QUELQUES RESULTATS EXPERIMENTAUX SUR L'ENGRAVEMENT ET<br>LES POSSIBILITES DE DEGRAVEMENT D'UNE RETENUE FORMEE PAR<br>UN BARRAGE EN LIT DE RIVIERE . . . . . | 1128 |
| Jaroslav Martinec, Jaroslav Urban                                                                                                                           |      |
| FLOW CONDITIONS ABOVE A HIGH DAM . . . . .                                                                                                                  | 1134 |
| Walenty Jarocki                                                                                                                                             |      |
| EFFECTS OF PIERS ON WATER STREAMS AND BED FROM . . . . .                                                                                                    | 1147 |
| J. Valembois                                                                                                                                                |      |
| ESTIMATION DE LA DISTANCE DE REMONTEE DU REMOUS D'UN<br>OBSTACLE DANS UNE RIVIERE EN CRUE . . . . .                                                         | 1153 |
| C. J. Posey, J. H. Sybert                                                                                                                                   |      |
| EROSION PROTECION OF PRODUCTION STRUCTURES . . . . .                                                                                                        | 1157 |
| J. S. Burgess                                                                                                                                               |      |
| SCOUR DOWNSTREAM OF A COMPOUND MEASURING WEIR . . . . .                                                                                                     | 1163 |
| A. De Leeuw                                                                                                                                                 |      |
| MODIFICATIONS OF NATURAL WATERCOURSES BY ENGINEERING<br>STRUCTURES IN DESERT OF SOUTHERN ISRAEL . . . . .                                                   | 1172 |
| Mushtaq Ahmad, Abid Ali                                                                                                                                     |      |
| EFFECT OF BARRAGES ON THE REGIME OF RIVERS OF INDUS BASIN . . . . .                                                                                         | 1177 |

L. Ivicsics

|                                                                                                 |      |
|-------------------------------------------------------------------------------------------------|------|
| SCALE MODEL INVESTIGATION OF SEDIMENTATION OVER REACHES<br>LYING UPSTREAM OF BARRAGES . . . . . | 1195 |
|-------------------------------------------------------------------------------------------------|------|

L. Escande, J. Nougaro, L. Castex, H. Barthet

|                                                                                                  |      |
|--------------------------------------------------------------------------------------------------|------|
| INFLUENCE DE QUELQUES PARAMETRES SUR UNE ONDE DE CRUE<br>SUBITE A L'AVANT D'UN BARRAGE . . . . . | 1198 |
|--------------------------------------------------------------------------------------------------|------|

Mihailo Vojinović

|                                                                                          |      |
|------------------------------------------------------------------------------------------|------|
| MODIFICATIONS DES COURANTS D'EAU CAUSEES PAR DES PRISES<br>D'EAU SANS BARRAGES . . . . . | 1207 |
|------------------------------------------------------------------------------------------|------|

J. Chabert, M. Remilieux, I. Spitz

|                                                                                                                            |      |
|----------------------------------------------------------------------------------------------------------------------------|------|
| APPLICATION DE LA CIRCULATION TRANSVERSALE A LA CORREC-<br>TION DES RIVIERES ET A LA PROTECTION DES PRISES D'EAU . . . . . | 1216 |
|----------------------------------------------------------------------------------------------------------------------------|------|

Slavoljub Jovanović

|                                                                                                       |      |
|-------------------------------------------------------------------------------------------------------|------|
| ADAPTATION DE L'OUVRAGE DE SEPARATION POUR LA PROTECTION<br>DES RETENUES CONTRE ENSABLEMENT . . . . . | 1224 |
|-------------------------------------------------------------------------------------------------------|------|

Isao Akikusa, Hideo Kikkawa

|                                                           |      |
|-----------------------------------------------------------|------|
| HYDRAULIC BEHAVIOR OF THE GROINS IN THE STREAMS . . . . . | 1234 |
|-----------------------------------------------------------|------|

*CONFERENCES GENERALES*  
*GENERAL LECTURES*

Bogić Knežević

|                                               |      |
|-----------------------------------------------|------|
| LES PROBLEMES HYDRAULIQUES DU KARST . . . . . | 1247 |
|-----------------------------------------------|------|

H. Schlichting

|                                                 |      |
|-------------------------------------------------|------|
| THREE-DIMENSIONAL BOUNDARY LAYER FLOW . . . . . | 1262 |
|-------------------------------------------------|------|

Hunter Rouse

|                                                        |      |
|--------------------------------------------------------|------|
| ENERGY TRANSFORMATION IN ZONES OF SEPARATION . . . . . | 1291 |
|--------------------------------------------------------|------|

*SEMINAIRE „A“ — SEMINAR „A“*  
*SEMINAIRE „B“ — SEMINAR „B“*

Živko Vladislavljević-Medak

|                                                                                    |      |
|------------------------------------------------------------------------------------|------|
| HYDRAULIQUE DE PETITES CONSTRUCTIONS DANS L'IRRIGATION<br>ET LE DRAINAGE . . . . . | 1305 |
|------------------------------------------------------------------------------------|------|

John McNown

|                                                                                  |      |
|----------------------------------------------------------------------------------|------|
| FUNDAMENTAL HYDRODYNAMICS OF FREE-SURFACE FLOW AND OF<br>UNSTEADY FLOW . . . . . | 1314 |
|----------------------------------------------------------------------------------|------|

J. V. Daily

|                                                         |      |
|---------------------------------------------------------|------|
| SIMPOSIUM ON HYDRAULIC MACHINERY . . . . .              | 1318 |
| LISTE DES AUTEURS — LIST OF AUTHORS . . . . .           | 1320 |
| LISTE DES PARTICIPANTS — LIST OF PARTICIPANTS . . . . . | 1332 |

# TURBULENT FLOW IN OPEN CHANNELS

N. V. CHANDRASEKHARA SWAMY

INDIAN INSTITUTE OF SCIENCE, BANGALORE 12, INDIA

The velocity distribution in open channels has been analytically studied by dividing the flow region into three zones — (i) the upper zone where the eddy viscosity due to turbulence is predominant, (ii) the intermediate zone where the eddy and the kinematic viscosities are of the same order, (iii) the lower zone where the eddy viscosity is negligible and only the kinematic viscosity and wall shear are the controlling factors. Series solutions have been derived for the first two zones and are found to agree with the Prandtl Universal Logarithmic distribution as well as experiment. The lower zone corresponds to the well-defined region of the turbulent boundary layer. Approximate limits have been indicated for the extent of the three zones.

Distribution des vitesses en canaux ouverts est analytiquement étudiée par la division de la région de l'écoulement en trois zones:

(1) zone supérieure où la viscosité du tourbillon due à la turbulence est prédominante;

(2) zone intermédiaire où la viscosité cinématique et celle du tourbillon sont du même ordre;

(3) zone inférieure où la viscosité du tourbillon étant négligeable, la viscosité cinématique et la traction à la paroi sont les seuls facteurs de contrôle.

Une suite de solutions en dérivent pour les deux premières zones, s'accordant avec la Distribution logarithmique universelle de Prandtl tout aussi bien qu'avec les expériences. La zone inférieure correspond à la zone bien définie de la couche limite tourbillante. Des limites approximatives sont indiquées pour l'extension des trois zones.

## INTRODUCTION

The application of the theories of turbulence to problems in hydraulics has been instrumental in solving many as yet unsolved problems in hydraulics. Hydraulics has till now been an empirical science, lacking in the conciseness and richness of the more logical sciences. Most problems of everyday occurrence in hydraulics have been studied only empirically without any analytical background. This has led to several controversies which could easily be averted if the subject is studied from a more rational point of view.

One such controversy in the field of hydraulics is the extent of a hydraulic jump. It is known that a hydraulic jump gets formed when a supercritical flow encounters a subcritical flow. But what exactly is the extent of this jump has been a disputable point. Innumerable empirical formulae are available

and are being proposed periodically which all the more confuses the practical designer. The reason for this state of affairs is that our knowledge of the phenomena occurring inside a hydraulic jump is negligible. Rouse, Siao and Nagaratnam [1] have made certain turbulence measurements in the hydraulic jump. On the basis of this work, the author has given an expression for the extent of a hydraulic jump [2], which has agreed well with experiments. This was made possible by incorporating certain basic ideas regarding turbulent flow into the mathematics of the hydraulic jump.

Another interesting but as yet unexplored field of research in hydraulics is the velocity distribution in an open channel. Even though a large amount of work has been done on the resistance to flow in open channels for smooth or rough conditions, it is surprising that very little attention seems to have been paid



The fundamental problem of velocity distribution in open channels. The universal logarithmic velocity distribution of Prandtl and von Karman based on the momentum transfer theory has been modified by Vanoni [3] for the case of an open channel in the form

$$u = V + \frac{1}{K} \sqrt{gys} \left( 1 + 2.3 \log_{10} \frac{y'}{y} \right) \quad (1)$$

where  $u$  is the velocity at any height  $y'$  from the bed,  $y$  the depth of flow in the channel,  $V$  the mean velocity for the entire section of flow and  $K$  is von Karman's constant having a value of 0.40. The agreement between the experimental values and equation (1) was found to be very close except near the bed where the logarithmic law fails. But this is not a rational analysis of the problem.

The present author [4] attempted to solve this problem of open channel flow by resorting to the fundamental Navier-Stokes equations of motion and simplifying them to a certain extent by making the usual assumptions of turbulent shear flow. The problem was attacked in two ways — (i) by considering the entire depth of flow to be governed by a single law, (ii) the flow region being divided into two regions, the upper portion with predominant eddy viscosity and the lower one with predominant kinematic viscosity. Even though the velocity distributions obtained agreed fairly well with experiment and the universal logarithmic velocity distributions, the analysis contained some serious shortcomings. The fundamental aspect of turbulence is momentum transfer which was not taken into considerations in the previous work.

In this paper, the problem has been attacked by an entirely different method by dividing the channel into three distinct regions. It is a well established fact that turbulent flow gives rise to an eddy viscosity which is not an intrinsic property of the fluid like the kinematic viscosity. Also, the eddy viscosity is not constant like the kinematic viscosity for any fluid at any given temperature, but happens to vary widely over the flow field. In spite of these complications, the eddy viscosity is a useful concept which has been often used in studies on turbulence.

In the case of an open channel, the eddy viscosity and the kinematic viscosity play important roles in any region depending upon the location of that region with respect to the boundary. Very near the boundary, the eddy viscosity is small and we can consider

only the kinematic viscosity. On the other hand, near the free surface, the eddy viscosity is much more predominant. In between, there is a region where both the viscosities are of the same order. On this basis, we divide the region of flow in an open channel into three distinct parts:

- (1) Region of high viscosity near the bed (Lower region).
- (2) Region of low viscosity near the free surface (upper region).
- (3) Region where the kinematic and eddy viscosities are of the same order (intermediate region).

It is the object of this paper to study these three regions separately by using the fundamental equations of motion.

## EQUATIONS OF MOTION

We assume that the Navier-Stokes equation holds good for both the mean — and turbulent — velocity components. We also assume the fluid to be incompressible. To eliminate density from the equation, kinematic quantities are used (5).

The Navier-Stokes equations are given in the tensorial form by

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \nu \nabla^2 u_i \quad (2)$$

where repeated indices denote summation from 1 to 3. Both the indices  $i$  and  $j$  range from 1 to 3. The velocities  $u_i$  and  $u_j$  and the pressure  $p$  can be considered to be made up of the mean and the turbulent components. Let

$$\begin{aligned} u_i &= U_i + u'_i \\ u_j &= U_j + u'_j \end{aligned} \quad (3a, b, c)$$

and

$$p = P + p'$$

where  $U_i, U_j$  and  $P$  are the mean components, and  $u'_i, u'_j$  and  $p'$  are the turbulent components. Substituting this in equation (2), we have

$$\begin{aligned} \frac{\partial}{\partial t} (U_i + u'_i) + (U_j + u'_j) \frac{\partial}{\partial x_j} (U_i + u'_i) \\ = - \frac{\partial}{\partial x_i} (P + p') + \nu \nabla^2 (U_i + u'_i) \end{aligned} \quad (4)$$