Pierluigi Crescenzi
Giuseppe Prencipe
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Fun with Algorithms

4th International Conference, FUN 2007
Castiglioncello, Italy, June 2007
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Preface

This volume contains the papers presented at the Fourth International Conference on Fun with Algorithms (FUN 2007), held June 3–5, 2007 in the beautiful Tuscanian coastal town of Castiglioncello, Italy.

FUN is a three-yearly conference dedicated to the use, design, and analysis of algorithms and data structures, focusing on results that provide amusing, witty but nonetheless original and scientifically profound contributions to the area. The previous three meetings were held on Elba Island, Italy, and special issues of the journals *Theoretical Computer Science* (FUN 1998), *Discrete Applied Mathematics* (FUN 2001), and *Theory of Computing Systems* (FUN 2004) feature extended versions of selected papers from the three conference programs.

In response to the Call for Papers for FUN 2007, we received 41 submissions from 25 countries. Each submission was reviewed by at least three Program Committee members. At the end of the selection process, the committee decided to accept 20 papers. The program also includes three invited talks by Giuseppe Di Battista (U. Rome III, Italy), Nicola Santoro (Carleton U., Canada), and Luca Trevisan (U.C. Berkeley, USA).

We wish to thank all the authors who submitted their papers to FUN 2007 and thus contributed to the creation of a high-quality program and entertaining meeting, as well as the colleagues who accepted to serve on the Program Committee and provided invaluable help with the reviewing process. We also wish to thank the external reviewers (listed on the following pages) including those who completed urgent reviews during the discussion phase. Paper submission, selection, and generation of the proceedings was greatly eased by the use of the public-domain *EasyChair* Conference System (`http://www.easychair.org`). We wish to thank the EasyChair creators and maintainers for their selfless committment to the scientific community. Finally, special thanks go to Vincenzo Gervasi, whose constant help and dedication was crucial in making FUN 2007 a successful event.

April 2007

Pierluigi Crescenzi
Giuseppe Prencipe
Geppino Pucci

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On Embedding a Graph in the Grid with the Maximum Number of Bends and Other Bad Features

Giuseppe Di Battista, Fabrizio Frati, and Maurizio Patrignani

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Abstract. Graph Drawing is (usually) concerned with the production of readable representations of graphs. In this paper, instead of investigating how to produce "good" drawings, we tackle the opposite problem of producing "bad" drawings. In particular, we study how to construct orthogonal drawings with many bends along the edges and with large area. Our results show surprising contact points, in Graph Drawing, between the computational cost of niceness and the one of ugliness.

1 Breaking the Graph Drawing Rules

Up to now, bad diagrams have been produced manually or with the aid of a graphic editor; in both cases placement of symbols and routing of connections are under responsibility of the designer. The goal of this work is to investigate how poor readability of diagrams can be achieved by means of automatic tools.

Indeed, although the opposite problem of automatically producing good quality drawings of graphs has been studied since, at least, three decades by a large research community, called Graph Drawing community, the problem of obtaining drawings where the main quality is unreadability has been, as far as we know, neglected.

One of the most important reference points for the Graph Drawing community is the seminal paper of Tamassia [13] devoted to the minimization of the number of bends in orthogonal drawings. Such a paper can be considered as the milestone of the topology-shape-metric approach (see also, [6,2,5]), in which the process of producing an orthogonal drawing is organized in three steps: in the Planarization step the topology of the drawing, is determined. Such a topology is described by a planar embedding, i.e., the order of the edges around each vertex. In this step the purpose is to minimize the number of crossings. The Orthogonalization step determines the drawing shape, in which vertices do not have coordinates and each edge is equipped with a list of angles, describing the bends featured by the orthogonal line representing the edge in the final drawing. The purpose of this step is the minimization of the total number of bends. The Compaction step determines the final coordinates of the vertices and bends. The target is to minimize the area and/or the total length of the edges.

We look at the topology-shape-metric approach from the opposite perspective. Namely, our purpose is to study how a bad orthogonal drawing of a graph can be constructed by interpreting on the negative side the three mentioned steps. More precisely, we concentrate on Orthogonalization and Compaction, leaving to further studies contributions on the Planarization step.

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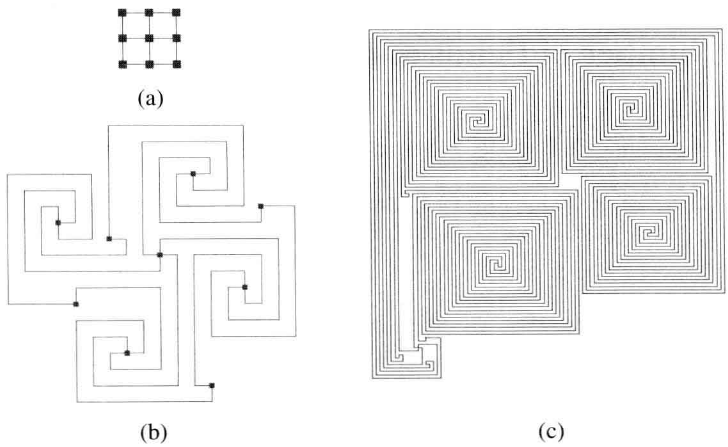

Fig. 1. A drawing of the 3×3 grid with 0 bends per edge (a), 5 bends per edge (b), and 30 bends per edge (c)

In a *planar orthogonal drawing* Γ of a plane graph G each edge is drawn as a polygonal chain of alternating horizontal and vertical segments. There are two types of angles in Γ [5]. Angles formed by edges incident on a common vertex, called *vertex-angles* and angles formed by bends (formed by consecutive segments on the same edge), called *bend-angles*. The sum of the measures of the vertex-angles around a vertex is 2π. Let f be an internal face. The sum of the measures of the vertex-angles and bend-angles inside f is $\pi(p-2)$, where p is the total number of such angles. If f is the external face, then the above sum is $\pi(p+2)$. A plane graph has a planar orthogonal drawing iff the degree of its vertices is at most 4.

Consider any edge e of Γ and an arbitrary direction for e. Try to add along e two more bends: a $\pi/2$ bend to the left and a consecutive $\pi/2$ bend to the right (or vice-versa). It is easy to see that there always exists an orthogonal drawing Γ' of G with the same shape of Γ plus the two mentioned extra bends. Hence, we can arbitrarily increase the ugliness of a drawing inserting consecutive pairs of left and right bends on each edge. However, the aesthetic effect of those bends is not "that bad", in the sense that the human eye can easily "virtually stretch" such two consecutive bends still being able to read the drawing. The effect of sequences of bends all to the left (right) is much worse. Hence, in the following we do not consider drawings that have an edge with two consecutive left-right or right-left bends.

Consider again edge e and try to add to e an arbitrary number of bends all to the left (right). Even in this case it is easy to see that there always exists an orthogonal drawing Γ' of G with the prescribed angles on e. This implies that, even if we neglect consecutive left-right and right-left bends, it is possible to draw G with a number of bends that is arbitrarily high. However, consider again Γ' from the aesthetic perspective. Even if e has now a large number of bends, we do not know anything on the remaining part of the drawing, that, maybe, has in Γ' still a nice sub-drawing. At this point it would be easy for the human eye to neglect the bad shape of e, concentrating on the remaining part of the drawing and preserving a "side view" of the adjacency expressed by e. Hence, to capture the notion of ugly drawing we need a more sophisticated model.

A k-bend drawing Γ of G is an orthogonal drawing where each edge e has exactly k bends. Traveling on e in any direction such bends are either all to the left or all to the right. We think that the notion of k-bend drawing captures very well the notion of bad drawing. Of course the highest is k the worst is the drawing. Examples of 0-bend, 5-bend, and 30-bend drawings are in Figure 1.

In Section 3 we study if it is possible to construct k-bend drawings. We show that, unfortunately, there are important classes of graphs that cannot be arbitrarily unpleasant from this perspective. On the other hand there are classes that have this interesting feature. Our results show surprising contact points between the computational cost of niceness and the one of ugliness.

Once the shape has been determined, the topology-shape-metric approach computes the final drawing. The *area* of a *grid drawing* Γ, where vertices and bends have integer coordinates, is the number of grid points of a minimum size rectangle with sides parallel to the axes that covers the drawing. Of course, a nice drawing is a drawing with limited area. Conversely, a bad drawing is a drawing with large area. Even in this case, to capture the idea of bad drawing it is not enough to simply maximize instead of minimize. In fact, it is easy to see that any grid drawing can be scaled-up to an arbitrarily large value of area. However, the aesthetic effect of this is negligible, since for the human eye is quite easy to re-scale down and to read the drawing. Hence, we adopt a different model. We consider only drawings that do not have "empty strips". Namely, in our drawings if x_m and x_M are the minimum and maximum x-coordinate of a vertex or of a bend of Γ, for each integer x_i with $x_m \leq x_i \leq x_M$ there is either a vertex or a bend in Γ with x-coordinate equal to x_i. The same holds for y-coordinates.

In Section 4 we study the problem of maximizing the area in an orthogonal drawing of a graph. Since in the topology-shape-metric approach the final coordinates are computed after the Orthogonalization step, we will assume that the orthogonal shape to draw has been already fixed. In this setting we will also consider the problem of maximizing the total edge length of an orthogonal drawing of a given shape.

Finally, in Section 5 we propose alternative models that can be studied in order to construct bad drawings of graphs and we suggest several open problems that we believe are worth of interest in a hypothetical Bad Graph Drawing community.

2 Orthogonal Representations and Flow Networks

To continue our discussion we need some definitions from Graph Drawing.

Let f be a face of a plane graph G of maximum degree four, and let Γ be an orthogonal drawing of G. Each pair of consecutive (possibly coinciding) segments of f can be associated with a value α, where $1 \leq \alpha \leq 4$, such that $\alpha \cdot \pi/2$ is the angle formed by the two segments into f.

An *orthogonal representation* or *orthogonal shape* H of G is the equivalence class of planar orthogonal drawings of G with the "same shape", that is, with the same α values associated with the angles of its faces.

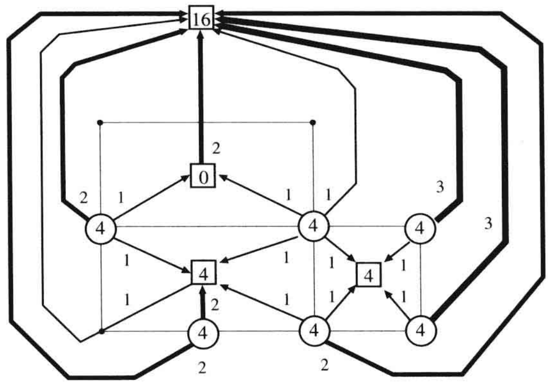

Fig. 2. An example of orthogonal shape and the corresponding flow network, where only non-null flows are represented. Vertices (circles) are labeled with the flow they produce. Faces (rectangles) are labeled with the flow they consume.

In [17,13] it is shown that an orthogonal representation of G corresponds to an assignment of α values to the angles such that $1 \leq \alpha \leq 4$ and the sum of α values around an internal (external) face f is 4 (-4, respectively).

In [13,5] it is shown that any orthogonal shape H of a degree four plane graph G is associated with a flow into a suitable flow network N defined as follows. N has a node n_v for each vertex v of G and a node n_f for each face f. Also, N has a directed arc (n_v, n_f) for each vertex v incident to a face f. Finally, for any pair of adjacent faces f and g, N has two arcs (f, g) and (g, f). In N each unit of flow is meant to represent a $\pi/2$ angle. Hence, each vertex is a producer of four units of flow and each face f of degree $a(f)$ consumes $2a(f) - 4$ units of flow, if f is internal, or $2a(f) + 4$ units of flow, if f is external. Each bend in H corresponds to one unit of flow across its incident faces. Therefore, by giving unit cost to the flow exchanged between adjacent faces, we have that a drawing with the minimum number of bends corresponds to a flow of minimum cost. This yields a polynomial-time algorithm for bend minimization. This technique was first presented in [13], with variations, refinements, and extensions given in [6,12,14,16]. Linear-time algorithms for constructing planar orthogonal drawings with $O(1)$ bends per edge, but that do not guarantee the minimum number of bends, are given in [15]. Note that it is NP-hard to minimize bends over all possible embeddings of a planar graph [7]. Polynomial-time algorithms exist only for special classes of planar graphs [1,9,11].

3 Maximizing the Number of Bends

In this section we deal with the maximization of the number of bends in orthogonal drawings. First, we show that, for all bipartite graphs that admit a straight-line orthogonal drawing, arbitrarily bad drawings can be constructed.

Theorem 1. *A bipartite graph admitting a 0-bend drawing admits a k-bend drawing, for any positive integer k.*

Proof: Let V be the vertex set of G, with $V = V_1 \cup V_2$ such that G contains only edges from vertices in V_1 to vertices in V_2. Suppose $G = (V, E)$ admits a 0-bend orthogonal representation H. Consider $|V_1|$ cuts such that each cut c_i consists of the edges incident to a distinct vertex in V_1. Since V is bipartite each edge belongs to exactly one cut. Cut c_i corresponds, in the flow network associated with H, to a cycle C_i, which can be assumed arbitrarily oriented. Increase the flow in each C_i of k units. It's easy to see that the obtained flow corresponds to an orthogonal representation H' with exactly k bends on each edge. □

For example, Figure 1 shows a 3×3 grid drawn with 0, 5, and 30 bends per edge.

Next, we show that when dealing with general planar graphs, the ugliness of the drawings cannot be arbitrarily high:

Theorem 2. *Let G be a non-bipartite plane graph. There exists an integer $k_0 > 0$ such that, for every integer $k \geq k_0$, G does not admit a k-bend drawing.*

Proof: Suppose that G is biconnected: the proof for the connected case is analogous. If G is not bipartite, then G has at least one face of odd degree. Consider the odd-degree face f that has the smallest number $2m + 1$ of vertices. Let $k_0 = 2m + 6$. Suppose, as a contradiction, that G admits a k_0-bend orthogonal representation H. Consider the network flow associated with H and node n_f associated with f, which is a sink of $2(2m + 1) - 4 = 4m - 2$ units of flow if f is an internal face, or is a sink of $2(2m + 1) + 4 = 4m + 6$ units of flow if f is the external face. Each edge e of f corresponds to one arc a_e^+ entering n_f and one arc a_e^- exiting n_f. Since H is a k_0-bend orthogonal representation of a k-bend drawing, one between a_e^+ and a_e^- carries k_0 units of flow, while the other carries none. Since f has an odd number of edges, the sum of such flows yields at least k_0 units either entering (Case 1) or exiting (Case 2) n_f. Also, the $2m+1$ vertices of G incident to f inject into n_f at least $2m+1$ and at most $6m+3$ units of flow. In Case 1 we have at least $(2m+6)+(2m+1) = 4m+7$ units entering n_f that needs at most $4m + 6$ units of flow. In Case 2 we have at most $6m + 3$ units injected by the vertices of f, while we need at least $(2m + 6) + (4m - 2) = 6m + 4$ units of flow to balance the flow in n_f. Since there is not a network flow associated with H that satisfies the above constraints, we have a contradiction. □

In the next theorem we show that for all planar bipartite graphs the possibility of obtaining bad drawings determines also the possibility of obtaining good drawings.

Theorem 3. *Let G be a bipartite plane graph. There exists an integer $k(G) > 0$ such that if G admits a $k(G)$-bend drawing, then G admits a 0-bend drawing.*

Proof: Suppose that G is biconnected: the proof for the connected case is analogous. Let $k(G) = M + 3$, where $2M$ is the greatest number of vertices incident to a face of G. Suppose G admits a $k(G)$-bend orthogonal representation H. Consider the network flow N associated with H and the node n_f associated with a face f with $2m \leq 2M$ edges, which, hence, is a sink of $2(2m) - 4 = 4m - 4$ units of flow if f is an internal face, or is a sink of $2(2m)+4 = 4m+4$ units of flow if f is the external face. Each edge e of f corresponds to one arc a_e^+ entering n_f and one arc a_e^- exiting n_f. Since H is a $k(G)$-bend orthogonal representation of a $k(G)$-bend drawing, one between a_e^+ and a_e^- carries $k(G)$ units of flow, while the other carries none. Since f has an even number of edges, the sum of such flows yields at least $2k(G) = 2M+6$ units entering n_f (Case 1), at least $2k(G) = 2M + 6$ units exiting n_f (Case 2), or exactly zero units entering n_f (Case 3). Also, the $2m$ vertices of G incident to f inject into n_f at least $2m$ and at most $6m$ units of flow. In Case 1 we have at least $(2M+6)+(2m) \geq 4m+6$ units entering n_f that needs at most $4m+4$ units of flow. In Case 2 we have at most $6m$ units injected by the vertices of f, while we need at least $(2M + 6) + (4m - 4) \geq 6m + 2$ units of flow to balance the flow in n_f. Hence, Case 3 is the only possible for each face of H, which implies that the $4m-4$ units of flow needed by each internal face and the $4m+4$ units of flow needed by the external face are balanced by the flow coming from their incident vertices. Therefore, we can obtain a network flow N' from N where, for each edge e, the flow on the arcs a_e^+ and a_e^- is equal to zero. The orthogonal representation associated with N' has zero bends. □

Notice that if the integer $k(G)$ of the above theorem exists such that G admits a $k(G)$-bend drawing, then Theorem 1 applies and G admits a k-bend drawing, for every $k \geq 0$.

4 Maximizing the Area of an Orthogonal Shape

In this section we deal with the problem of obtaining orthogonal drawings of a shape with maximum area. First, we show that both for biconnected and for simply-connected orthogonal shapes the area requirement cannot be arbitrarily high.

Theorem 4. *The maximum area of an orthogonal drawing of a connected graph with n vertices and b bends such that every vertex has degree at least 2 is $\lfloor \frac{n+b}{2} \rfloor \cdot \lceil \frac{n+b}{2} \rceil$.*

Proof: Consider any orthogonal drawing Γ of a graph G. Replace each bend with a dummy vertex, obtaining an orthogonal drawing Γ' with $n' = n + b$ vertices and no bend. For every vertex u that has only two incident edges (u, u_1) and (u, u_2) that are both vertical, remove u, insert an edge (u_1, u_2), and, if there is no other vertex on the same horizontal grid line R of u, delete R (all the edges cutting R will be shortened consequently). Analogously, for every vertex u that has only two incident edges (u, u_1) and (u, u_2) that are both horizontal, remove u, insert an edge (u_1, u_2), and, if there is no other vertex on the same vertical grid line C of u, then delete C (all the edges cutting C will be shortened consequently). Let r and c be the number of horizontal and vertical deleted grid lines, respectively. The resulting n''-vertex orthogonal drawing Γ'', with $n'' \leq n'$, is still such that every vertex has degree at least 2. Moreover, there are at least two vertices for each horizontal and for each vertical grid line of the drawing. Hence, the maximum area of Γ'' is $(\lfloor n''/2 \rfloor) \times (\lfloor n''/2 \rfloor)$. Observe that the area of Γ' is at most $(c + \lfloor n''/2 \rfloor) \times (r + \lfloor n''/2 \rfloor) = (rc + (r + c)\lfloor n''/2 \rfloor + (\lfloor n''/2 \rfloor)^2)$ and recall that $n' = n'' + r + c$. For every n'' the area of Γ' is maximized when rc is maximal, that is: (i) when $r = c = \frac{n'-n''}{2}$, in the case in which $r + c$ is even; in this case the maximum area of Γ' is $(\frac{n'-n''}{2} + \lfloor \frac{n''}{2} \rfloor)^2$, that is equal to $(\frac{n'}{2})^2$ if n'' and n' are even