

经 典 原 版 书 库

金融数学

(英文版)

The Mathematics of Finance: *Modeling and Hedging*

Joseph Stampfli
Victor Goodman

THE BROOKS/COLE SERIES IN
ADVANCED MATHEMATICS
Paul J. Sally, Jr., EDITOR

(美) Joseph Stampfli Victor Goodman 著
印 第 安 纳 大 学



机械工业出版社
China Machine Press

经典原版书库

金融数学

(英文版)

The Mathematics of Finance
Modeling and Hedging

(美) Joseph Stampfli Victor Goodman 著
印第安纳大学



机械工业出版社
China Machine Press

Joseph Stampfli and Victor Goodman: The Mathematics of Finance: Modeling and Hedging (ISBN: 0-534-37776-9).

Original edition copyright © 2001 by Brooks/Cole, a division of Thomson Learning.

First published by Brooks/Cole, an imprint of Thomson Learning, United States of America.

All rights reserved.

Reprinted for the People's Republic of China by Thomson Asia Pte Ltd and China Machine Press under the authorization of Thomson Learning. No part of this book may be reproduced in any form without the express written permission of Thomson Learning Asia and China Machine Press.

本书英文影印版由汤姆森学习出版社与机械工业出版社合作出版。未经出版者书面许可,不得以任何方式复制或抄袭本书内容。

版权所有,侵权必究。

本书版权登记号: 图字: 01-2003-1018

图书在版编目(CIP)数据

金融数学(英文版)/(美)斯坦普夫里(Stampfli, J.), (美)古德曼(Goodman, V.)著. —北京:机械工业出版社, 2003.4

(经典原版书库)

书名原文: The Mathematics of Finance: Modeling and Hedging

ISBN: 7-111-11912-6

I. 金… II. ①斯… ②古… III. 金融数学—高等学校—教材—英文 IV. F830

中国版本图书馆CIP数据核字(2003)第023796号

机械工业出版社(北京市西城区百万庄大街22号 邮政编码 100037)

责任编辑: 杨海玲

北京瑞德印刷有限公司印刷·新华书店北京发行所发行

2003年4月第1版·2003年6月第2次印刷

787mm×1092mm 1/16·16.75印张

印数: 2 001-4 000册

定价: 35.00元

凡购本书,如有倒页、脱页、缺页,由本社发行部调换

PREFACE

Throughout the nineties, we have seen the synergistic union of mathematics, finance, the computer, and the global economy. Currency markets trade two trillion dollars per day, and sophisticated financial derivatives such as options, swaps, and quantos are commonplace.

Since the appearance of the Black-Scholes formula in 1973, the financial community has embraced an abundant and ever-expanding array of mathematical tools and models. Enrollment in courses presenting these applications of mathematical finance has exploded at schools everywhere. It is driven by the attraction of the material, coupled with enormous employment demand. We expect that the twenty-first century will see even greater growth in these areas, following Kurzweil's law of accelerating returns. The practical analysis of a broad range of market transactions and activities has converted many market devotees to this mode of thinking.

This textbook explains the basic financial and mathematical concepts used in modeling and hedging. Each topic is introduced with the assumption that the reader has had little or no previous exposure to financial matters or to the activities that are common to major equity markets. Exercises and examples illustrate these topics. Often an exercise or example uses real market data.

To the Instructor

A complete, well-balanced course at the undergraduate level can be based on Chapters 2, 3, 5, 6, 7, 8, and 9. An instructor might touch only briefly on Chapter 1 as an introduction to the financial terminology and to strategies that are employed in trading equity shares. You might wish to return to Chapter 1 repeatedly as you progress through the textbook; the chapter is always there as a convenient reference for market transactions and terminology.

Most undergraduate students seem to be very comfortable with computers, and they appear to pick up the ins and outs of software packages such as Maple™, *Mathematica*™, and Microsoft® Excel very quickly. Each instructor will have to evaluate the proficiency of his or her own students in this area. For example, we have found that Excel is readily available on the Indiana University campus and that students are comfortable in preparing data and reports using this software.

Acknowledgments

We would like to thank the National Science Foundation for support while preparing some of the material used in this textbook. In particular, we owe a great debt of gratitude to Dan Maki and Bart Ng, principal investigator on the NSF grant, “Mathematics Throughout the Curriculum,” for encouraging us to write the book and for their continued support, financial and personal, during the period of creation. We wish to thank our reviewers: Rich Sowers, University of Illinois; William Yin, La Grange College; and John Chadam, University of Pittsburgh.

In November 1999, Joseph Stampfli presented several lectures on financial mathematics at a workshop on this topic in Bangkok, Thailand, sponsored by Mahidol University. We would like to thank the university and, in particular, Professor Yongwemon Lenbury and Ponchai Matangkasombut, then Dean and now President of the university, for their gracious hospitality throughout the visit. It was a truly memorable experience.

We would also like to thank the editorial and production teams at Brooks/Cole for their continuous and timely help. In particular, Gary Ostedt and Carol Benedict did everything an editorial team can do and more. Several unexpected crises arose as the book progressed, and Gary guided us through them with patience, wisdom, and humor. We would also like to thank the other members of the Brooks/Cole team: Mary Vezilich, Production Coordinator; Karin Sandberg, Marketing Manager; Sue Ewing, Permissions Editor; and Samantha Cabaluna, Marketing Communications. We would also like to thank Kris Engberg of Publication Services, who helped us solve hundreds of problems, both large and small; Jerome Colburn, whose contributions as copy editor turned limp doggerel into sparkling prose; and Jason Brown and his production team.

Victor Goodman wishes to thank Devraj Basu for his personal input during the early stages of the manuscript preparation. In addition, Joseph Stampfli would like to thank Jeff Gerlach, a graduate student in Economics at Indiana University. Chapter 11 is entirely due to Jeff’s efforts, and he provided solutions to most of the exercises.

How to Reach Us

Readers are encouraged to bring errors and suggestions to our attention. E-mail is excellent for this purpose. Our addresses are

goodmanv@indiana.edu
stampfli@indiana.edu

A web site for this book is maintained at <http://www.indiana.edu/~iubmtc/mathfinance/>

Victor Goodman
Joseph Stampfli

CONTENTS

1	Financial Markets	1
1.1	Markets and Math	1
1.2	Stocks and Their Derivatives	2
1.2.1	Forward Stock Contracts	3
1.2.2	Call Options	7
1.2.3	Put Options	9
1.2.4	Short Selling	11
1.3	Pricing Futures Contracts	12
1.4	Bond Markets	15
1.4.1	Rates of Return	16
1.4.2	The U.S. Bond Market	17
1.4.3	Interest Rates and Forward Interest Rates	18
1.4.4	Yield Curves	19
1.5	Interest Rate Futures	20
1.5.1	Determining the Futures Price	20
1.5.2	Treasury Bill Futures	21
1.6	Foreign Exchange	22
1.6.1	Currency Hedging	22
1.6.2	Computing Currency Futures	23

2	Binomial Trees, Replicating Portfolios, and Arbitrage	25
2.1	Three Ways to Price a Derivative	25
2.2	The Game Theory Method	26
2.2.1	Eliminating Uncertainty	27
2.2.2	Valuing the Option	27
2.2.3	Arbitrage	27
2.2.4	The Game Theory Method—A General Formula	28
2.3	Replicating Portfolios	29
2.3.1	The Context	30
2.3.2	A Portfolio Match	30
2.3.3	Expected Value Pricing Approach	31
2.3.4	How to Remember the Pricing Probability	32
2.4	The Probabilistic Approach	34
2.5	Risk	36
2.6	Repeated Binomial Trees and Arbitrage	39
2.7	Appendix: Limits of the Arbitrage Method	41
3	Tree Models for Stocks and Options	44
3.1	A Stock Model	44
3.1.1	Recombining Trees	46
3.1.2	Chaining and Expected Values	46
3.2	Pricing a Call Option with the Tree Model	49
3.3	Pricing an American Option	52
3.4	Pricing an Exotic Option—Knockout Options	55
3.5	Pricing an Exotic Option—Lookback Options	59
3.6	Adjusting the Binomial Tree Model to Real-World Data	61
3.7	Hedging and Pricing the N -Period Binomial Model	66
4	Using Spreadsheets to Compute Stock and Option Trees	71
4.1	Some Spreadsheet Basics	71
4.2	Computing European Option Trees	74
4.3	Computing American Option Trees	77
4.4	Computing a Barrier Option Tree	79
4.5	Computing N -Step Trees	80

5	Continuous Models and the Black-Scholes Formula	81
5.1	A Continuous-Time Stock Model	81
5.2	The Discrete Model	82
5.3	An Analysis of the Continuous Model	87
5.4	The Black-Scholes Formula	90
5.5	Derivation of the Black-Scholes Formula	92
5.5.1	The Related Model	92
5.5.2	The Expected Value	94
5.5.3	Two Integrals	94
5.5.4	Putting the Pieces Together	96
5.6	Put-Call Parity	97
5.7	Trees and Continuous Models	98
5.7.1	Binomial Probabilities	98
5.7.2	Approximation with Large Trees	100
5.7.3	Scaling a Tree to Match a GBM Model	102
5.8	The GBM Stock Price Model—A Cautionary Tale	103
5.9	Appendix: Construction of a Brownian Path	106
6	The Analytic Approach to Black-Scholes	109
6.1	Strategy for Obtaining the Differential Equation	110
6.2	Expanding $V(S, t)$	110
6.3	Expanding and Simplifying $V(S_t, t)$	111
6.4	Finding a Portfolio	112
6.5	Solving the Black-Scholes Differential Equation	114
6.5.1	Cash or Nothing Option	114
6.5.2	Stock-or-Nothing Option	115
6.5.3	European Call	116
6.6	Options on Futures	116
6.6.1	Call on a Futures Contract	117
6.6.2	A PDE for Options on Futures	118
6.7	Appendix: Portfolio Differentials	120
7	Hedging	122
7.1	Delta Hedging	122
7.1.1	Hedging, Dynamic Programming, and a Proof that Black-Scholes Really Works in an Idealized World	123
7.1.2	Why the Foregoing Argument Does Not Hold in the Real World	124
7.1.3	Earlier Δ Hedges	125

7.2	Methods for Hedging a Stock or Portfolio	126
7.2.1	Hedging with Puts	126
7.2.2	Hedging with Collars	127
7.2.3	Hedging with Paired Trades	127
7.2.4	Correlation-Based Hedges	127
7.2.5	Hedging in the Real World	128
7.3	Implied Volatility	128
7.3.1	Computing σ_I with Maple	128
7.3.2	The Volatility Smile	129
7.4	The Parameters Δ , Γ , and Θ	130
7.4.1	The Role of Γ	131
7.4.2	A Further Role for Δ , Γ , Θ	133
7.5	Derivation of the Delta Hedging Rule	134
7.6	Delta Hedging a Stock Purchase	135
8	Bond Models and Interest Rate Options	137
8.1	Interest Rates and Forward Rates	137
8.1.1	Size	138
8.1.2	The Yield Curve	138
8.1.3	How Is the Yield Curve Determined?	139
8.1.4	Forward Rates	139
8.2	Zero-Coupon Bonds	140
8.2.1	Forward Rates and ZCBs	140
8.2.2	Computations Based on $Y(t)$ or $P(t)$	142
8.3	Swaps	144
8.3.1	Another Variation on Payments	147
8.3.2	A More Realistic Scenario	148
8.3.3	Models for Bond Prices	149
8.3.4	Arbitrage	150
8.4	Pricing and Hedging a Swap	152
8.4.1	Arithmetic Interest Rates	153
8.4.2	Geometric Interest Rates	155
8.5	Interest Rate Models	157
8.5.1	Discrete Interest Rate Models	158
8.5.2	Pricing ZCBs from the Interest Rate Model	162
8.5.3	The Bond Price Paradox	165
8.5.4	Can the Expected Value Pricing Method Be Arbitraged?	166
8.5.5	Continuous Models	171
8.5.6	A Bond Price Model	171

8.5.7	A Simple Example	174
8.5.8	The Vasicek Model	178
8.6	Bond Price Dynamics	180
8.7	A Bond Price Formula	181
8.8	Bond Prices, Spot Rates, and HJM	183
8.8.1	Example: The Hall-White Model	184
8.9	The Derivative Approach to HJM: The HJM Miracle	186
8.10	Appendix: Forward Rate Drift	188
9	Computational Methods for Bonds	190
9.1	Tree Models for Bond Prices	190
9.1.1	Fair and Unfair Games	190
9.1.2	The Ho-Lee Model	192
9.2	A Binomial Vasicek Model: A Mean Reversion Model	200
9.2.1	The Base Case	201
9.2.2	The General Induction Step	202
10	Currency Markets and Foreign Exchange Risks	207
10.1	The Mechanics of Trading	207
10.2	Currency Forwards: Interest Rate Parity	209
10.3	Foreign Currency Options	211
10.3.1	The Garman-Kohlhagen Formula	211
10.3.2	Put-Call Parity for Currency Options	213
10.4	Guaranteed Exchange Rates and Quantos	214
10.4.1	The Bond Hedge	215
10.4.2	Pricing the GER Forward on a Stock	216
10.4.3	Pricing the GER Put or Call Option	219
10.5	To Hedge or Not to Hedge—and How Much	220
11	International Political Risk Analysis	221
11.1	Introduction	221
11.2	Types of International Risks	222
11.2.1	Political Risk	222
11.2.2	Managing International Risk	223
11.2.3	Diversification	223
11.2.4	Political Risk and Export Credit Insurance	224
11.3	Credit Derivatives and the Management of Political Risk	225
11.3.1	Foreign Currency and Derivatives	225
11.3.2	Credit Default Risk and Derivatives	226

XII CONTENTS

11.4	Pricing International Political Risk	228
11.4.1	The Credit Spread or Risk Premium on Bonds	229
11.5	Two Models for Determining the Risk Premium	230
11.5.1	The Black-Scholes Approach to Pricing Risky Debt	230
11.5.2	An Alternative Approach to Pricing Risky Debt	234
11.6	A Hypothetical Example of the JLT Model	238
	Answers to Selected Exercises	241
	Index	247

CHAPTER 1

FINANCIAL MARKETS

*If you can look into the seeds of time,
And say which grain will grow and which will not,
Speak then to me...*

Shakespeare, *Macbeth*, Act I, Scene ii

Note: This chapter is intended to be a glossary. It is designed to introduce concepts, ideas, and definitions as they are needed. We do not recommend that one work through the entire chapter as a unit. Visit it sparingly as needed.

1.1 MARKETS AND MATH

Nearly everyone has heard of the New York, London, and Tokyo stock exchanges. Reports of the trading activity in these markets frequently make the front page of newspapers and are often featured on evening television newscasts. There are many other financial markets. Each of these has a character determined by the type of financial objects being exchanged.

The most important markets to be discussed in this book are *stock* markets, *bond* markets, *currency* markets, and *futures and options* markets. These financial terms will be explained later. But first we draw your attention to the fact that every item that is exchanged, or **traded**, on some market is of one of two types.

The traded item may be a **basic equity**, such as a stock, a bond, or a unit of currency. Or the item's value may be indirectly *derived from* the value of some other traded equity. If so, its future price is tied to the price of another equity on a future date. In this case, the item is a **financial derivative**; the equity it refers to is termed the **underlying equity**.

This chapter contains many examples of financial derivatives. Each example will be thoroughly explained in order to make the derivative concept clear to the

reader. Our examples will be options based on stocks, bonds, and currencies. Also, we will discuss *futures* and options on futures.

Mathematics enters this subject in a serious way when we try to relate a derivative price to the price of the underlying equity. Mathematically based arguments give surprisingly accurate estimates of these values.

The main objective of this book is to explain the process of computing derivative prices in terms of underlying equity prices.

We also wish to provide the reader with the mathematical tools and techniques to carry out this process. Through developing an understanding of this process, you will gain insights into how derivatives are used, and you will comprehend the risks associated with creating or trading these assets. These insights into derivative trading provide extra knowledge of how modern equity markets work.

The mathematics in this book will emphasize two financial concepts that have had a startling impact over the last two decades on the way the financial industry views derivative trading.

We will emphasize investments that **replicate** equities, and we will explore mathematical models of how equities behave in the **absence of arbitrage opportunities**.

The combination of these two concepts furnishes a powerful tool for finding prices. An example is overdue at this point. In the next section, we present an example in which a *replicating investment* and the *lack of arbitrage opportunities* give us a price for a derivative. This example is worth careful reading.

1.2 STOCKS AND THEIR DERIVATIVES

A company that needs to raise money can do so by selling its shares to investors. The company is *owned* by its shareholders. These owners possess **shares** or **equity certificates** and may or may not receive **dividends**, depending on whether the company makes a profit and decides to share this with its owners.

What is the value of the company's stock? Its value reflects the views or predictions of investors about the likely dividend payments, future earnings, and resources that the company will control. These uncertainties are resolved (each trading day) by buyers and sellers of the stock. They exercise their views by trading shares in **auction markets** such as the New York, London, and Tokyo stock exchanges. That is, most of the time a stock's value is judged by what someone else is willing to pay for it on a given day.

What is a stock derivative? It is a specific contract whose value at some future date will depend *entirely* on the stock's future values. The person or firm who formulates this contract and offers it for sale is termed the **writer**. The person or firm who purchases the contract is termed the **holder**. The stock that the contract is based on is termed the **underlying equity**.

What is a derivative worth? The terms of such a contract are crucial in any estimation of its value. As our first example, we choose a derivative with a simple structure so that our main financial concepts, *replicating an equity* and *lack of arbitrage opportunities*, can be easily explained. These will give us a price for this derivative.

We will also explain several trading opportunities in this example that are important for understanding concepts you will encounter later on.

1.2.1 Forward Stock Contracts

It is sometimes convenient to have the assurance that, on some specific future date, one will buy a share of stock for a guaranteed price. This *obligation* to buy in the future is known as a

FORWARD CONTRACT

Here are the contract conditions:

- On a specific date, termed the **expiration date**, the holder of this contract **must** pay a prescribed amount of money, the **exercise price**, to the writer of the contract.
- The writer of the contract **must** deliver one share of stock to the holder on the expiration date.

Figure 1.1 is a pictorial view of the exchange of stock and cash in a forward contract, often called a **forward**.

This contract can either be a good deal for the holder on the day of delivery or be a bad one. The outcome depends on the stock price on the expiration date.

Profit or Loss at Expiration

To state things quantitatively, we will denote the price on that date by S_T and the required exercise price by X . The exercise price, X , is a known quantity. It is also called the **strike price**, and often the expiration date is referred to as the **strike date**.

The profit or loss to the holder at time T is expressed as

$$S_T - X$$

Is there some way to find a profit or loss price formula that will be useful before the contract expires? The question of what the contract should be worth is not an

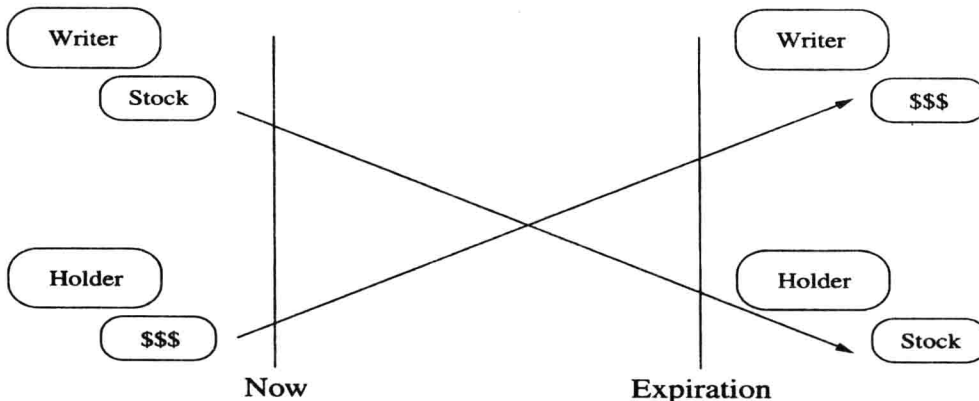


FIGURE 1.1
Forward contract

academic one. Modern markets allow a contract holder to sell the contract in the market or purchase new ones during any trading day. In other words, these instruments are **traded**.

You might imagine that the contract has a high value if today's price is much higher than the exercise price, X , and expiration is not too far away. On the other hand, if today's stock price is quite low, then perhaps it is nearly worthless. We obtain information about its price by creating an investment that *replicates* some other equity.

Replicating Investment

Form an investment choice, called a **portfolio**, that consists of one contract (worth f) and the following amount of cash:

$$Xe^{-r(T-t)}$$

Now the net worth is

$$f + Xe^{-r(T-t)} \quad (1.1)$$

The exponential factor counteracts interest income.

Any **cash amount** in a portfolio grows by a factor of $e^{r(T-t)}$ in the time from now to expiration. This is reasonable, since the cash will be invested safely. Here, "safely" means that it is placed where the capital will not be at risk from market price changes and can be extracted immediately if needed for some better investment choice. The r denotes the *current* interest rate return on such an investment. During the spring of 2000, an r value of 0.055 per year was used for interest on short-term monetary investments.

The cash amount is adjusted to produce a desired target value for this portfolio on the expiration date of the forward contract.

On that date the portfolio gains a share of stock and pays the exercise price, but the contrived cash amount has grown into exactly the exercise price. In effect, the cash part of the investment disappears and there is no fee.

We can say that on the expiration date this portfolio *replicates* a share of stock. Certainly, the price is correct, since

$$\text{Contract value} + \text{cash amount} = \text{one share of stock}$$

Trading a Portfolio

Now, here is a surprising aspect of modern markets. Their structure allows this portfolio to be traded *before the expiration date* as though it were an equity. In fact, one can buy this type of contract at any time and set aside some cash; this amounts to *purchasing* one unit of the portfolio.

On the other hand, one usually can sell this contract in a market *even if one does not own it*. An investor serves as the writer of a contract and has the same obligations as a writer. When one sells a basic equity such as a stock without *owning it first*, and then purchases the stock later to make delivery, this activity is referred to as **shorting** or **short selling** the equity. It is possible to sell almost any stock short. Clearly, one can "short" the amount of *cash* in the portfolio just by borrowing some

money at the short-term interest rate r . In effect, we can *sell* one unit of this portfolio, even if we do not own it to begin with.

We have just explained that one can buy and sell an instrument, the portfolio, that replicates a share of stock on a future date. This leads us to a comparison of prices on an earlier date. We will apply a second major financial principle, *the absence of arbitrage opportunities*, to equate some prices.

First Arbitrage Opportunity

Suppose that today's price is not consistent with its future value. In fact, let us look at the case when

$$\text{Contract value} + \text{cash amount} < \text{one share of stock}$$

This creates a gold mine. The investor can **sell short** quite large amounts of the stock today. This produces instant cash for any investor who is bold enough to sell something that he or she does not own. An investor could use some of the cash to form the correct number of portfolio units to **cover** the short selling.

That is, when the expiration date arrives, the investor neutralizes all the short sales of stock using the **replicating portfolio** value as a stock value. You can see that he or she pockets some cash at the beginning, *regardless of future market behavior*, since it was cheap to cover the short selling.

Second Arbitrage Opportunity

If the reverse situation holds, that is,

$$\text{Contract value} + \text{cash amount} > \text{stock price}$$

the investor could *sell* units of the portfolio *short*, as we explained when we discussed trading the portfolio. Similar arithmetic shows that an investor who covers these short sales with *cheap* stock, purchased immediately after selling short, will still be able to sleep at night.

An investor will not become nervous as the expiration date approaches, because he or she knows that each *short* unit of portfolio will serve only as the liability for exactly one *short* share of stock on this date. Again, some cash is earned at the beginning, *regardless of future market behavior*.

Real markets would not allow either of these money-making schemes to work. We will discuss the reason for this momentarily, but for now, consider the consequence of not having either of the two price inequities discussed above. We obtain the following

No-Arbitrage Price Equation

$$\text{Today's contract value} + \text{cash amount} = \text{today's stock price}$$

We may substitute the values from the net worth equation (1.1) to obtain the formula

$$f + Xe^{-r(T-t)} = S_t$$

The relation can be restated as

$$f = S_t - Xe^{-r(T-t)} \quad (1.2)$$

Equation (1.2) shows that we have “solved” for f . To obtain today’s price for this contract, one obtains quotes of today’s stock price and the short-term interest rate. Once these ingredients are substituted into the formula above, one has a price for a forward stock contract beginning today.

The price should be recomputed each day as the stock price changes and the time to expiration decreases. The r value is unlikely to change in practice, because forward contracts usually expire within 90 days of their initiation. Since this time span is so short, the return on cash invested is far more sensitive to the time to expiration than it is to the rate for one-month or three-month cash investments.

Example Suppose we have a forward for Eli Lilly stock that will expire 40 days from now. If the exercise price is \$65, and if today’s stock price is $\$64\frac{3}{4}$, what is the contract price today?

We will use an r value of 0.055 per year. The quantities we substitute into equation (1.2) are

$$T - t = 40/365 = 0.1096 \text{ (so that } e^{-r(T-t)} = 0.994\text{)}$$

and the two quotes, which are

$$S_t = 64.75 \quad \text{and} \quad X = 65$$

The end result is

$$f = 64.75 - 65(0.994) = 64.75 - 64.61 = \$0.14$$

Another insight this formula gives is that, for the same strike and stock price in the example, a longer-term contract (say, for six months) will have a larger price, because the $e^{-r(T-t)}$ term changes to 0.974. This illustrates the usefulness of equation (1.2). It allows us to compare prices of forward contracts for various expiration dates and various strike prices.

Why Do Arguments Based on Replication and No Arbitrage Work?

The price formula given by equation (1.2) and the market price of a forward contract cannot be substantially different. Any sizable difference would induce people to follow one of the two investment tricks we discussed. The *guarantee* of profit would induce them to invest enormous sums of money in one of these schemes. Their activities would, in turn, move prices until the *arbitrage opportunity* was driven out of existence by changes in the underlying stock price. For example, if a large amount of a stock is sold short, its present value goes down because so much of it is offered for sale.

Put another way, the market pressures generated by the investment schemes of people who are *certain* to profit would force stock and contract prices into equilibrium values, where the arbitrage opportunity is missing.