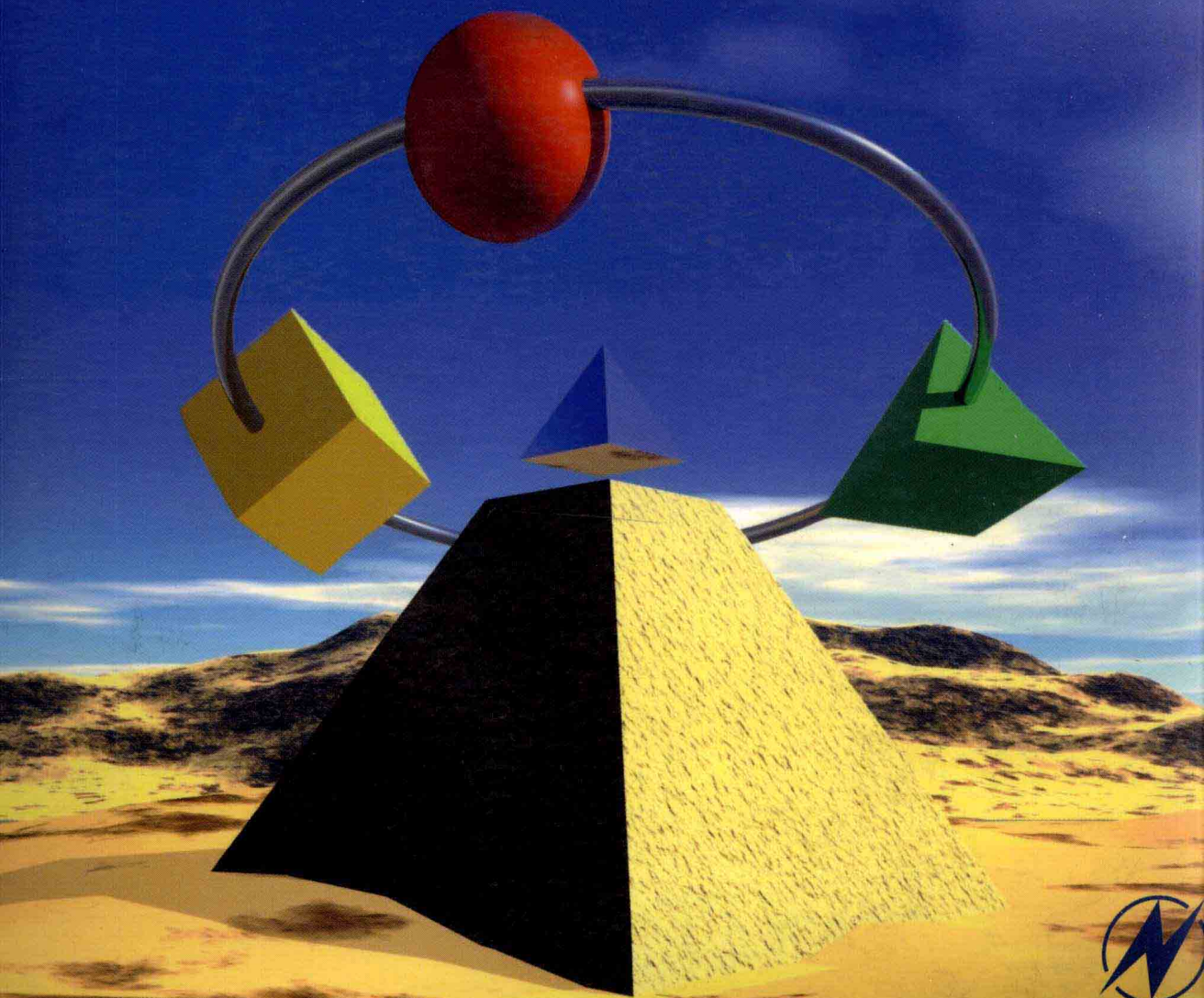


**4<sup>th</sup>**  
edition

# Engineering Mathematics

**John Bird**



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# Engineering Mathematics

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*In memory of Elizabeth*

# Preface

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This fourth edition of '*Engineering Mathematics*' covers a wide range of syllabus requirements. In particular, the book is most suitable for the latest **National Certificate and Diploma courses and Vocational Certificate of Education syllabuses in Engineering.**

This text will provide a foundation in mathematical principles, which will enable students to solve mathematical, scientific and associated engineering principles. In addition, the material will provide engineering applications and mathematical principles necessary for advancement onto a range of Incorporated Engineer degree profiles. It is widely recognised that a students' ability to use mathematics is a key element in determining subsequent success. First year undergraduates who need some remedial mathematics will also find this book meets their needs.

In *Engineering Mathematics 4<sup>th</sup> Edition*, theory is introduced in each chapter by a simple outline of essential definitions, formulae, laws and procedures. The theory is kept to a minimum, for problem solving is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

For clarity, the text is divided into **ten topic areas**, these being: number and algebra, mensuration, trigonometry, graphs, vectors, complex numbers, statistics, differential calculus, integral calculus and further number and algebra.

This new edition will cover the following syllabuses:

- (i) **Mathematics for Technicians**, the core unit for **National Certificate/Diploma** courses in Engineering, to include all or part of the following chapters:

1. **Algebra:** 2, 4, 5, 8–13, 17, 19, 27, 30
2. **Trigonometry:** 18, 21, 22, 24
3. **Statistics:** 36, 37
4. **Calculus:** 44, 46, 47, 54

- (ii) **Further Mathematics for Technicians**, the optional unit for **National Certificate/Diploma** courses in Engineering, to include all or part of the following chapters:

1. **Algebraic techniques:** 10, 14, 15, 28–30, 34, 59–61
2. **Trigonometry:** 22–24, 26
3. **Calculus:** 44–49, 52–58
4. **Statistical and probability:** 36–43

- (iii) **Applied Mathematics in Engineering**, the compulsory unit for **Advanced VCE** (formerly Advanced GNVQ), to include all or part of the following chapters:

1. **Number and units:** 1, 2, 4
2. **Mensuration:** 17–20
3. **Algebra:** 5, 8–11
4. **Functions and graphs:** 22, 23, 27
5. **Trigonometry:** 21, 24

- (iv) **Further Mathematics for Engineering**, the optional unit for **Advanced VCE** (formerly Advanced GNVQ), to include all or part of the following chapters:

1. **Algebra and trigonometry:** 5, 6, 12–15, 21, 25
2. **Graphical and numerical techniques:** 20, 22, 26–31
3. **Differential and integral calculus:** 44–47, 54

- (v) The Mathematics content of **Applied Science and Mathematics for Engineering**, for **Intermediate GNVQ**
- (vi) **Mathematics for Engineering**, for **Foundation and Intermediate GNVQ**
- (vii) **Mathematics 2 and Mathematics 3** for **City & Guilds Technician Diploma in Telecommunications and Electronic Engineering**
- (viii) Any **introductory/access/foundation course** involving Engineering Mathematics at University, Colleges of Further and Higher education and in schools.

Each topic considered in the text is presented in a way that assumes in the reader little previous knowledge of that topic.

'*Engineering Mathematics 4<sup>th</sup> Edition*' provides a follow-up to '*Basic Engineering Mathematics*' and a lead into '*Higher Engineering Mathematics*'.

This textbook contains over **900 worked problems**, followed by some **1700 further problems** (all **with answers**). The further problems are contained within some **208 Exercises**; each Exercise follows on directly from the relevant section of work, every two or three pages. In addition, the text contains **234 multiple-choice questions**. Where at all possible, the problems mirror practical situations found in engineering and science. **500 line diagrams** enhance the understanding of the theory.

At regular intervals throughout the text are some **16 Assignments** to check understanding. For example, Assignment 1 covers material contained in Chapters 1 to 4, Assignment 2 covers the material in Chapters 5 to 8, and so on. These Assignments do not have answers given since it is envisaged that

lecturers could set the Assignments for students to attempt as part of their course structure. Lecturers may obtain a complimentary set of solutions of the Assignments in an **Instructor's Manual** available from the publishers via the internet—full worked solutions and mark scheme for all the Assignments are contained in this Manual, which is available to lecturers only. To obtain a password please e-mail [j.blackford@elsevier.com](mailto:j.blackford@elsevier.com) with the following details: course title, number of students, your job title and work postal address.

To download the Instructor's Manual use the following direct URL: <http://books.elsevier.com/manualsprotected/0750657766/>

'**Learning by Example**' is at the heart of '*Engineering Mathematics 4<sup>th</sup> Edition*'.

**John Bird**

*University of Portsmouth*

# Contents

---

*Preface xi*

## **Part 1 Number and Algebra 1**

### **1 Revision of fractions, decimals and percentages 1**

- 1.1 Fractions 1
- 1.2 Ratio and proportion 3
- 1.3 Decimals 4
- 1.4 Percentages 7

### **2 Indices and standard form 9**

- 2.1 Indices 9
- 2.2 Worked problems on indices 9
- 2.3 Further worked problems on indices 11
- 2.4 Standard form 13
- 2.5 Worked problems on standard form 13
- 2.6 Further worked problems on standard form 14

### **3 Computer numbering systems 16**

- 3.1 Binary numbers 16
- 3.2 Conversion of binary to decimal 16
- 3.3 Conversion of decimal to binary 17
- 3.4 Conversion of decimal to binary via octal 18
- 3.5 Hexadecimal numbers 20

### **4 Calculations and evaluation of formulae 24**

- 4.1 Errors and approximations 24
- 4.2 Use of calculator 26
- 4.3 Conversion tables and charts 28
- 4.4 Evaluation of formulae 30

#### **Assignment 1 33**

### **5 Algebra 34**

- 5.1 Basic operations 34
- 5.2 Laws of Indices 36
- 5.3 Brackets and factorisation 38
- 5.4 Fundamental laws and precedence 40
- 5.5 Direct and inverse proportionality 42

### **6 Further algebra 44**

- 6.1 Polynomial division 44
- 6.2 The factor theorem 46
- 6.3 The remainder theorem 48

### **7 Partial fractions 51**

- 7.1 Introduction to partial fractions 51
- 7.2 Worked problems on partial fractions with linear factors 51
- 7.3 Worked problems on partial fractions with repeated linear factors 54
- 7.4 Worked problems on partial fractions with quadratic factors 55

### **8 Simple equations 57**

- 8.1 Expressions, equations and identities 57
- 8.2 Worked problems on simple equations 57
- 8.3 Further worked problems on simple equations 59
- 8.4 Practical problems involving simple equations 61
- 8.5 Further practical problems involving simple equations 62

#### **Assignment 2 64**

### **9 Simultaneous equations 65**

- 9.1 Introduction to simultaneous equations 65
- 9.2 Worked problems on simultaneous equations in two unknowns 65
- 9.3 Further worked problems on simultaneous equations 67
- 9.4 More difficult worked problems on simultaneous equations 69
- 9.5 Practical problems involving simultaneous equations 70

### **10 Transposition of formulae 74**

- 10.1 Introduction to transposition of formulae 74
- 10.2 Worked problems on transposition of formulae 74
- 10.3 Further worked problems on transposition of formulae 75
- 10.4 Harder worked problems on transposition of formulae 77

### **11 Quadratic equations 80**

- 11.1 Introduction to quadratic equations 80
- 11.2 Solution of quadratic equations by factorisation 80

- 11.3 Solution of quadratic equations by 'completing the square' 82
- 11.4 Solution of quadratic equations by formula 84
- 11.5 Practical problems involving quadratic equations 85
- 11.6 The solution of linear and quadratic equations simultaneously 87
- 12 Logarithms 89**
- 12.1 Introduction to logarithms 89
- 12.2 Laws of logarithms 89
- 12.3 Indicial equations 92
- 12.4 Graphs of logarithmic functions 93
- Assignment 3 94**
- 13 Exponential functions 95**
- 13.1 The exponential function 95
- 13.2 Evaluating exponential functions 95
- 13.3 The power series for  $e^x$  96
- 13.4 Graphs of exponential functions 98
- 13.5 Napierian logarithms 100
- 13.6 Evaluating Napierian logarithms 100
- 13.7 Laws of growth and decay 102
- 14 Number sequences 106**
- 14.1 Arithmetic progressions 106
- 14.2 Worked problems on arithmetic progression 106
- 14.3 Further worked problems on arithmetic progressions 107
- 14.4 Geometric progressions 109
- 14.5 Worked problems on geometric progressions 110
- 14.6 Further worked problems on geometric progressions 111
- 14.7 Combinations and permutations 112
- 15 The binomial series 114**
- 15.1 Pascal's triangle 114
- 15.2 The binomial series 115
- 15.3 Worked problems on the binomial series 115
- 15.4 Further worked problems on the binomial series 117
- 15.5 Practical problems involving the binomial theorem 120
- 16 Solving equations by iterative methods 123**
- 16.1 Introduction to iterative methods 123
- 16.2 The Newton–Raphson method 123
- 16.3 Worked problems on the Newton–Raphson method 123
- Assignment 4 126**
- Multiple choice questions on chapters 1 to 16 127**
- Part 2 Mensuration 131**
- 17 Areas of plane figures 131**
- 17.1 Mensuration 131
- 17.2 Properties of quadrilaterals 131
- 17.3 Worked problems on areas of plane figures 132
- 17.4 Further worked problems on areas of plane figures 135
- 17.5 Worked problems on areas of composite figures 137
- 17.6 Areas of similar shapes 138
- 18 The circle and its properties 139**
- 18.1 Introduction 139
- 18.2 Properties of circles 139
- 18.3 Arc length and area of a sector 140
- 18.4 Worked problems on arc length and sector of a circle 141
- 18.5 The equation of a circle 143
- 19 Volumes and surface areas of common solids 145**
- 19.1 Volumes and surface areas of regular solids 145
- 19.2 Worked problems on volumes and surface areas of regular solids 145
- 19.3 Further worked problems on volumes and surface areas of regular solids 147
- 19.4 Volumes and surface areas of frusta of pyramids and cones 151
- 19.5 The frustum and zone of a sphere 155
- 19.6 Prismoidal rule 157
- 19.7 Volumes of similar shapes 159
- 20 Irregular areas and volumes and mean values of waveforms 161**
- 20.1 Areas of irregular figures 161
- 20.2 Volumes of irregular solids 163
- 20.3 The mean or average value of a waveform 164
- Assignment 5 168**
- Part 3 Trigonometry 171**
- 21 Introduction to trigonometry 171**
- 21.1 Trigonometry 171
- 21.2 The theorem of Pythagoras 171
- 21.3 Trigonometric ratios of acute angles 172



- 21.4 Fractional and surd forms of trigonometric ratios 174
- 21.5 Solution of right-angled triangles 175
- 21.6 Angles of elevation and depression 176
- 21.7 Evaluating trigonometric ratios of any angles 178
- 21.8 Trigonometric approximations for small angles 181

## 22 Trigonometric waveforms 182

- 22.1 Graphs of trigonometric functions 182
- 22.2 Angles of any magnitude 182
- 22.3 The production of a sine and cosine wave 185
- 22.4 Sine and cosine curves 185
- 22.5 Sinusoidal form  $A \sin(\omega t \pm \alpha)$  189
- 22.6 Waveform harmonics 192

## 23 Cartesian and polar co-ordinates 194

- 23.1 Introduction 194
- 23.2 Changing from Cartesian into polar co-ordinates 194
- 23.3 Changing from polar into Cartesian co-ordinates 196
- 23.4 Use of  $R \rightarrow P$  and  $P \rightarrow R$  functions on calculators 197

### Assignment 6 198

## 24 Triangles and some practical applications 199

- 24.1 Sine and cosine rules 199
- 24.2 Area of any triangle 199
- 24.3 Worked problems on the solution of triangles and their areas 199
- 24.4 Further worked problems on the solution of triangles and their areas 201
- 24.5 Practical situations involving trigonometry 203
- 24.6 Further practical situations involving trigonometry 205

## 25 Trigonometric identities and equations 208

- 25.1 Trigonometric identities 208
- 25.2 Worked problems on trigonometric identities 208
- 25.3 Trigonometric equations 209
- 25.4 Worked problems (i) on trigonometric equations 210
- 25.5 Worked problems (ii) on trigonometric equations 211
- 25.6 Worked problems (iii) on trigonometric equations 212

- 25.7 Worked problems (iv) on trigonometric equations 212

## 26 Compound angles 214

- 26.1 Compound angle formulae 214
- 26.2 Conversion of  $a \sin \omega t + b \cos \omega t$  into  $R \sin(\omega t + \alpha)$  216
- 26.3 Double angles 220
- 26.4 Changing products of sines and cosines into sums or differences 221
- 26.5 Changing sums or differences of sines and cosines into products 222

### Assignment 7 224

## Multiple choice questions on chapters 17 to 26 225

## Part 4 Graphs 231

### 27 Straight line graphs 231

- 27.1 Introduction to graphs 231
- 27.2 The straight line graph 231
- 27.3 Practical problems involving straight line graphs 237

### 28 Reduction of non-linear laws to linear form 243

- 28.1 Determination of law 243
- 28.2 Determination of law involving logarithms 246

### 29 Graphs with logarithmic scales 251

- 29.1 Logarithmic scales 251
- 29.2 Graphs of the form  $y = ax^n$  251
- 29.3 Graphs of the form  $y = ab^x$  254
- 29.4 Graphs of the form  $y = ae^{kx}$  255

### 30 Graphical solution of equations 258

- 30.1 Graphical solution of simultaneous equations 258
- 30.2 Graphical solution of quadratic equations 259
- 30.3 Graphical solution of linear and quadratic equations simultaneously 263
- 30.4 Graphical solution of cubic equations 264

### 31 Functions and their curves 266

- 31.1 Standard curves 266
- 31.2 Simple transformations 268
- 31.3 Periodic functions 273
- 31.4 Continuous and discontinuous functions 273
- 31.5 Even and odd functions 273
- 31.6 Inverse functions 275

### Assignment 8 279

**Part 5 Vectors 281****32 Vectors 281**

- 32.1 Introduction 281
- 32.2 Vector addition 281
- 32.3 Resolution of vectors 283
- 32.4 Vector subtraction 284

**33 Combination of waveforms 287**

- 33.1 Combination of two periodic functions 287
- 33.2 Plotting periodic functions 287
- 33.3 Determining resultant phasors by calculation 288

**Part 6 Complex Numbers 291****34 Complex numbers 291**

- 34.1 Cartesian complex numbers 291
- 34.2 The Argand diagram 292
- 34.3 Addition and subtraction of complex numbers 292
- 34.4 Multiplication and division of complex numbers 293
- 34.5 Complex equations 295
- 34.6 The polar form of a complex number 296
- 34.7 Multiplication and division in polar form 298
- 34.8 Applications of complex numbers 299

**35 De Moivre's theorem 303**

- 35.1 Introduction 303
- 35.2 Powers of complex numbers 303
- 35.3 Roots of complex numbers 304

**Assignment 9 306****Part 7 Statistics 307****36 Presentation of statistical data 307**

- 36.1 Some statistical terminology 307
- 36.2 Presentation of ungrouped data 308
- 36.3 Presentation of grouped data 312

**37 Measures of central tendency and dispersion 319**

- 37.1 Measures of central tendency 319
- 37.2 Mean, median and mode for discrete data 319
- 37.3 Mean, median and mode for grouped data 320
- 37.4 Standard deviation 322
- 37.5 Quartiles, deciles and percentiles 324

**38 Probability 326**

- 38.1 Introduction to probability 326
- 38.2 Laws of probability 326

- 38.3 Worked problems on probability 327
- 38.4 Further worked problems on probability 329
- 38.5 Permutations and combinations 331

**39 The binomial and Poisson distribution 333**

- 39.1 The binomial distribution 333
- 39.2 The Poisson distribution 336

**Assignment 10 339****40 The normal distribution 340**

- 40.1 Introduction to the normal distribution 340
- 40.2 Testing for a normal distribution 344

**41 Linear correlation 347**

- 41.1 Introduction to linear correlation 347
- 41.2 The product-moment formula for determining the linear correlation coefficient 347
- 41.3 The significance of a coefficient of correlation 348
- 41.4 Worked problems on linear correlation 348

**42 Linear regression 351**

- 42.1 Introduction to linear regression 351
- 42.2 The least-squares regression lines 351
- 42.3 Worked problems on linear regression 352

**43 Sampling and estimation theories 356**

- 43.1 Introduction 356
- 43.2 Sampling distributions 356
- 43.3 The sampling distribution of the means 356
- 43.4 The estimation of population parameters based on a large sample size 359
- 43.5 Estimating the mean of a population based on a small sample size 364

**Assignment 11 368****Multiple choice questions on chapters 27 to 43 369****Part 8 Differential Calculus 375****44 Introduction to differentiation 375**

- 44.1 Introduction to calculus 375
- 44.2 Functional notation 375
- 44.3 The gradient of a curve 376
- 44.4 Differentiation from first principles 377

- 44.5 Differentiation of  $y = ax^n$  by the general rule 379
- 44.6 Differentiation of sine and cosine functions 380
- 44.7 Differentiation of  $e^{ax}$  and  $\ln ax$  382
- 45 Methods of differentiation 384**
- 45.1 Differentiation of common functions 384
- 45.2 Differentiation of a product 386
- 45.3 Differentiation of a quotient 387
- 45.4 Function of a function 389
- 45.5 Successive differentiation 390
- 46 Some applications of differentiation 392**
- 46.1 Rates of change 392
- 46.2 Velocity and acceleration 393
- 46.3 Turning points 396
- 46.4 Practical problems involving maximum and minimum values 399
- 46.5 Tangents and normals 403
- 46.6 Small changes 404
- Assignment 12 406**
- Part 9 Integral Calculus 407**
- 47 Standard integration 407**
- 47.1 The process of integration 407
- 47.2 The general solution of integrals of the form  $ax^n$  407
- 47.3 Standard integrals 408
- 47.4 Definite integrals 411
- 48 Integration using algebraic substitutions 414**
- 48.1 Introduction 414
- 48.2 Algebraic substitutions 414
- 48.3 Worked problems on integration using algebraic substitutions 414
- 48.4 Further worked problems on integration using algebraic substitutions 416
- 48.5 Change of limits 416
- 49 Integration using trigonometric substitutions 418**
- 49.1 Introduction 418
- 49.2 Worked problems on integration of  $\sin^2 x$ ,  $\cos^2 x$ ,  $\tan^2 x$  and  $\cot^2 x$  418
- 49.3 Worked problems on powers of sines and cosines 420
- 49.4 Worked problems on integration of products of sines and cosines 421
- 49.5 Worked problems on integration using the  $\sin \theta$  substitution 422
- 49.6 Worked problems on integration using the  $\tan \theta$  substitution 424
- Assignment 13 425**
- 50 Integration using partial fractions 426**
- 50.1 Introduction 426
- 50.2 Worked problems on integration using partial fractions with linear factors 426
- 50.3 Worked problems on integration using partial fractions with repeated linear factors 427
- 50.4 Worked problems on integration using partial fractions with quadratic factors 428
- 51 The  $t = \frac{\theta}{2}$  substitution 430**
- 51.1 Introduction 430
- 51.2 Worked problems on the  $t = \tan \frac{\theta}{2}$  substitution 430
- 51.3 Further worked problems on the  $t = \tan \frac{\theta}{2}$  substitution 432
- 52 Integration by parts 434**
- 52.1 Introduction 434
- 52.2 Worked problems on integration by parts 434
- 52.3 Further worked problems on integration by parts 436
- 53 Numerical integration 439**
- 53.1 Introduction 439
- 53.2 The trapezoidal rule 439
- 53.3 The mid-ordinate rule 441
- 53.4 Simpson's rule 443
- Assignment 14 447**
- 54 Areas under and between curves 448**
- 54.1 Area under a curve 448
- 54.2 Worked problems on the area under a curve 449
- 54.3 Further worked problems on the area under a curve 452
- 54.4 The area between curves 454
- 55 Mean and root mean square values 457**
- 55.1 Mean or average values 457
- 55.2 Root mean square values 459
- 56 Volumes of solids of revolution 461**
- 56.1 Introduction 461
- 56.2 Worked problems on volumes of solids of revolution 461

- 56.3 Further worked problems on volumes of solids of revolution 463
- 57 Centroids of simple shapes 466**
- 57.1 Centroids 466
- 57.2 The first moment of area 466
- 57.3 Centroid of area between a curve and the  $x$ -axis 466
- 57.4 Centroid of area between a curve and the  $y$ -axis 467
- 57.5 Worked problems on centroids of simple shapes 467
- 57.6 Further worked problems on centroids of simple shapes 468
- 57.7 Theorem of Pappus 471
- 58 Second moments of area 475**
- 58.1 Second moments of area and radius of gyration 475
- 58.2 Second moment of area of regular sections 475
- 58.3 Parallel axis theorem 475
- 58.4 Perpendicular axis theorem 476
- 58.5 Summary of derived results 476
- 58.6 Worked problems on second moments of area of regular sections 476
- 58.7 Worked problems on second moments of areas of composite areas 480
- Assignment 15 482**
- Part 10 Further Number and Algebra 483**
- 59 Boolean algebra and logic circuits 483**
- 59.1 Boolean algebra and switching circuits 483
- 59.2 Simplifying Boolean expressions 488
- 59.3 Laws and rules of Boolean algebra 488
- 59.4 De Morgan's laws 490
- 59.5 Karnaugh maps 491
- 59.6 Logic circuits 495
- 59.7 Universal logic circuits 500
- 60 The theory of matrices and determinants 504**
- 60.1 Matrix notation 504
- 60.2 Addition, subtraction and multiplication of matrices 504
- 60.3 The unit matrix 508
- 60.4 The determinant of a 2 by 2 matrix 508
- 60.5 The inverse or reciprocal of a 2 by 2 matrix 509
- 60.6 The determinant of a 3 by 3 matrix 510
- 60.7 The inverse or reciprocal of a 3 by 3 matrix 511
- 61 The solution of simultaneous equations by matrices and determinants 514**
- 61.1 Solution of simultaneous equations by matrices 514
- 61.2 Solution of simultaneous equations by determinants 516
- 61.3 Solution of simultaneous equations using Cramers rule 520
- Assignment 16 521**
- Multiple choice questions on chapters 44–61 522**
- Answers to multiple choice questions 526**
- Index 527**

# Part 1 Number and Algebra

## 1

### Revision of fractions, decimals and percentages

#### 1.1 Fractions

When 2 is divided by 3, it may be written as  $\frac{2}{3}$  or  $2/3$ .  $\frac{2}{3}$  is called a **fraction**. The number above the line, i.e. 2, is called the **numerator** and the number below the line, i.e. 3, is called the **denominator**.

When the value of the numerator is less than the value of the denominator, the fraction is called a **proper fraction**; thus  $\frac{2}{3}$  is a proper fraction. When the value of the numerator is greater than the denominator, the fraction is called an **improper fraction**. Thus  $\frac{7}{3}$  is an improper fraction and can also be expressed as a **mixed number**, that is, an integer and a proper fraction. Thus the improper fraction  $\frac{7}{3}$  is equal to the mixed number  $2\frac{1}{3}$ .

When a fraction is simplified by dividing the numerator and denominator by the same number, the process is called **cancelling**. Cancelling by 0 is not permissible.

Problem 1. Simplify  $\frac{1}{3} + \frac{2}{7}$

The lowest common multiple (i.e. LCM) of the two denominators is  $3 \times 7$ , i.e. 21

Expressing each fraction so that their denominators are 21, gives:

$$\begin{aligned}\frac{1}{3} + \frac{2}{7} &= \frac{1}{3} \times \frac{7}{7} + \frac{2}{7} \times \frac{3}{3} = \frac{7}{21} + \frac{6}{21} \\ &= \frac{7+6}{21} = \frac{13}{21}\end{aligned}$$

Alternatively:

$$\frac{1}{3} + \frac{2}{7} = \frac{\begin{array}{c} \text{Step (2)} \\ \downarrow \\ (7 \times 1) \end{array} + \begin{array}{c} \text{Step (3)} \\ \downarrow \\ (3 \times 2) \end{array}}{\begin{array}{c} 21 \\ \uparrow \\ \text{Step (1)} \end{array}}$$

Step 1: the LCM of the two denominators;

Step 2: for the fraction  $\frac{1}{3}$ , 3 into 21 goes 7 times,  $7 \times$  the numerator is  $7 \times 1$ ;

Step 3: for the fraction  $\frac{2}{7}$ , 7 into 21 goes 3 times,  $3 \times$  the numerator is  $3 \times 2$ .

Thus  $\frac{1}{3} + \frac{2}{7} = \frac{7+6}{21} = \frac{13}{21}$  as obtained previously.

Problem 2. Find the value of  $3\frac{2}{3} - 2\frac{1}{6}$

One method is to split the mixed numbers into integers and their fractional parts. Then

$$\begin{aligned}3\frac{2}{3} - 2\frac{1}{6} &= \left(3 + \frac{2}{3}\right) - \left(2 + \frac{1}{6}\right) \\ &= 3 + \frac{2}{3} - 2 - \frac{1}{6} \\ &= 1 + \frac{4}{6} - \frac{1}{6} = 1\frac{3}{6} = 1\frac{1}{2}\end{aligned}$$

Another method is to express the mixed numbers as improper fractions.

$$\text{Since } 3 = \frac{9}{3}, \text{ then } 3\frac{2}{3} = \frac{9}{3} + \frac{2}{3} = \frac{11}{3}$$

$$\text{Similarly, } 2\frac{1}{6} = \frac{12}{6} + \frac{1}{6} = \frac{13}{6}$$

$$\text{Thus } 3\frac{2}{3} - 2\frac{1}{6} = \frac{11}{3} - \frac{13}{6} = \frac{22}{6} - \frac{13}{6} = \frac{9}{6} = \frac{1}{2}$$

as obtained previously.

**Problem 3.** Determine the value of

$$4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5}$$

$$\begin{aligned} 4\frac{5}{8} - 3\frac{1}{4} + 1\frac{2}{5} &= (4 - 3 + 1) + \left(\frac{5}{8} - \frac{1}{4} + \frac{2}{5}\right) \\ &= 2 + \frac{5 \times 5 - 10 \times 1 + 8 \times 2}{40} \\ &= 2 + \frac{25 - 10 + 16}{40} \\ &= 2 + \frac{31}{40} = 2\frac{31}{40} \end{aligned}$$

**Problem 4.** Find the value of  $\frac{3}{7} \times \frac{14}{15}$

Dividing numerator and denominator by 3 gives:

$$\frac{1\cancel{3}}{7} \times \frac{14}{\cancel{15}_5} = \frac{1}{7} \times \frac{14}{5} = \frac{1 \times 14}{7 \times 5}$$

Dividing numerator and denominator by 7 gives:

$$\frac{1 \times \cancel{14}^2}{\cancel{7}_1 \times 5} = \frac{1 \times 2}{1 \times 5} = \frac{2}{5}$$

This process of dividing both the numerator and denominator of a fraction by the same factor(s) is called **cancelling**.

**Problem 5.** Evaluate  $1\frac{3}{5} \times 2\frac{1}{3} \times 3\frac{3}{7}$

Mixed numbers **must** be expressed as improper fractions before multiplication can be performed. Thus,

$$\begin{aligned} 1\frac{3}{5} \times 2\frac{1}{3} \times 3\frac{3}{7} \\ = \left(\frac{5}{5} + \frac{3}{5}\right) \times \left(\frac{6}{3} + \frac{1}{3}\right) \times \left(\frac{21}{7} + \frac{3}{7}\right) \end{aligned}$$

$$\begin{aligned} &= \frac{8}{5} \times \frac{1\cancel{7}}{\cancel{3}} \times \frac{\cancel{24}^8}{\cancel{7}_1} = \frac{8 \times 1 \times 8}{5 \times 1 \times 1} \\ &= \frac{64}{5} = 12\frac{4}{5} \end{aligned}$$

**Problem 6.** Simplify  $\frac{3}{7} \div \frac{12}{21}$

$$\frac{3}{7} \div \frac{12}{21} = \frac{3}{7} \times \frac{21}{12}$$

Multiplying both numerator and denominator by the reciprocal of the denominator gives:

$$\frac{3}{7} \times \frac{21}{12} = \frac{3}{7} \times \frac{\cancel{21}^3}{\cancel{12}_4} = \frac{3}{4}$$

This method can be remembered by the rule: invert the second fraction and change the operation from division to multiplication. Thus:

$$\frac{3}{7} \div \frac{12}{21} = \frac{3}{7} \times \frac{21}{12} = \frac{3}{4} \text{ as obtained previously.}$$

**Problem 7.** Find the value of  $5\frac{3}{5} \div 7\frac{1}{3}$

The mixed numbers must be expressed as improper fractions. Thus,

$$5\frac{3}{5} \div 7\frac{1}{3} = \frac{28}{5} \div \frac{22}{3} = \frac{28}{5} \times \frac{3}{22} = \frac{42}{55}$$

**Problem 8.** Simplify

$$\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$$

The order of precedence of operations for problems containing fractions is the same as that for integers, i.e. remembered by **BODMAS** (Brackets, Of, Division, Multiplication, Addition and Subtraction). Thus,

$$\frac{1}{3} - \left(\frac{2}{5} + \frac{1}{4}\right) \div \left(\frac{3}{8} \times \frac{1}{3}\right)$$

$$= \frac{1}{3} - \frac{4 \times 2 + 5 \times 1}{20} \div \frac{8^1}{24_8} \quad (\text{B})$$

$$= \frac{1}{3} - \frac{13}{20} \times \frac{8^2}{1} \quad (\text{D})$$

$$= \frac{1}{3} - \frac{26}{5} \quad (\text{M})$$

$$= \frac{(5 \times 1) - (3 \times 26)}{15} \quad (\text{S})$$

$$= \frac{-73}{15} = -4\frac{13}{15}$$

**Problem 9.** Determine the value of

$$\frac{7}{6} \text{ of } \left(3\frac{1}{2} - 2\frac{1}{4}\right) + 5\frac{1}{8} \div \frac{3}{16} - \frac{1}{2}$$

$$\frac{7}{6} \text{ of } \left(3\frac{1}{2} - 2\frac{1}{4}\right) + 5\frac{1}{8} \div \frac{3}{16} - \frac{1}{2}$$

$$= \frac{7}{6} \text{ of } 1\frac{1}{4} + \frac{41}{8} \div \frac{3}{16} - \frac{1}{2} \quad (\text{B})$$

$$= \frac{7}{6} \times \frac{5}{4} + \frac{41}{8} \div \frac{3}{16} - \frac{1}{2} \quad (\text{O})$$

$$= \frac{7}{6} \times \frac{5}{4} + \frac{41}{8} \times \frac{16^2}{3} - \frac{1}{2} \quad (\text{D})$$

$$= \frac{35}{24} + \frac{82}{3} - \frac{1}{2} \quad (\text{M})$$

$$= \frac{35 + 656}{24} - \frac{1}{2} \quad (\text{A})$$

$$= \frac{691}{24} - \frac{1}{2} \quad (\text{A})$$

$$= \frac{691 - 12}{24} \quad (\text{S})$$

$$= \frac{679}{24} = 28\frac{7}{24}$$

Now try the following exercise

### Exercise 1 Further problems on fractions

Evaluate the following:

1. (a)  $\frac{1}{2} + \frac{2}{5}$  (b)  $\frac{7}{16} - \frac{1}{4}$

$$\left[ \text{(a) } \frac{9}{10} \quad \text{(b) } \frac{3}{16} \right]$$

2. (a)  $\frac{2}{7} + \frac{3}{11}$  (b)  $\frac{2}{9} - \frac{1}{7} + \frac{2}{3}$

$$\left[ \text{(a) } \frac{43}{77} \quad \text{(b) } \frac{47}{63} \right]$$

3. (a)  $10\frac{3}{7} - 8\frac{2}{3}$  (b)  $3\frac{1}{4} - 4\frac{4}{5} + 1\frac{5}{6}$

$$\left[ \text{(a) } 1\frac{16}{21} \quad \text{(b) } \frac{17}{60} \right]$$

4. (a)  $\frac{3}{4} \times \frac{5}{9}$  (b)  $\frac{17}{35} \times \frac{15}{119}$

$$\left[ \text{(a) } \frac{5}{12} \quad \text{(b) } \frac{3}{49} \right]$$

5. (a)  $\frac{3}{5} \times \frac{7}{9} \times 1\frac{2}{7}$  (b)  $\frac{13}{17} \times 4\frac{7}{11} \times 3\frac{4}{39}$

$$\left[ \text{(a) } \frac{3}{5} \quad \text{(b) } 11 \right]$$

6. (a)  $\frac{3}{8} \div \frac{45}{64}$  (b)  $1\frac{1}{3} \div 2\frac{5}{9}$

$$\left[ \text{(a) } \frac{8}{15} \quad \text{(b) } \frac{12}{23} \right]$$

7.  $\frac{1}{2} + \frac{3}{5} \div \frac{8}{15} - \frac{1}{3}$   $\left[ 1\frac{7}{24} \right]$

8.  $\frac{7}{15} \text{ of } \left(15 \times \frac{5}{7}\right) + \left(\frac{3}{4} \div \frac{15}{16}\right)$   $\left[ 5\frac{4}{5} \right]$

9.  $\frac{1}{4} \times \frac{2}{3} - \frac{1}{3} \div \frac{3}{5} + \frac{2}{7}$   $\left[ -\frac{13}{126} \right]$

10.  $\left(\frac{2}{3} \times 1\frac{1}{4}\right) \div \left(\frac{2}{3} + \frac{1}{4}\right) + 1\frac{3}{5}$   $\left[ 2\frac{28}{55} \right]$

## 1.2 Ratio and proportion

The ratio of one quantity to another is a fraction, and is the number of times one quantity is contained in another quantity **of the same kind**. If one quantity is **directly proportional** to another, then as one quantity doubles, the other quantity also doubles. When a quantity is **inversely proportional** to another, then as one quantity doubles, the other quantity is halved.

**Problem 10.** A piece of timber 273 cm long is cut into three pieces in the ratio of 3 to 7 to 11. Determine the lengths of the three pieces

The total number of parts is  $3 + 7 + 11$ , that is, 21.  
Hence 21 parts correspond to 273 cm

$$1 \text{ part corresponds to } \frac{273}{21} = 13 \text{ cm}$$

$$3 \text{ parts correspond to } 3 \times 13 = 39 \text{ cm}$$

$$7 \text{ parts correspond to } 7 \times 13 = 91 \text{ cm}$$

$$11 \text{ parts correspond to } 11 \times 13 = 143 \text{ cm}$$

i.e. **the lengths of the three pieces are 39 cm, 91 cm and 143 cm.**

(Check:  $39 + 91 + 143 = 273$ )

**Problem 11.** A gear wheel having 80 teeth is in mesh with a 25 tooth gear. What is the gear ratio?

$$\text{Gear ratio} = 80:25 = \frac{80}{25} = \frac{16}{5} = 3.2$$

i.e. gear ratio = **16 : 5** or **3.2 : 1**

**Problem 12.** An alloy is made up of metals A and B in the ratio 2.5 : 1 by mass. How much of A has to be added to 6 kg of B to make the alloy?

$$\text{Ratio A : B} = 2.5 : 1 \text{ (i.e. A is to B as 2.5 is to 1)}$$

$$\text{or } \frac{A}{B} = \frac{2.5}{1} = 2.5$$

$$\text{When } B = 6 \text{ kg, } \frac{A}{6} = 2.5 \text{ from which,}$$

$$A = 6 \times 2.5 = \mathbf{15 \text{ kg}}$$

**Problem 13.** If 3 people can complete a task in 4 hours, how long will it take 5 people to complete the same task, assuming the rate of work remains constant

The more the number of people, the more quickly the task is done, hence inverse proportion exists.

3 people complete the task in 4 hours,

1 person takes three times as long, i.e.

$$4 \times 3 = 12 \text{ hours,}$$

5 people can do it in one fifth of the time that one person takes, that is  $\frac{12}{5}$  hours or **2 hours 24 minutes.**

**Now try the following exercise**

### Exercise 5 Further problems on ratio and proportion

1. Divide 621 cm in the ratio of 3 to 7 to 13.  
[81 cm to 189 cm to 351 cm]
2. When mixing a quantity of paints, dyes of four different colours are used in the ratio of 7:3:19:5. If the mass of the first dye used is  $3\frac{1}{2}$  g, determine the total mass of the dyes used. [17 g]
3. Determine how much copper and how much zinc is needed to make a 99 kg brass ingot if they have to be in the proportions copper : zinc : 8 : 3 by mass.  
[72 kg : 27 kg]
4. It takes 21 hours for 12 men to resurface a stretch of road. Find how many men it takes to resurface a similar stretch of road in 50 hours 24 minutes, assuming the work rate remains constant. [5]
5. It takes 3 hours 15 minutes to fly from city A to city B at a constant speed. Find how long the journey takes if
  - (a) the speed is  $1\frac{1}{2}$  times that of the original speed and
  - (b) if the speed is three-quarters of the original speed.
 [(a) 2 h 10 min (b) 4 h 20 min]

## 1.3 Decimals

The decimal system of numbers is based on the **digits** 0 to 9. A number such as 53.17 is called a **decimal fraction**, a decimal point separating the integer part, i.e. 53, from the fractional part, i.e. 0.17



A number which can be expressed exactly as a decimal fraction is called a **terminating decimal** and those which cannot be expressed exactly as a decimal fraction are called **non-terminating decimals**. Thus,  $\frac{3}{2} = 1.5$  is a terminating decimal, but  $\frac{4}{3} = 1.33333\dots$  is a non-terminating decimal.  $1.33333\dots$  can be written as  $1.\dot{3}$ , called 'one point-three recurring'.

The answer to a non-terminating decimal may be expressed in two ways, depending on the accuracy required:

- (i) correct to a number of **significant figures**, that is, figures which signify something, and
- (ii) correct to a number of **decimal places**, that is, the number of figures after the decimal point.

The last digit in the answer is unaltered if the next digit on the right is in the group of numbers 0, 1, 2, 3 or 4, but is increased by 1 if the next digit on the right is in the group of numbers 5, 6, 7, 8 or 9. Thus the non-terminating decimal  $7.6183\dots$  becomes 7.62, correct to 3 significant figures, since the next digit on the right is 8, which is in the group of numbers 5, 6, 7, 8 or 9. Also  $7.6183\dots$  becomes 7.618, correct to 3 decimal places, since the next digit on the right is 3, which is in the group of numbers 0, 1, 2, 3 or 4.

**Problem 14.** Evaluate  
 $42.7 + 3.04 + 8.7 + 0.06$

The numbers are written so that the decimal points are under each other. Each column is added, starting from the right.

$$\begin{array}{r} 42.7 \\ 3.04 \\ 8.7 \\ 0.06 \\ \hline 54.50 \end{array}$$

Thus  $42.7 + 3.04 + 8.7 + 0.06 = 54.50$

**Problem 15.** Take 81.70 from 87.23

The numbers are written with the decimal points under each other.

$$\begin{array}{r} 87.23 \\ -81.70 \\ \hline 5.53 \end{array}$$

Thus  $87.23 - 81.70 = 5.53$

**Problem 16.** Find the value of  
 $23.4 - 17.83 - 57.6 + 32.68$

The sum of the positive decimal fractions is

$$23.4 + 32.68 = 56.08$$

The sum of the negative decimal fractions is

$$17.83 + 57.6 = 75.43$$

Taking the sum of the negative decimal fractions from the sum of the positive decimal fractions gives:

$$56.08 - 75.43$$

$$\text{i.e. } -(75.43 - 56.08) = -19.35$$

**Problem 17.** Determine the value of  
 $74.3 \times 3.8$

When multiplying decimal fractions: (i) the numbers are multiplied as if they are integers, and (ii) the position of the decimal point in the answer is such that there are as many digits to the right of it as the sum of the digits to the right of the decimal points of the two numbers being multiplied together. Thus

$$\begin{array}{r} \text{(i)} \quad 743 \\ \quad 38 \\ \hline 5944 \\ 22290 \\ \hline 28234 \end{array}$$

- (ii) As there are  $(1 + 1) = 2$  digits to the right of the decimal points of the two numbers being multiplied together, ( $74.\underline{3} \times 3.\underline{8}$ ), then

$$74.3 \times 3.8 = 282.34$$

**Problem 18.** Evaluate  $37.81 \div 1.7$ , correct to (i) 4 significant figures and (ii) 4 decimal places