SUMMARY OF EDUCATIONAL INVESTIGATIONS RELATING TO ARITHMETIC

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PREFACE

The summary contained in this monograph constitutes a preliminary phase of a general investigation of arithmetic which the authors are engaged in making with the aid of a subvention from the Commonwealth Fund. It is published in advance of the later phases of the study in order to serve the purposes of a subcommittee appointed by the Educational Research Committee of the Commonwealth Fund and charged with the duty of inquiring into the possibilities of reorganizing the administrative units of the American school system. The subcommittee has undertaken to collect and summarize the scientific studies in two fields of elementary instruction and learning. The present monograph covers the field of number work in the elementary schools; a later publication will deal in a similar way with reading.

The plan for the present summary, the collection of the materials, and the annotations in the bibliography are the joint work of the authors. In the preparation of the text, Mr. Buswell was primarily responsible for chapters v, vi, and vii; Mr. Judd, for the other chapters.

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CHAPTER I

INTRODUCTION

This monograph summarizes the articles and books which report scientific investigations of the methods and results of the teaching of arithmetic. No effort has been made to include investigations which deal with algebra or geometry even though these subjects are sometimes taught in the elementary school. Nor have investigations made in Europe been included, except in the case of a few British studies. The great majority of the European continental investigations which are applicable to American conditions have been repeated in this country and are therefore made indirectly a part of the American work. In most instances, the difference between European methods of teaching, which are largely those of oral arithmetic, and American methods, which employ in very much larger degree written exercises, renders European studies of school results inapplicable to American conditions.

Within the limits defined in the foregoing paragraph, an attempt has been made to approach completeness so far as possible. With a few exceptions of articles which are out of print and not included in the files of periodicals, it is believed that all references have been read in the preparation of this monograph. Not all of the materials canvassed, however, are here summarized. The problem confronting the authors was that of excluding mere speculation and mere opinion in the effort to report only strictly scientific material without falling into the error of incompleteness. If error has been committed, the authors believe that it has been on the side of too great inclusiveness rather than on the side of too rigid interpretation of the requirement that every reference be scientific.

The practical procedure in dealing with the material was as follows: A bibliography was built up through a canvass of every available source of references. In the final stages of the compilation of their list the authors had the benefit of the co-operation of J. C. Brown, who was good enough to compare his bibliography, collected through many years, with that compiled by the authors of this monograph. The bibliography did not at any stage attempt to cover the textbooks in arithmetic; nor did it include state and city courses of study except in the rare instances where such courses are based on original investigations and contain explicit statements of the methods and results of these investigations.

While the bibliography was being collected, an effort was made to secure a copy of each report. This was possible in the cases of somewhat more than six hundred references. There were only two references which could not be located.

After reading all of the materials collected, the authors eliminated all of the items which seemed to them not to merit the description scientific or historical. The final bibliography of publications summarized includes 307 references. A complete list of these references is presented with brief notations at the end of the monograph. While the preparation of the monograph was in progress, a number of publications appeared. These could not be included in the summary but are included in the bibliography in a supplementary list.

The principles on which the authors proceeded in rejecting available materials from their final summary were as follows: (1) Books and articles which contain nothing but opinion or unsupported recommendations were eliminated. (2) Purely routine applications of arithmetic tests in local surveys were disregarded. (3) Articles which report the use of number problems for the purpose of testing general mental conditions, as, for example, fatigue or diurnal variations, were excluded unless they throw light on the arithmetical processes as well as on the general conditions investigated.

The application of these principles of exclusion left a list of references which can be described as reporting objective data, more or less critically gathered, on the various problems which arise in the organization of arithmetic teaching.

The scientific work done in the field here under investigation has extended through a sufficiently long period of time to justify a brief treatment of it as an important chapter in the history of the science of education. Table I shows the number of references in the bibliography belonging to each quadrennium. It will be noted from this table that very little work of a scientific type was done earlier than 1893, the date of the report of the Committee of Ten. The report of that committee and also the report of the Committee of Fifteen, which was presented to the National Education Association in 1895, may very properly be pointed to as the first impressive evidences of the beginning of a new period in education. These committees did not, of course, take up their work altogether spontaneously. They were appointed in response to a demand, which grew out of general conditions, for improved and expanded education. The committees, however, brought the new problems of an expanding educational program clearly to consciousness and formulated tendencies in a new and emphatic way.

The Committee of Fifteen was so fully convinced that arithmetic had been overemphasized in the schools that its report contains the following explicit statement. "Your committee believes that, with the right methods and a wise use of time in preparing the arithmetic lesson in and out of school, five years are sufficient for the study of mere arithmetic—the five years beginning with the second school year and ending with the close of the sixth year."[1]

Other forces were at work during the nineties creating a critical attitude toward the formal methods of instruction common in American schools. G. Stanley Hall and his students were vigorously engaged in the psychological study of children and were developing what was known as

TABLE I

Date	Number of Books	Number of Periodicals	Number of Bulletins	Total
Prior to 1892.....	2	4	0	6
1893–96.....	1	8	0	9
1897–1900.....	2	13	0	15
1901–4.....	2	7	1	10
1905–8.....	3	13	0	16
1909–12.....	5	23	1	29
1913–16.....	9	54	20	83
1917–20.....	9	49	15	73
1921–24.....	12	42	12	66
Total.....	45	213	49	307

the child-study movement. Hall was advocating the postponement of all formal teaching of arithmetic to a much later stage in the school program than was common, holding that the earliest school years should be devoted to the accumulation of concrete experiences.

John Dewey was advocating the introduction into the curriculum of life-activities as the exercises best adapted to the stimulation of natural curiosity and interest in such social arts as reading, writing, and arithmetical calculation. In 1895 he collaborated with McLellan in the preparation of a book on *The Psychology of Number* (**160**). The main thesis of this book is that measurement is the practical motive which, in the course of racial evolution, has led to the development of number ideas and the best means of introducing pupils to such ideas.

In the quadrennium from 1897 to 1900 came the stimulating articles by J. M. Rice containing his pioneer contributions to the measurement of

[1] *Journal of Proceedings and Addresses of the National Education Association*, 1895, p. 300.

educational results. For several years prior to 1897, Rice had been publishing in the *Forum*, of which he was editor, accounts of the leading school systems of the country. In 1897 he published an article in which he supplied exact information with regard to existing wastes in education by reporting the results of certain tests in spelling which he had given in various school systems. He used the striking title "The Futility of the Spelling Grind" for this article. In 1902 he published a report of extensive measurements in arithmetic.

There is perhaps no more impressive example afforded in the history of American schools of a shock to pedagogical complacence than that which was produced by Rice's writings on spelling and arithmetic (**217, 218, 219**). The method which he adopted in collecting his materials is described in the following paragraphs:

> In the July issue of the *Forum*, I showed that up to the present time the elementary schools had been conducted altogether on lines dictated by theories, that the ways and means of different schools had varied in accordance with different theories, and that no attempt had been made to discover the comparative value of different processes by comparing the results. To remedy this I proposed the substitution of facts for theories as a basis for guidance, I discussed the manner in which such facts might be ascertained, and stated that the Department of Educational Research would be open to contributions destined to throw light on the educational problem from the standpoint of facts.
>
> In the present paper I shall present the facts secured by a test in arithmetic, and I shall concentrate attention chiefly upon the two fundamental questions by which teachers are confronted whenever a subject is incorporated in the school programme: (1) What results shall be accomplished? and (2) How much time shall be devoted to the branch? [**217**: 281]

The data supplied by Rice's tests (**217, 219**) made it clear that in many schools where a great deal of time was being devoted to the study of arithmetic results were no more satisfactory than in schools which were giving much less time to the subject. Rice also showed that in many cases the general attainments of pupils were unsatisfactory. On the positive side, he concluded that the essential condition determining success is adequate supervision based on objective examination of results.

The conclusions reached by Rice were accepted very slowly by school officers. There was a strong disposition in many quarters to reject his methods, his results, and his interpretations of his findings. The sheer objectivity of his methods, however, commanded attention, and other investigators followed his example of applying tests to schools in the effort to measure results.

It was not until 1908, however, when Stone published his results (**248**), that the measurement movement in arithmetic can be said to have really gained sanction. In 1909 Courtis published his first articles (**53**), and from this date on a flood of articles and monographs have appeared.

The agitation for the elimination of formal materials, the introduction of measurements, and the demands for reform resulting from measurements have led, since 1910, to various new lines of inquiry, which are aimed at the determination of the topics which shall be retained in arithmetic and the discovery of any new material which should be included. We may distinguish these studies which deal with the content of the course as curriculum studies. They differ from measurement investigations in method and degree of applicability to the problems of classroom teaching.

Curriculum studies were undertaken during the nineties, but these early studies were less systematic than those which followed the use of exact methods of measuring results. The interest in the curriculum which began to manifest itself about a decade ago may accordingly be described as a critical interest growing out of the measurement movement. It is a current interest of great importance and is engaging, perhaps more than the measurement movement itself, the energies of educational workers, especially those who are most intimately related to practical school operations.

The testing movement has matured rapidly since 1910 and has contributed to the determination of the content of the curriculum through comparisons of different school systems and the resultant establishment of general standards.

These efforts to set up standards revealed individual differences as they had never been revealed before. It became apparent that pupils of the same age and grade differ from one another in their grasp of the whole subject of arithmetic and in their mastery of particular phases of number work. Pupils were found to differ from the standards in many ways; sometimes the same child showed marked deficiencies in subtraction and division while making excellent scores in addition and multiplication.

The first systematic studies in what we have called the period of curriculum investigation were devoted to finding out what topics are considered by administrators as desirable or undesirable for ordinary school use. During the same period came investigations of the time allotments and other administrative practices of school systems. The later studies of this period are aimed at the discovery of principles of curriculum determination on psychological and social grounds.

The efforts to find a positive scientific basis for the course in arith-

metic are paralleled by similar efforts in other fields. Indeed, it may be said that the chief interest of the educational world at the present time is in curriculum studies and curriculum reconstruction. In general, the present movement can be described as very much more constructive than any which has preceded it. Educators are not merely aiming to eliminate waste and to standardize practice; they are exploring in every direction in the effort to find useful content and efficient methods which will improve the work of the schools.

There are three distinct lines along which the investigations in arithmetic are now moving. First, inquiries are being made as to the various kinds of arithmetical processes which ought to be included in a course if the course is to be logically complete. Such studies are cataloguing every possible combination in addition, subtraction, multiplication, and division; every type of carrying; every possible mathematical term which should be included in the child's vocabulary; and other similar matters of content.

Second, investigations are being carried on to determine the various kinds of demand which pupils will encounter in school and in adult life for the use of arithmetic. These investigations may be described as enumerations of social demands. They deal with such matters as the various kinds of counting which pupils are called on to do in out-of-school activities, the extent to which people must be able to add and multiply in making change and in other common activities of life, and the demands for arithmetical knowledge made by different industries and by courses of study in institutions above the level of the elementary school.

The third line of inquiry deals with the way in which pupils' minds work in assimilating number ideas and mastering number combinations. Investigations of this type ask: What are the most difficult combinations for pupils to master? Why? What is the proper order in which ideas are to be taught? What are the common errors which pupils make?

In trying to answer these questions, investigators are using all of the methods of analysis at hand. They are very commonly using tests. These tests, however, are used not, as formerly, merely to criticize or validate existing courses of study but in the effort constructively to revise the course and produce something entirely free from traditional restrictions. They are using laboratory methods and the methods of the social survey. In short, the present period is one which is not dominated by a single method but is aiming to make the broadest general study of a curriculum problem.

Along with these investigations dealing directly with the content and arrangement of arithmetic, there are investigations in certain related fields

which aim to improve the administration of this content after it has been determined. Questions are raised as to the amount of time which can be allotted to arithmetic in a program which must include also reading, history, geography, penmanship, and other subjects. Inquiry is being made as to when and under what conditions higher mathematics may be brought into the program.

There is one very striking fact regarding scientific studies of arithmetic revealed by Table I. In spite of the great need for improvement in the teaching of arithmetic, as shown by the fact that this subject is the chief source of non-promotion in the elementary school, there has been a noticeable falling off in the number of scientific studies in this field during the last two quadrenniums. This falling off is not paralleled in reading, as shown by a similar table[1] based on a bibliography of scientific studies in this subject. This suggests that new methods of scientific inquiry are probably required before arithmetic can be fully reconstructed in such a way as to meet the needs of pupils.

It is the purpose of this monograph to contribute to the discovery of new lines of thinking and investigation and to a more vigorously constructive program in arithmetic by bringing together, in as compact a summary as possible, the suggestions which are to be found in the literature of educational tests and curriculum and administrative studies.

In order to avoid the distractions which are inevitable in an extensive summary, the authors have in many instances omitted from their discussion the names of investigators and have presented results in a wholly impersonal form. It is hoped that this will not be interpreted as due to a lack of recognition of the rights of scientific priority. The common form of summaries in the field of education is that which enumerates under an investigator's name a list of his findings. In such a case there is no lack of credit for scientific work, but there is often loss in systematic presentation because results are frequently brought together which relate to wholly different phases of the school course. The present monograph has organized the discussion around topics rather than authors. It has attempted to make full acknowledgment of the rights of investigators by appending an annotated bibliography and referring constantly in the text to the items in this bibliography. At points it has seemed proper to mention the results or opinion of a particular investigator, especially where a balance is being attempted between conflicting views. In the main, however, the treatment of topics is impersonal.

[1] This table will appear in a monograph by William Scott Gray, entitled, *A Summary of Investigations Relating to Reading.*

Finally, a word may be added regarding the headings under which the discussions have been organized. These were derived by a strictly empirical method from the materials themselves. The whole body of reports was canvassed in a preliminary way in order to discover the divisions of the monograph that would be necessary in order to cover all of the contributions and at the same time bring together the individual references which belong together. The chapter headings and the subheadings are therefore, in an important sense, parts of the summary and not in any sense outside headings superimposed upon the material.

The authors of the monograph have attempted to avoid any appearance of imposing their own theories with regard to arithmetic on the body of results here presented. They may have influenced, in some measure, the total impression produced through the emphasis which they have given to topics and through the selections which they made in arriving at the final working bibliography. Even in these matters, however, an attempt was made to accept the suggestions derived from the materials themselves. Wherever comment has been made, care has been taken to draw a sharp distinction between the summary of findings and the interpretations added by the authors.

The authors have reserved for later publications statements of their own investigations and of the conclusions to which they have been led regarding the proper constitution of the course in arithmetic.

CHAPTER II

CONTENT AND ADMINISTRATION OF THE CURRICULUM IN ARITHMETIC

There is a sense in which every investigation which deals with arithmetic contributes to the construction of the course of study in this subject. When the results of class work are measured and when individual differences are ascertained and classified, steps are taken which vitally affect the content and administration of instruction. There are, however, certain investigations which contribute in a narrower and more intimate sense to the determination of what shall be included in the course of study. Such are the investigations which attempt to determine the topics which should be included in the work of the schools and of each of the grades and the investigations which deal with the time allotments appropriate to arithmetic as a whole and to its various phases. This chapter will be devoted to a summary of reports of investigations of the second or more limited type.

THE OVEREXPANSION OF ARITHMETIC

For three-quarters of a century after Warren Colburn published the first distinctively American arithmetic in 1821 there was a steady expansion of the content of the subject. Denominate numbers were dealt with in increasing variety; practical and theoretical problems accumulated and found their way into the textbooks. Drill exercises were given increased time in the school program. Arithmetic may very properly be described as a favorite subject in the district school which flourished during the middle of the last century. It is a subject in which definite problems can be set and in which the attainments of pupils can be accurately determined. Pupils and teachers alike accepted it as an important part of the school work and were satisfied to have it consume an appreciable part of the school day.

THE BEGINNINGS OF RECONSTRUCTION

During the later decades of the nineteenth century there came a general change in the conception of education. Society began to demand a broader type of schooling. The result was a general expansion of the curriculum. This expansion raised numerous questions as to the possi-

bility of making room for new subjects by eliminating waste in the traditional courses of study. Educators began to question every topic included in the curriculum. Reference has been made to the report of the Committee of Fifteen, which is full of such questioning. At a later date (1904) the question of waste was forcefully presented again by McMurry in an address delivered before the National Education Association.

The agitation was intensified by the work of Hall, Dewey, and Rice, which was mentioned in the Introduction.

COMPARISON OF COURSES IN VARIOUS SCHOOL SYSTEMS

The earliest studies dealing strictly with curriculum reconstruction were those which compared courses of study with a view to discovering common practices and those which compared textbooks for a like purpose. So important are textbooks in their effects on the curriculum and on methods of teaching that their analysis will be made a special subject of consideration in the next chapter.

The comparison of courses of study was one of the earliest methods of studying the curriculum employed. A paper reporting the results of a study made by this method was published as early as 1900 (**246**). The statements made in this article show that the reconstruction of the course in arithmetic was being vigorously discussed at that time, although there were few positive studies on which to base the reconstruction. It is also made evident by the results reported in this first reference and in numerous subsequent references that throughout the history of American schools down to the present time there has been great lack of uniformity in the practices of school systems in different cities. Later studies using this same comparative method have taken up in great detail the practices of a few cities (**86**) with reference to the distribution of the major topics of arithmetic or have collected less detailed materials on an extensive scale (**120, 122, 125, 287**). All of these studies show that there is the greatest variety in emphasis and some differences even in the major topics taught.

A report by Jessup, published in 1913, summarizes the results of an analysis of courses of study as follows:

> An analysis of the material specifically eliminated in the courses of study in arithmetic in these 150 cities brought out the following information:

DISTRIBUTION OF TOPICS SPECIFICALLY MENTIONED AS BEING ELIMINATED

Name of Topics	Number of Cases
Cube root	29
Compound proportion	11
Troy weight	10
Apothecaries' weight	8
Metric system	8
Partial payments	8
Compound interest	7
All interest except 6 per cent	6
Domestic exchange	5
Least common multiple and greatest common divisor	5
Foreign exchange	5
Foreign money	5
Partitive proportion	5
Longitude and time	4
Present worth	4
True discount	4
Stocks and bonds	4
Scattering topics mentioned three times	4
Scattering topics mentioned two times	14
Scattering topics mentioned once	38

From the foregoing table it is seen that there are 206 cases of specific eliminations in the list, distributed over seventy-three different topics. With the exception of cube root, there seems to be practically no agreement as to just what topics to eliminate. The fact that there are so many instances of elimination, showing such a wide variation as to actual topics, should be taken into consideration in any general plan for elimination of material as a factor in the economy of time. School men are clearly conscious of the need of elimination. Experimentation is going on in many different directions. If adequate means of testing the results of these experimentations were available, we might have a definite program of elimination established [**120**: 467–68].

In 1914 Jessup extended his investigations and reported the results of a study made by a different method. He collected the judgments of superintendents with regard to the topics which should be included in courses in arithmetic. He summarized his findings as follows:

This study reveals the fact that there is an overwhelming tendency on the part of half of the superintendents in this country in favor of either eliminating or lessening the attention to be given to certain subjects in arithmetic such as alligation, cube root, unreal fractions, progression, and certain obsolete tables such as folding paper, surveyors' tables, etc. Again, it reveals an overwhelming

attitude in favor of increased emphasis on such fundamental subjects as addition, multiplication, subtraction, and division. There is also a decidedly strong disposition to favor increased emphasis on the application of arithmetic to the social and economic conditions of the day, such as the saving and loaning of money, taxation, public expenditures, life insurance, etc. [121: 222]

Other writers have enumerated in similar fashion topics which should be eliminated from the course in arithmetic (231) or have outlined on the basis of their own judgments the topics which should be included (23, 146, 228).

ARITHMETIC AND SOCIAL DEMANDS

While comparative studies of the type referred to in the foregoing paragraphs furnish a sounder basis for the organization of the course than does individual opinion, they do not supply the fundamental principles on which practice must ultimately rest if it is to satisfy the demand that the curriculum be scientifically defensible. A number of studies undertaken since 1913 have attempted to supply a strictly objective basis for the course in arithmetic by investigating the demands which are made in practical life for a knowledge of number and number combinations.

In the New York City school survey (1913) Courtis reports tests which he made of the arithmetical abilities of a number of employees of a commercial establishment in New York City and recommends explicitly that the practices of the schools be governed by such findings. Typical extracts from his report are as follows:

> Exactly the same tests and time allowances were used as with the children, and the same procedure in conducting the examination and in scoring and tabulating the papers was followed throughout. Forty-one complete records were eventually obtained.
>
> The subjects represented seven different types of positions in the store (a large department store), and in number were as follows:
>
> | | |
> |---|---|
> | Auditing department | 5 |
> | Bill clerks | 5 |
> | Cashiers | 8 |
> | Clerks | 13 |
> | Salesmen | 7 |
> | Typists | 3 |
> | Total | 41 |
>
> . . . . . .
>
> The results show that addition is the ability most generally affected by the work in the store, although there are marked changes in subtraction and in

the abstract work as well. Multiplication and division are very slightly affected, and in speed of copying figures the deviations from the average are slight. In Test 7, abstract work, the agreement with the standard is close, except for the two types of work in which the greatest amount of practice occurs, the bill clerks and the auditors. In general, therefore, on the basis of these results, a child attaining the standard scores in the tests for abstract work would have at least an average equipment with which to enter upon business life.

. . . . . .

However, accepting for speed and accuracy the average of this group as representative of the degree of ability demanded by the store of those who apply for positions, it is possible to measure the efficiency of the public schools in terms of this measure. In Test 7, for instance, the average score of the adult group was 14.4 examples attempted and 10.4 examples right. Of the 1,187, 8 B boys and 1,053, 8 B girls measured, 610 or only 51 per cent of the one and 596 or only 56 per cent of the other equal or exceed these records. On this basis alone, therefore, the store would reject nearly one-half of even those applicants who had completed the full grammar school course.

The most important conclusion to be drawn from the preceding discussions is that although the individual variation shown in these results is of the same character as that found in the scores of children, apparently for many of the tests the range of variation is much smaller. That is, a class of fifty bill clerks, for example, selected as of equal ability by the head of the department would show less difference between the highest and lowest scores made than a class of fifty children selected as of equal ability by the promotion machinery of the school. The size of the different groups is, of course, too small to admit of any valid judgments as to the range of variation, but as soon as definite degrees of ability are proved by measurement to be required for given social and industrial positions, the lack of standards which now exist in school work in arithmetic will be appreciated at its true importance. This test apparently shows plainly that greater ability is required for some positions than for others and that the degree of ability is changed by practical experience. On the other hand, the range of variation that is found even in these small groups proves that arithmetical ability is but one of many factors in determining the efficiency of an employee. The slow inaccurate worker, by conscientious checking, may produce results superior to those of every other member of the force, while the most gifted and best trained employee as far as arithmetical ability is concerned may be utterly unreliable and worthless because of his low ideals. That is, the supreme work of the schools should be to develop in children those ideals of personal responsibility and service that make for both growth and efficient discharge of social duties. It is unfortunate that our schools are not organized as yet on such a basis. At the same time it should not be forgotten that the efficiency of the individual inspired by even the highest ideals is conditioned by his mental equipment. If all the auditors, for example, were actuated by the same worthy

motives, it is extremely probable, in view of these results, that their relative value in the auditing work could be approximated closely from a comparison of their scores in these tests. In other words, in this case, as in so many other of the points discussed in this report, the results but serve to prove the possibility of, and the need for, more extended and more detailed investigations of the same kind [**55**: 495–502].

In 1917 Wilson published the earliest report now in print of the work which he had carried on in Connersville, Indiana, and in various centers in Iowa in the effort to find out from business men and other classes of citizens what arithmetic they actually employ in everyday life. The conclusions set forth in this report are as follows:

1. The opinions of business men and of educators that many arithmetical processes consuming much time could be omitted from the course without loss, is borne out quite fully by this survey of the social and business usage of arithmetic. The facts go farther than the opinions and are a safer guide.

2. The problems solved in actual usage are brief and simple. They chiefly require the more fundamental and more easily mastered processes.

3. In actual usage, few problems of an abstract nature are encountered. The problems are concrete and relate to business situations. They require simple reasoning and a decision as to the processes to be employed.

4. The study justifies careful consideration of the following question: after the development of reasonable speed and accuracy in the fundamentals and the mastery of the simple and more useful arithmetical processes, should the arithmetic work not be centered largely around those problems which furnish the basis for much business information?

5. Another question: May we not hope through the use of large informational problems and situations in the upper grades, to secure a more intelligent application of arithmetic to actual life situations, i.e., to secure the use of more arithmetic in the productive work of the kitchen, in intelligent buying, in proper farm accounting, in intelligent saving and investing, etc.?

6. Aside from the work implied by the questions raised in 4 and 5 above, it is evident that the necessary work in arithmetic can be mastered in much less time than is now being devoted to it [**283**: 141–42].

Wilson continued the use of the method of study which he had adopted and has published a number of articles and books (**285, 286, 287**). The volume published in 1922 (**287**) brings together all of the results which he has secured through his investigations and proposes a course to be followed throughout the grades. This volume is described as a third edition of the Connersville course of study, which was prepared ten years before, the earlier editions of which are out of print.