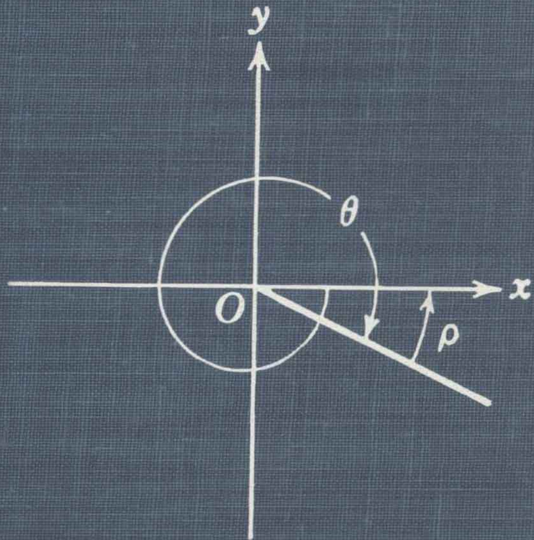

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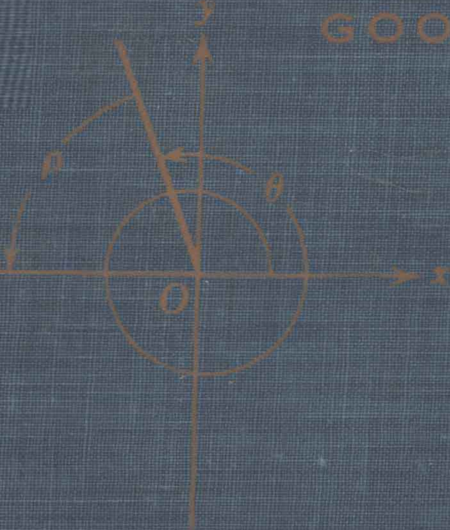
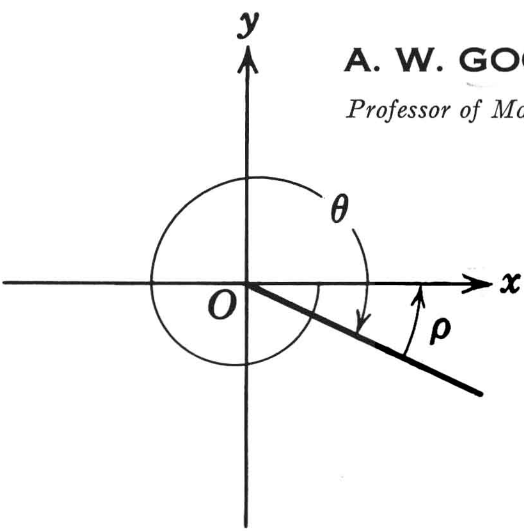


PLANE TRIGONOMETRY



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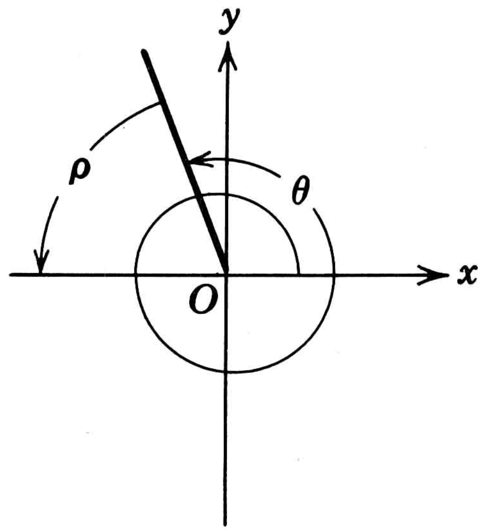

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PREFACE FOR THE STUDENT

★ Don't be frightened. Trigonometry is an easy subject, and you too can learn it. All that is required is a little effort on your part.

If you have had trouble with your mathematics courses in the past, there are several possible causes. It is easy for you to claim that you had poor teachers who were either boring, incompetent, overbearing, or repulsive, or a combination of these. You might even be right. But if you had a good textbook as you probably did, you really have no excuse. By studying a good textbook, you could and should learn any subject, despite a poor teacher. This brings us to the central point, "studying."

You probably just don't know how to study. No one has ever told you. Do you sit with the book open in front of you for an hour, and then claim you have been studying for an hour? Rubbish! I may as well sit with my feet dangling in the pool, and then claim I have been swimming for an hour. Or I may as well listen to Rubinstein piano recordings on the phonograph for an hour, and then claim I have been practicing piano for sixty minutes. Obviously, whether it is playing piano, swimming, or studying mathematics, you must actively participate in order to really learn a subject.

Here is a list of hints that may help you do this when you study mathematics. Since we are all individuals, some of the items in this list may not apply to you, but you may be able to add to this list some that do seem to help you.

1. After each sentence that you read, ask yourself: Does it make sense? Is it correct?

2. If the sentence or equation doesn't make sense, is there a misprint you can spot and correct for yourself?

3. If the sentence or equation still doesn't make sense, after a reasonable effort on your part, go on to the next item, and keep coming back to the difficult part. It may be that later parts of the exposition will clarify the earlier muddy sentence or equation.

4. After reading a section or paragraph, close your eyes and see how much you can remember. If you have just read a proof of a theorem, you should be able to reproduce it. Close the book, take a blank sheet of paper, and see if you can write down the proof. Open the book and see how far off you are.

5. After you have read several pages covering a number of related ideas, close the book and see if you can make an outline of the material. Make a diagram using arrows to show how the various ideas are connected.

6. Memorize word for word the statements of the theorems. Definitions of new words should receive similar respectful treatment. Although memorization without understanding is worthless, you should memorize any item that appears to be essential at the same time that you are trying to understand it, because memorization and understanding reinforce one another.

7. In working problems do not work toward the answer in the back of the book. Postpone looking at the answer until you have finished all the numerical work.

8. Must you study with the radio or the television set turned on? Most professional scholars prefer quiet when they are studying, although some of the truly great intellects seem to be able to work well even when it is noisy. If you are doing poor work, why not try for one semester to have absolute quiet when studying? See if you don't do better. If not, then at the end of the semester turn your radio or your roommate back on again.

9. Not all studying is done at a desk with a book. There are many times during the day when the mind is free to think: while brushing your teeth, while waiting for a bus, or riding the bus, while walking from one classroom to the next one, while delivering a paper route, etc. During such periods you should try to recall the important formulas, theorems, and proofs. Are there blank spots and fuzzy places? When you have a chance to look in the book, fill in the blank spaces and clear up the fuzzy ones.

The brain is very much like a muscle, and like a muscle you must use it in order to develop it. If you don't use it, it can atrophy. The man who waits until he gets to college to start studying has already done damage to his brain that he can never completely repair, although he can partly recover the lost ground by very hard work. Those who advise students in grade school and high school to avoid mathematics either because it is hard or useless are sabotaging the educational program of their country.

Now a word about this particular book. If you are working through this book without the guidance of a teacher, then you might very well omit the starred (★) sections without too much loss. The starred material is either very difficult material, which you might well skip completely or postpone for a time; or it is material that is not absolutely essential to the study of trigonometry.

The starred problems are somewhat difficult, and the double starred (★★) problems are still more difficult. If you find that trigonometry is hard for you, skip all the starred problems, and concentrate on the others. If, on the other hand, most of the problems seem to be too easy, the starred problems will probably keep you busy. One would expect an A student to try all of the starred problems, and to solve most of them.

When a new word or phrase is introduced, it is set in italics and the sentence in which it is used defines it. For example, in Chapter 4 the sentence,

"An angle is said to be in *standard position* if the vertex of the angle coincides with the origin and the initial side falls on the positive x-axis,"

defines the phrase *standard position* for an angle. You should realize that this is a definition giving the meaning of this phrase, and the italics are an automatic warning device telling you that you should learn this new word or phrase and its meaning. Later, if you have forgotten the meaning of some word or phrase you can look it up in the index. Here you will find a reference to the page on which the concept is defined, and in addition a list of other pages of the book where this concept is used. The index is there to help you, and it can help you only if you use it.

I have set in boxes those formulas you should memorize because of their importance. Of course, your teacher may have a different opinion on the amount of memory work that should be required of his students. In case you are in doubt as to which formulas to memorize, you should consult him.

Students frequently ask me why they should learn this topic or that proof, etc. They want to know where the item in question will be used, and although they don't say so, what they frequently want to know is, can it be used to make money. Instead of trying to answer the specific question, it is better to shift the subject matter and then ask the question again. Why should you do anything at all in life? Why should you learn to swim? Why should you learn to play a musical instrument? Why learn to appreciate classical music by listening to it? Basically, the answer is the same in all cases. You do these things because they are fun. Or if they aren't fun at first, you hope that as you acquire proficiency, they will become fun. Only a few professionals make money from playing basketball, or swimming, or listening to classical music, but nearly everyone enjoys at least one of these activities, and some fortunate individuals enjoy all three. If exercising your muscles by swimming or pole vaulting can give pleasure, there is no reason why you can't also enjoy exercising your brain by doing mathematics.

Mathematics is the greatest game ever invented by man.

Why don't you learn to play the game a little bit, and see if you don't enjoy it?

A. W. Goodman

Lexington, Kentucky
March, 1959

PREFACE FOR THE TEACHER

★ This book is neither ultramodern nor purely conventional. Most of trigonometry is reasonably well standardized, so I will mention only those few items in which this book differs from the majority of the textbooks currently on the market.

In Chapter 1 the trigonometric functions are defined for an acute angle as ratios of sides in a right triangle. Please don't jump. I am well aware that most textbooks now begin immediately with the general angle. This modern treatment is superior, so its advocates claim, because it disposes of the definitions once and for all, and saves the trouble of later changing the definitions. But how does the student regard this sophisticated approach? The answer is obvious: he doesn't even know it is sophisticated and he cares less. All that he sees is a confusing array of ratios which are unmotivated and therefore uninteresting to him. Very likely this sets up in him a subconscious resistance to learning the ratios. On the other hand, with the classical presentation, the importance of the ratios is first illustrated with suitable practical problems, and then after the student is convinced of their usefulness, names are given to the ratios, and he begins the study of their properties.

A really clever teacher will hint to his class that these definitions, given only for acute angles will later be enlarged to include all angles, and he will stimulate the more active students to wonder how this can be done. Only another Descartes in the class will actually anticipate the answer, but some of the more curious might look ahead in the book, and even those who merely wait patiently to be shown must admire the skillful way in which the definitions of the trigonometric functions for acute angles are extended to the general case.

Actually the modern treatment defines the trigonometric functions only for real θ, and not for complex θ. The teacher who is really sincere about his desire to give a definition of $\sin \theta$ valid for all θ should start his trigonometry course with some material on infinite series and then define $\sin \theta$ by its Maclaurin series

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

Of course this is obviously impractical. It is far better to give the student a narrow definition valid only in a limited range, and then as his knowledge of mathematics expands, modify the definition to fit the enlarged picture. Indeed this process of modifying a definition is one of the central features of modern mathematics, and it seems a shame to deny a student the opportunity to observe this process in trigonometry where it occurs in a very simple and clear form.

In Chapter 7, the addition formulas for the trigonometric functions are proved. The proofs are based ultimately on the invariance of the distance between two points under a rotation of the plane. The advantage of this type of proof is that it is valid for all real angles. The traditional proof given by stacking one triangle on top of another is valid only for angles in a certain restricted range, and the extension of this range is always a nuisance. In most cases the student is expected to accept on faith the general validity of the addition formulas.

In Chapter 5, I present a duality principle for trigonometric identities. Although this material is extremely simple and is perhaps known to many mathematicians, I have never seen it in print before. There is a possibility that this duality principle represents a small but new contribution to trigonometry.

This book can be used in a variety of ways as indicated.

Course of 45 Lessons

Chapter	1	2	3	4	5	6	7	8	9	10	11	13	14	Exams
Number of lessons	5	5	2	2	4	4	5	2	2	2	2	1	3	6

A short course in which the emphasis is on the applications of trigonometry could use the following plan:

Course of 30 Lessons

Chapter	1	2	3	4	5	6	8	13	Exams
Number of lessons	5	5	2	2	4	4	2	2	4

If the students have had trigonometry in high school, so that the practical applications can be safely ignored, the course could begin with Chapter 4:

Course of 30 Lessons—Analytrical Trigonometry

Chapter	4	5	6	7	8	9	10	11	13	14	Exams
Number of lessons	2	4	2	5	2	2	2	2	1	3	5

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March, 1959

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