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Multiple Criteria Decision Making Theory and Application

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PREFACE

The objective of the Third International Conference on Multiple Criteria Decision Making was to enable an exchange of recent ideas and experiences in this domain regarding theory and practice. After the earlier two conferences, organized by H. Thiriez and S. Zionts 1975 in Jouy-en-Josas, France, and by S. Zionts 1977 in Buffalo NY, USA, with this conference something like a tradition has been generated, which will possibly be continued in 1980 by Joel Morse in Newark/ Delaware, USA.

The past conferences were meetings of a high intensity, because in all three cases the group of the invited participants were kept quite small - 60 persons on an average. At this last conference in Hagen and Königswinter 37 papers were presented, distributed over 6 sections. With respect to the young history of the theory of multicriteria decision making it was no wonder, that the two equally large sections Theoretical Concepts and Methods with altogether 24 contributions were represented very strongly in comparison with the other 4 sections. But nevertheless 7 papers were devoted to the section Economic Applications, and the section of recently developed Group Decision Concepts showed already 3 contributions.

Bertil Tell and Jyrki Wallenius helped us preparing the conference in the organization committee. We appreciate the time and effort they devoted to travelling, reading the submitted papers and organizing, and we thank them very much for coworking.

The conference could not had been realized without the generous financial support we received from our Sponsors. For this we want to express our deep gratitude to the German Research Foundation, the Ministry of Science and Research of the State Nordrhein-Westfalen, the office of Naval Research, which was represented during the whole conference by Herbert Solomon, and the Fernuniversität, the administration staff of which has been very helpful for us.

A great help for us was also the assistance of our secretaries, Mrs. Krafczyk and Mrs. Olek, and our co-workers, especially Dr. H. Gehring, A. Prasiswa, B. Vogeler, A. Kruse und H.-J. Kruse.

Of course, the participants themselves supported us realizing the conference by their activities. Each of them was acting as discussant and some did the work of session-chairmen. We appreciate this and thank the speakers for their well prepared and interesting contributions and their efforts typing the papers for the Proceedings. The list of participants is found on page 567 and the conference program on page 559. The contributions are ordered alphabetically by the name of the first author.

Hagen, November 1979

Günter Fandel
Tomas Gal

PERSPECTIVES OF THE DEVELOPMENT IN MULTIPLE CRITERIA DECISION MAKING

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The fact that business administration is concerned with decision problems with multiple objectives may be attributed to the knowledge gained from numerous empirical investigations that, generally, the goal catalogue of the enterprise is in reality only inadequately described merely by profit maximization. Thus, for example in the end of the fifties in the USA KAPLAN, DIRLAM and LANZILOTTI 1) could state in their study of the objectives relating to the price policy of American companies that reaching an adequate profitability, stabilizing prices and profit margins, securing and improving the share of the market as well as adapting to the competitive behaviour of the competitors were the most important features of company policy. For none of the firms concerned merely one of these components prevailed. In a similar way BAUMOL 2) arrived at the opinion that oligopolistic companies in particular will prefer the increase in sales to profit maximization. In the German literature in the beginning of the sixties many authors have considerably questioned the undue restriction of the management goals to profit maximization, at the same time commenting in detail on the goal system of management. At an earlier date already GUTENBERG 3) had pointed out that industrial firms mostly consider several aspects with regard to replacing capital assets and increasing capital investments. Taking his empirical investigations about the goal catalogues of Danish firms as a basis JOHNSEN 4) presented a detailed study in multiobjective decision models in the end of the sixties.

The more analytic interest in the optimal solution of decision problems with multiple objectives has primarily been aroused by CHARNES and COOPER. The latter have stated that problems of this kind are formally

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1) See: KAPLAN, A., DIRLAM, J. and LANZILOTTI, R.: Pricing in Big Business, Washington 1958, p. 128.

2) See: BAUMOL, J.: Business Behavior, Value and Growth, New York 1959, p. 45 ff.

3) See: GUTENBERG, E.: Untersuchungen über die Investitionsentscheidungen industrieller Unternehmungen, Köln und Opladen 1959, p. 220 ff.

4) See: JOHNSEN, E.: Studies in Multiobjective Decision Models, Lund 1968.

equivalent to the mathematical vector maximum problem, and that in these cases the notion of efficiency with respect to the various objective functions considered, may easily be modified 1) by that of functional efficiency. Thus it is possible to exclude, from the beginning, those decision alternatives from further solution considerations that are obviously bad (dominated) with respect to the goals pursued. The theoretical and practical solution approaches that followed subsequently, emphasizing at the same time the importance of such problems with regard to business administration, have meanwhile been dealt with at five international workshops and conferences. The results have been published in the corresponding proceedings 2). Assuming that there is no perfect solution 3) to the multiobjective decision problems discussed - i.e. there are no decision alternatives maximizing all objective functions simultaneously - and that thus the problems cannot yet be regarded as principally solved, two ways of research may be distinguished.

The one group of authors 4) deals with the question how the complete solution 5) of the problems that consist either of the set of all functional-efficient decision alternatives or equivalent to that, of the set of all efficient goal vectors, can be determined, or at least selected elements of this set of solution can be characterized by certain information useful to the decision maker. Thus, with respect

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1) See: CHARNES, A. and COOPER, W.: Management Models and Industrial Applications of Linear Programming, Vol. 1, New York 1961, especially p. 321.

2) See: COCHRANE, J.L. and ZELENY, M. (Editors): Multiple Criteria Decision Making, Columbia/South Carolina 1973; KEENEY, R.L. and RAIFFA, H. (Editors): Proceedings of a workshop on decision making with multiple conflicting objectives, Laxenburg 1975; ZELENY, M. (Editor): Multiple Criteria Decision Making, Berlin-Heidelberg-New York 1976; THIRIEZ, H. and ZIONTS, S. (Editors): Multiple Criteria Decision Making, Berlin-Heidelberg-New York 1976; ZIONTS, S. (Editor): Multiple Criteria Problem Solving, Berlin-Heidelberg-New York 1978.

3) This notion of solution has been introduced by GEOFFRION, A.M.: A parametric programming solution to the vector maximum problem, with applications to decisions under uncertainty, Stanford/California 1965, p. 2.

4) Thus for example: GEOFFRION, A.M., loc. cit.; ECKER, J.G. and KOUADA, I.A.: Finding Efficient Points for Linear Multiple Objective Programs, in: Mathematical Programming 1975, pp. 375-377, GAL, T.: A General Method for Determining the Set of All Efficient Solutions to a Linear Vectormaximum Problem, in: European Journal of Operational Research 1977, p. 307-322; ZELENY, M.: Linear Multiobjective Programming, Berlin-Heidelberg-New York 1974; ISERMANN, H.: Duality in Multiple Objective Linear Programming, in: ZIONTS, S. (Editor), loc. cit., pp. 274-285.

5) DINKELBACH, W.: Über einen Lösungsansatz zum Vektormaximumproblem, in: BECKMANN, M. (Editor): Unternehmensforschung heute, Berlin-Heidelberg-New York 1971, p. 2

to linear problem structures it was first tried to discover the connection between efficient goal vectors and optimal solutions of the corresponding linear parametric programming problem that starts from the maximization of the convex combination of all objective functions. Similarly, a procedure could be followed that consists in finding out all efficient extreme points of the goal set and determining the thus spanned facets as parts of the efficient border of this set, in order to be able to conclude from these upon the structure of the complete solution or even determine it totally. It was only natural that these two methods should be linked up, which resulted in the effort to decompose the weight set of the objective functions underlying the parametric programme into subsets, in such a way that those can be identified with certain ranges of efficient goal vectors. The disadvantage which some of these approaches have in common consists in the fact that, if more complex problems are dealt with, they require numerical efforts to an increasing degree. Thus, in most cases it is impossible to determine the complete solution exactly. That is why, meanwhile, some authors restrict themselves to revealing the objective interdependencies locally in the form of substitution rates for certain efficient extreme points of the objective set by using duality approaches. If the complete solution does not incidentally consist of one element only - it would then equal the perfect solution - the above strategies will rather have to be regarded as belonging to the field of decision preparation than to that of the decision-making process. They solely aim at determining the set of the efficient goal vectors of the decision problem out of which, under economical aspects with objectives conflicting, logically only the possibly unique optimal compromise solution can be chosen. But, especially in practical cases, the choice of such a compromise solution will by no means be released. To determine it by additional informations, required of the individual decision maker, is the main concern of the second way of research. In the following the solution approaches and strategies that have so far been developed for this purpose in literature will be outlined under uniformed aspects.

In a series of approaches 1) the existence of a complete preference relation defined on the objective set is assumed by which the decision maker has to determine the optimal compromise solution by a pairwise

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1) Among others: WILHELM, J.: Objectives and Multi-Objective Decision Making under Uncertainty, Berlin-Heidelberg-New York 1975.

comparison of efficient goal vectors. This approach, however, is not workable with respect to nondenumerable sets of efficient goal vectors and, moreover, with the increasing number of components the method of comparison surcharges the decision maker's judgement. Therefore, in order to simplify the task of gathering information within the framework of these approaches, methods have been worked out, which are based on paired comparisons of systematically selected goal vectors with an increasing preference sequence, confining themselves always to such goal vectors that differ in only two goal components each. Using additional data of this kind the solution approaches based on preference relations could be made practicable decision concepts for problems with multiple objectives. Moreover, if the preference relation can be represented by a special linear, quadratic or hyperbolic utility function of the decision maker, they allow to draw conclusions from the location of the optimal solution upon the characteristic structural parameter of this utility function, and thus to express it explicitly.

Another category of solution approaches 1) is based on the further assumption that the decision maker has an utility function relative to the goal system the shape of which, however, can only be locally determined for certain points. Generally, the problem then consists in using the locally available information in a way that allows an iterative determination of the optimal compromise solution. With reference to linear utility functions it has for example been tried to approximate the goal weights by the least-squares-method while approximating a finite preference sequence of goal vectors chosen at random by a corresponding utility function. Other approaches integrate such statements of preferences immediately into the simplex method in order to determine the optimal solution; by evaluating adjacent efficient

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1) See for example: ZIONTS, S. and WALLENIUS, J.: An Interactive Programming Method for Solving the Multiple Criteria Problem, in: Management Science 1976, p. 652-663; NIEVERGELT, E.: Ein Beitrag zur Lösung von Entscheidungsproblemen mit mehrfacher Zielsetzung, in: Die Unternehmung 1971, p. 101-126; GEOFFRION, A.M., loc. cit. and GEOFFRION, A.M.: Vector Maximal Decomposition Programming, Paper Presented at the 7th Mathematical Programming Symposium 1970, Den Haag 1970; DYER, J.S.: A Time-Sharing Computer Programm for the Solution of the Multiple Criteria Problem, in: Management Science 1973, pp. 1379-1383; MARGLIN, S.A.: Objectives of Water-Resource-Development: A General Statement, in: MAASS, A. (Editor): Design of Water-Resource-Systems, Cambridge/Massachusetts 1966, pp. 17-87.

extreme points, which are successively submitted, as well as the rates of transformation between the goals connected therewith, the above approaches aim at reducing the set of the potential goal weights. By applying the method iteratively, after finitely many steps the optimal compromise solution can be determined simultaneously by means of goal weights. Taking nonlinear concave utility functions as a basis and assuming that with respect to his utility function the decision maker is able to state the marginal rates of substitution between the individual goal components existing locally at each feasible goal vector, in the literature some authors will apply the gradient method adopted from nonlinear programming, in order to solve decision problems with multiple objectives. Using this method the gradient, which at the corresponding point is orthogonal to the vector expressing the marginal rates of substitution, indicates the direction of the steepest increase of the utility function in form of local goal weighting. This orientation will serve as a basis for choosing further goal vectors with increasing utility, with regard to which the method will iteratively be continued. This thereby generated sequence of goal vectors will due to the concave utility function converge toward the optimal solution. Practical objections to this formally correct process - such as the statement that the interdependence of the marginal rates ob substitution, particularly with respect to higher dimensioned problems, will considerably complicate their determination - have led to the development of corresponding auxiliary methods, which have already been mentioned in the last paragraph. In this case too, an increasing number of authors confine themselves to paired comparisons of vectors which are showing only little deviations in two components each, thereby getting the information necessary for the process of iteration from the decision maker, without asking too much of his judgement. A last category of utility concepts tries to deduce the local goal weights from the formulation of a chance-constrained-programming problem, in which one objective function will be maximized subject to restrictions for the remaining ones. The Lagrangian multipliers appearing in this connection will then be interpreted as goal weights. The quality of these goal weights will have to be evaluated by the decision maker. Depending on this evaluation the restrictions will be sharpened or relaxed until the optimal solution and the weight vector characterizing it are sufficiently approximated.

A number of solution approaches, goal programming belongs to them, tries to determine the optimal compromise solution by using distance functions [1]. For this purpose the decision maker will introduce a goal vector to which one will have to come geometrically as near as possible by choosing a suitable decision alternative. From the economical point of view this vector may be regarded as the goal levels aimed at by the decision maker. In this connection the general question has been discussed to what extent (provided a given vector) the complete solution as well as different decision behaviours at the same time may be described by different metrics. The result showed that, as a rule, not all efficient goal vectors can be generated by the distance concept and the assumed decision behaviour cannot be analyzed with respect to its assumptions implied.

As against the solution approaches outlined so far, which tried to combine the multidimensional goal system into a onedimensional optimization function, there are those approaches starting from the assumption that the decision maker shows a decision behaviour immediately referred to the individual components of the goal system [2]. The common feature of all these approaches is the fact that in order to find the optimal compromise solution the information, which has to be given by the decision maker, is always required for efficient goal vectors only and will be systematically used. The gradient of the efficient border, which at these points coincides with the local goal weights, or the corresponding marginal rates of transformation between the goal components may serve as directives. Taking this as a basis, for example, one may start from the fact that at each point of the efficient border the decision maker can designate an objective function the value of which has to be reduced from this point in order to increase the values of the other objective functions or at least preserve them. In other cases the decision maker is asked to specify those objective functions for which a reduction from the efficient point considered is not permissible, and at the same time to name the levels of all objective

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1) See: DINKELBACH, W., loc. cit.; CHARNES, A. and COOPER, W., loc. cit., pp. 215-223; IJIRI, Y.: Management Goals and Accounting for Control, Amsterdam 1965, p. 44 ff.

2) AUBIN, J.-P. and NÄSLUND, B.: An Exterior Branching Algorithm, Working-paper 72-42, European Institute for Advanced Studies in Management, Brüssel 1972, BENAYOUN, R., DE MONTGOLFIER, J., TERGNY, J. and LARITCHEV, O.: Linear Programming with Multiple Objective Functions: Step Method (STEM), in: Mathematical Programming 1971, pp. 366-375; FANDEL, G.: Optimale Entscheidung bei mehrfacher Zielsetzung, Berlin-Heidelberg-New York 1972, pp. 56-85.

functions to fall short of which is not allowed. Eventually, a further modification is reached by assuming that - provided there is a goal conflict - the decision maker, with respect to each goal vector given, has clear conceptions concerning those components for which losses cannot be accepted any more, and indicates these components. By this the weakest requirements to the judgement of the decision maker seem to be characterized that have to be met within the framework of the solution approaches discussed. In order to be able to use the solution approaches outlined also when dealing with decision problems with multiple objectives under uncertainty two different methods have been followed 1). First, efforts were directed at transforming the stochastic objective functions into certainty equivalences and thus taking the problem under uncertainty into an equivalent one under certainty on which the solution approaches outlined can immediately be applied. On the other hand, the possibility presented itself to determine the utility function of the decision maker on the basis of standardized decision situations under certainty and then to solve the problem under uncertainty by using the BERNOULLI-concept. The latter procedure, however, is suitable only for certain structures of utility functions, since it requires the complete knowledge of the utility function concerned.

Formal investigations on the rationality of solution approaches 2) have shown that methods basing on utility concepts are without any problems with respect to this aspect; usually, this is not true of the distance and goal component-oriented models without special additional conditions. Contrary to these results empirical tests 3) have shown, however, that due to the information requirements different in quality, decision makers, in order to express their choice behaviour, seem to prefer the

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1) See also: WILHELM, J., loc. cit.

2) See: FANDEL, G. and WILHELM, J.: Zur Entscheidungstheorie bei mehrfacher Zielsetzung, in: Zeitschrift für Operations Research 1976, pp. 1-21.

3) See: DYER, J.: An Empirical Investigation of a Man-Machine Interactive Approach to the Solution of the Multiple Criteria Problem, University of South Carolina Press 1972; WALLENIUS, J.: Comparative Evaluation of Some Interactive Approaches to Multicriterion Optimization, Working-paper 73-30, European Institute for Advanced Studies in Management, Brüssel 1973.

goal-oriented methods to the utility approaches. Procedures of this kinds have been applied to real decision problems [1], such as in the field of university and management planning as well as in the consumer market. Recent theoretical studies deal with integer or mixed-integer problems which cannot easily be solved by the approaches outlined, since in these cases the efficient goal vectors will not always belong to the border points of a convex set. More fascinating and more promising as well, however, seem to be studies dealing with possibilities of extension, such as the solution of group decisions as decision problems with multiple objectives and with several decision makers, as against the so far discussed cases with only one decision maker. So one may be curious of the development perspectives this conference will set up for multiple criteria decision making.

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1) See the literature cited in FANDEL, G. and WILHELM, J., loc. cit., p. 18, and particularly the papers published in the proceedings ZIONTS, S. (Editor), l c. cit.

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CHARACTERIZATION OF PARETO AND LEXICOGRAPHIC OPTIMAL SOLUTIONS

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I. INTRODUCTION

Two important solution concepts in the theory of multicriteria decision making are Pareto optimum and Lexicographic optimum. Given a finite collection of functions $\{f^k : R^n \rightarrow R : k = 1, \ldots, m\}$ a point $x \in Y \subset R^n$ is a Pareto minimum (abbreviated P-minimum) over Y if there is no other $x \in Y$ such that

$$(1) \qquad \begin{cases} f^k(x) \leq f^k(x^*) \quad k = 1, \ldots, m \\ \text{with at least one strict inequality.} \end{cases}$$

The point $x^* \in Y$ is a Lexicographic minimum (abbreviated L-minimum) if

$$(2) \qquad f(x^*) \prec f(x) \quad \forall x \in Y,$$

where $f(\cdot) = (f^1(\cdot), f^2(\cdot), \ldots, f^m(\cdot))$. Here $\prec$ is the lexicographic ordering:

$u \prec v$ if and only if either $u = v$ or the first nonzero component of $u - v$ is negative.

In this paper we consider the convex case, i.e. the functions f^k are assumed to be convex.

Pareto optimum (also called efficient point, nondominated solution, admissible decision rule), derives its name from the Italian economist Pareto who in 1896 introduced the concept within the framework of welfare economics. Since then, Pareto optimality, in particular its relationships to competative equilibrium, has been studied extensively by economists such as Arrow [2], Koopmans [13], and Debreu [9]. The concept also plays an important role in statistical decision theory, especially with reference to optimal mixed strategies. See e.g. Karlin [12] and Ferguson [11].

Lexicographic optimization arises in those pratical situations where optimal policies are determined by making decision successively.