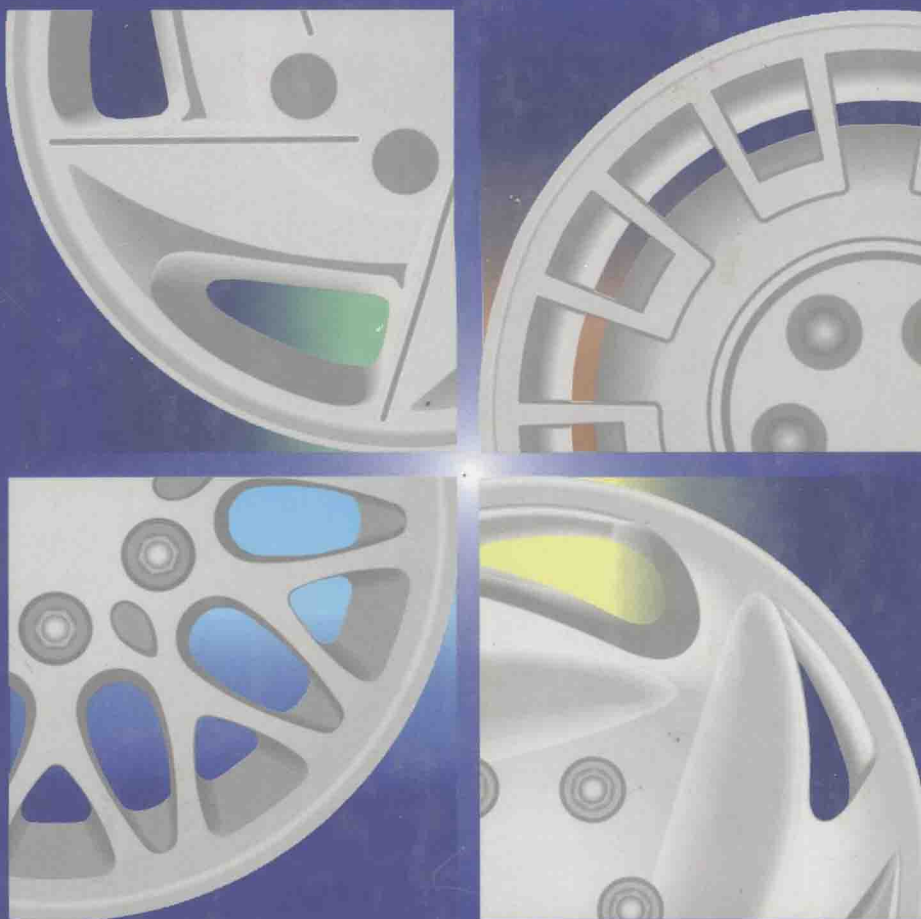


SIXTH EDITION

A Problem Solving Approach to

MATHEMATICS

for Elementary School Teachers



BILLSTEIN LIBESKIND LOTT

A Problem Solving Approach to **Mathematics for** **Elementary School Teachers**

Sixth Edition

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 **ADDISON-WESLEY**

An imprint of Addison Wesley Longman, Inc.

Reading, Massachusetts • Menlo Park, California • New York • Harlow, England
Don Mills, Ontario • Sydney • Mexico City • Madrid • Amsterdam

Senior Editor • *Bill Poole*
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Cover Designer • *Barbara Atkinson*
Composition/Prepress Services • *Typo-Graphics, Inc.*
Illustrations • *Typo-Graphics, Inc., James A. Bryant*

Library of Congress Cataloging-in-Publication Data

Billstein, Rick.

A problem solving approach to mathematics for elementary school teachers / Rick Billstein, Shlomo Libeskind, Johnny W. Lott.—6th ed.

p. cm.

Sixth ed. also published under title: A problem solving approach to mathematics.

Includes index.

ISBN 0-201-56649-4 (hardcover)

1. Mathematics — Study and teaching (Elementary) 2. Problem solving. I. Libeskind, Shlomo. II. Lott, Johnny W., 1944— III. Billstein, Rick. Problem solving approach to mathematics. 6th ed. IV. Title.

QA135.5.B49 1996b

372.7—dc20

96-26205

CIP

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1 2 3 4 5 6 7 8 9 10 - DOW - 99989796

In memory of Lee Yunker, a former student, colleague, and friend whose enthusiasm and hard work has left math education much richer.

RWB

In memory of my father Mendel, my uncles Janek and Moshe, and my aunt Chancia.

SL

To Thomas J. Brieske, who helped shape my personal vision as a teacher; to my co-authors, Rick and Shlomo, who share with me a cooperative vision of mathematics; to the Montana Council of Teachers of Mathematics for broadening my vision of mathematics education; and to the National Council of Teachers of Mathematics for providing a vision of mathematics education for the new century.

JWL

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Preface

The sixth edition of *A Problem Solving Approach to Mathematics for Elementary School Teachers* continues to focus on the goals of the previous editions, but extends its reach to meet the expectations for mathematics education for elementary teachers in the next century.

Standards of the NCTM

In the sixth edition, we continue to focus on two National Council of Teachers of Mathematics (NCTM) publications, the 1989 *Curriculum and Evaluation Standards of School Mathematics* (hereafter referred to as the *Standards*), and the 1991 *Professional Standards for Teaching Mathematics* (hereafter referred to as the *Teaching Standards*). Of primary importance to us is the standards (*Standards*, p. 253): “Prospective teachers must be taught in a manner similar to how they are to teach—by exploring, conjecturing, communicating, reasoning, and so forth.” In addition, “all teachers need an understanding of both the historical development and current applications of mathematics. Furthermore, they should be familiar with the power of technology.” In particular, the *Teaching Standards* (p. 3) emphasize the need for the following shifts in the teaching of mathematics.

- Toward logic and mathematical evidence as verification—away from the teacher as the sole authority for right answers;
- Toward mathematical reasoning—away from merely memorizing procedures;
- Toward conjecturing, inventing, and problem solving—away from an emphasis on mechanistic answer-finding;
- Toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures.

To achieve the aforementioned shifts, the sixth edition allows instructors a variety of approaches to teaching, encourages discussion and collaboration among students and with their instructors, allows for the integration of projects into the curriculum, and promotes discovery and active learning.

Continuing Goals

In the sixth edition our goals remain

- To present appropriate mathematics in an intellectually honest and mathematically correct manner.
- To use the heuristic of problem solving as an integral part of mathematics.
- To present the topics in the context of the *Standards*.
- To approach mathematics in a sequence that instills confidence, then challenges students.
- To identify and use the various problem solving strategies.

New Goals

For this edition, we have additional goals:

- To provide opportunity for alternate forms of teaching and learning, such as discovery methods, and group interaction and open-ended problem solving.

- To provide and expand communication problems so that future teachers can develop writing skills and practice the explanation of their thinking.
- To allow for additional integration of technology tools where appropriate.
- To revise the coverage of several topics, in order to make them more accessible.
- To provide core mathematics for prospective elementary teachers in a way that they are challenged to determine why mathematics is done as it is.
- To provide core mathematics that allows instructors to use methods integrated with content.

Problem Solving in the Sixth Edition



We showcase problem solving skills by:

- Devoting Chapter 1 entirely to problem solving skills, and expanding coverage in the sixth edition to place added emphasis on deductive versus inductive reasoning.
- Using a four-step problem solving process to solve problems in each chapter.
- Beginning each chapter with a preliminary problem that poses a question students can answer with the skills mastered from that particular chapter.

We encourage teachers to point out and discuss the preliminary problem at the beginning of each chapter to show how the techniques therein are necessary to solve the problem.

Features Retained in this Edition

Wherever possible, we present topics in ways that could be used in actual classrooms. Further, we have incorporated various study aids and features to facilitate learning.

- **Historical Notes** add context and humanize the mathematics.
- **Brain Teasers** provide a different avenue for problem solving. They are solved in the Instructor's Guide, and may be assigned or used by the teacher to challenge students.
- **Laboratory Activities** are integrated throughout the book to provide hands-on learning exercises. In each non-optional geometry section, van Hiele-type laboratory activities are included.
- **Cartoons** teach or emphasize important material and add levity.
- **Key Terms** are presented in the margins for quick review.
- **Definitions** are either set off in text or presented as key terms in the margin for quick review.
- **Review Problems** are included at the end of each non-optional section.
- **Optional Sections** as well as problems based on these sections are marked with an asterisk (*); more difficult problems are marked with a star (★). Problems numbered in color have answers at the back of the book.
- **Questions from the Classroom** have been continued. We strongly recommend that instructors use these questions posed by actual students in actual classrooms when building a course syllabus. Instructors may require students to write two answers to the questions—one mathematical and one pedagogical—using student texts and professional journals for research.
- **Chapter Outlines** at the end of each chapter help students review the chapter.
- **Chapter Reviews** at the end of each chapter allow students to test themselves.
- **Selected Bibliographies** have been updated and revised. They are at the end of each chapter.
- **Problem Solving Strategies** are often highlighted in italics, and indicated by .
- **Relevant Quotes** from the *Standards* and *Teaching Standards* are incorporated throughout the text, and marked by the standard icon .

- In view of the *Standards*' emphasis on communicating mathematical ideas, problem sets contain numerous problems in which students are asked to explain or justify their answers.
- **Full Color** has been used for pedagogical reasons and to help students visualize concepts. Figures are more modern, attractive, and easy to follow. All of the pages taken from elementary mathematics texts are presented in full color.

Features New to this Edition

- **Problem Sets** now contain six different types of problems, (1) on-going assessment, (2) communication, (3) open-ended, (4) cooperative learning, (5) technology, and (6) review. **Communication**, **Open-ended**, and **Cooperative Learning** are new problem sections introduced to conform with the major points stressed in the *Standards*.
- **More Relevant and Realistic Problems** have been added to the problem sets to appeal to and be more accessible to students of diverse backgrounds.
- **Technology Corners** now replace the feature formerly called "Computer Corners," and have been expanded and enhanced to include use of Logo, spreadsheets, both graphing and scientific calculators, Geometer's Sketchpad, and computer activities.
- **Investigations** appear throughout each chapter, and are intended to help students become actively involved in their learning, to facilitate the development and improvement of their critical thinking and problem solving skills, and to stimulate both in-class and out-of-class discussion.

Content

The sixth edition has been slightly reorganized and streamlined, with some new content added.

Chapter 1 An Introduction to Problem Solving

Chapter 1 has been expanded and reorganized. More emphasis is placed on inductive versus deductive reasoning. The calculator section has been expanded to include fraction calculators.

Chapter 2 Sets, Functions, and Logic

Section 2-3 on functions has been rewritten from a more concrete and application-oriented point of view. Relations are introduced in the problem set as generalizations of functions. In the optional section on logic (Sec. 2-4), more emphasis is placed on the understanding of conditionals and biconditionals through concrete examples and analogy between these concepts and set inclusion.

Chapter 3 Numeration Systems for the Ages

In Chapter 3, numeration systems and other number bases have been combined into one coherent package; students can now see where other number bases are actually used.

Chapter 4 Integers and Number Theory

Chapters 4 and 5 from the previous edition have been combined into one chapter. The section on solving equations (previously Sec. 4-3) has been reworked and is now found in the new Chapter 7.

Chapter 5 Rational Numbers as Fractions

Chapter 5 has been shortened and reorganized. The topic of comparing rational numbers has been placed at the beginning of the chapter. In addition, this chapter contains more pictures to help describe the concepts and to assist students with visualization. The presentation of division of rationals has been modified. The sections on ratio and proportion and exponents have been moved to later chapters.

Chapter 6 Exponents and Decimals

Chapter 6 now provides a full treatment of positive and negative integer exponents immediately before they are needed for the development of decimals.

Chapter 7 Applications of Mathematics

Besides a commitment to increased applications in the examples and problem sets throughout the text, the sixth edition has a new chapter with emphasis on applications. Much of the material is new. Coordinate systems are introduced in Section 7-3 and used in subsequent sections. Section 7-4 on ratio and proportion (previously in Chap. 5) includes new topics such as scale drawing, direct and inverse variation, and levers.

Chapter 9 Statistics: An Introduction

Coverage in Chapter 9 has been expanded to include an introduction on different types of graphs, such as double bar graphs. There is more material on box plots and additional uses for box plots. The section on abuses of statistics is no longer marked “optional.”

Chapter 10 Introductory Geometry

Section 10-3 on linear measure and circumference of a circle is new, although some of the material in this section appeared in Chapter 13 of the previous edition. Section 10-4, More about Angles, has been completely rewritten from a more intuitive, application-oriented approach.

Chapter 11 Constructions and Similarity

Chapter 11 includes a reorganized discussion of constructions. The separate section on circles and spheres has been eliminated and the material moved into other sections. Some material on coordinate geometry has been moved into a section on similarity to show how slope is developed. A new section on trigonometry ratios via similarity has also been added.

Chapter 12 More Concepts of Measurement

Linear measure, previously covered in this chapter, has been moved to Chapter 10 (Sec. 10-3) as it was needed earlier. (Chap. 12 was previously Chap. 13.)

Chapter 13 Motion Geometry and Tessellations

In Section 13-1, coordinate representation of translations is a new topic. In addition, slopes of perpendicular lines are investigated as an application of rotations. In Section 13-2, reflections in a coordinate system, a topic not presented in the previous edition, is introduced. New coverage also includes treatment of light reflecting from a surface. In Section 13-3, new material includes applications of size transformations, such as perspective drawing and photography.

Appendices

Three new appendices have been added to the sixth edition, including

- Graphing Calculators,
 - Geometry Utility (based on Geometer's Sketchpad), and
 - Using a Spreadsheet (based on Microsoft Excel).
- The Logo Turtle Graphics appendix has been streamlined and shortened from two sections to one section.

Calculator Usage

As prescribed in the *Standards*, coverage of calculators is necessary and timely. Calculators are introduced in Chapter 1, and a discussion of the use of scientific/fraction calculators appears in that chapter. The use of the graphing calculator is presented, where relevant, in the new Technology Corners, and problems involving the use of both scientific/fraction and graphing calculators appear in the problem sets.

Supplements for the Student

Student's Solutions Manual, by Louis Levy and Edward Fritz, contains detailed solutions to all odd-numbered exercises.

Activities Manual—*Mathematics Activities for Elementary School Teachers: A Problem Solving Approach, Third Edition*, by Daniel Dolan, Jim Williamson, and Mary Muri. This revised edition features activities that can be used to develop, reinforce, and/or apply mathematical concepts. The activities for each concept are ordered by developmental level within each chapter.

Supplements for the Instructor

Instructor's Resource Guide includes: Answers to all problems in the text; two forms of chapter assessments with answers for each chapter; suggested answers to Questions from the Classroom; Solutions to the Brain Teasers, and suggested answers to the Chapter Investigations.

Instructor's Solutions Manual, by Louis Levy and Edward Fritz, contains detailed solutions to all exercises.

Instructor's Guide to *Mathematics Activities for Elementary School Teachers: A Problem Solving Approach, Third Edition*, by Daniel Dolan, Jim Williamson, and Mary Muri, contains answers for all activities, as well as additional teaching suggestions for some activities.

OmniTest³ is available in DOS-based and Macintosh formats. This new version of this easy-to-use software was developed for Addison-Wesley by ips Publishing, a leader in computerized testing and assessment.

- DOS-based and Macintosh formats.
- The Macintosh format makes full use of the Macintosh graphical user interface.

- DOS user interface is easy to learn and operate. Its Windows look-alike structure lets you easily choose and control the items as well as the format for each test.
- You can quickly and easily create make-up exams, customized homework assignments, and multiple test forms.
- OmniTest³ is algorithm driven—meaning the program can automatically insert new numbers into the same equation—creating hundreds of variations of that equation.
 - The numbers are constrained to keep answers reasonable, so that you can create a virtually endless supply of parallel versions of the same test.
 - With this new version of OmniTest you can “lock in” the values shown in the model problem, if you wish.
- OmniTest³ is keyed section by section to the text, allowing you to select questions that test individual objectives from that section.
- You can also enter your own questions by way of OmniTest³'s sophisticated editor—complete with mathematical notation.

Acknowledgments

Reviewers of This and Previous Editions

The authors wish to thank the following for their helpful comments and suggestions for this and previous editions of the text.

Leon J. Ablon	Herbert E. Kasube
G. L. Alexanderson	Sarah Kennedy
Bernadette Antkowiak	Steven D. Kerr
Jane Barnard	Leland Knauf
Joann Becker	Margret F. Kothmann
Cindy Bernlohr	Hester Lewellen
James Bierden	Ralph A. Liguori
Jim Boone	Don Loftsgaarden
Sue Boren	Stanley Lukawecki
Beverly R. Broomell	Barbara Moses
Maurice Burke	Charles Nelson
David Bush	Glenn Nelson
Laura Cameron	Dale Oliver
Louis J. Chatterley	Keith Peck
Phyllis Chinn	Barbara Pence
Donald J. Dessart	Glenn L. Pfeifer
Ronald Dettmers	Jack Porter
Jackie Dewar	Edward Rathnell
Amy Edwards	Helen R. Santiz
Margaret Ehringer	Jane Schielack
Albert Filano	Barbara Shabell
Marjorie Fitting	M. Geralda Shaefer
Michael Flom	Gwen Shufelt
Martha Gady	Ron Smit
Sandy Geiger	Joe K. Smith
Glenadine Gibb	William Sparks
Elizabeth Gray	Virginia Strawderman
Alice Guckin	Viji Sundar
Boyd Henry	C. Ralph Verno
Alan Hoffer	Hubert Voltz
E. John Hornsby, Jr.	John Wagner
Judith E. Jacobs	Virginia Warfield
Jerry Johnson	Mark F. Weiner
Wilburn C. Jones	Grayson Wheatley
Robert Kalin	Jerry L. Young

BRIEF CONTENTS

Chapter 1	An Introduction to Problem Solving	1
Chapter 2	Sets, Functions, and Logic	51
Chapter 3	Numeration Systems for the Ages	107
Chapter 4	Integers and Number Theory	175
Chapter 5	Rational Numbers as Fractions	251
Chapter 6	Exponents and Decimals	293
Chapter 7	Applications of Mathematics	337
Chapter 8	Probability	411
Chapter 9	Statistics: An Introduction	471
Chapter 10	Introductory Geometry	537
Chapter 11	Constructions and Similarity	615
Chapter 12	More Concepts of Measurement	677
Chapter 13	Motion Geometry and Tessellations	749
Appendix I	Logo Turtle Graphics	815
Appendix II	Graphing Calculators	835
Appendix III	Using a Geometry Drawing Utility	845
Appendix IV	Using a Spreadsheet	853
	Answers to Selected Problems	861
	Index	909

Solution to the cover puzzle follows the index.

CONTENTS

1 An Introduction to Problem Solving 1

- Preliminary Problem 1
- 1-1 Explorations with Patterns 3
- 1-2 Mathematics and Problem Solving 19
- 1-3 Using a Calculator as a Problem-Solving Tool 39
- Solution to the Preliminary Problem 46
- Questions from the Classroom 47
- Chapter Outline 48
- Chapter Review 48
- Selected Bibliography 50

2 Sets, Functions, and Logic 51

- Preliminary Problem 51
- 2-1 Describing Sets 52
- 2-2 Other Set Operations and Their Properties 64
- 2-3 Functions 74
- *2-4 Logic: An Introduction 90
- Solution to the Preliminary Problem 101
- Questions from the Classroom 102
- Chapter Outline 103
- Chapter Review 104
- Selected Bibliography 106

3 Numeration Systems for the Ages 107

- Preliminary Problem 107
- 3-1 Numeration Systems 108
- 3-2 Addition and Subtraction of Whole Numbers 119
- 3-3 Multiplication and Division of Whole Numbers 129
- 3-4 Algorithms, Mental Math, and Estimation for Whole-number Addition and Subtraction 140
- 3-5 Algorithms for Whole-number Multiplication and Division 156
- Solution to the Preliminary Problem 169

Questions from the Classroom	170
Chapter Outline	171
Chapter Review	171
Selected Bibliography	172

4 Integers and Number Theory 175

Preliminary Problem	175
4-1 Integers and the Operations of Addition and Subtraction	176
4-2 Multiplication and Division of Integers	190
4-3 Divisibility	199
4-4 Prime and Composite Numbers	211
4-5 Greatest Common Divisor and Least Common Multiple	227
*4-6 Clock and Modular Arithmetic	238
Solution to the Preliminary Problem	245
Questions from the Classroom	246
Chapter Outline	247
Chapter Review	248
Selected Bibliography	249

5 Rational Numbers as Fractions 251

Preliminary Problem	251
5-1 The Set of Rational Numbers	252
5-2 Addition and Subtraction of Rational Numbers	263
5-3 Multiplication and Division of Rational Numbers	277
Solution to the Preliminary Problem	288
Questions from the Classroom	290
Chapter Outline	290
Chapter Review	291
Selected Bibliography	292

6 Exponents and Decimals 293

Preliminary Problem	293
6-1 Integer Exponents and Decimals	294
6-2 Operations on Decimals	307
6-3 Real Numbers	325
Solution to the Preliminary Problem	333
Questions from the Classroom	334
Chapter Review	334
Chapter Outline	335
Selected Bibliography	336

7 Applications of Mathematics 337

- Preliminary Problem 337
- 7-1** Algebraic Thinking 338
- 7-2** Word Problems 351
- 7-3** Lines in a Cartesian Coordinate System 358
- 7-4** Ratio and Proportion 374
- 7-5** Percents 387
- *7-6** Computing Interest 399
- Solution to the Preliminary Problem 405
- Chapter Outline 407
- Questions from the Classroom 408
- Chapter Review 408
- Selected Bibliography 410

8 Probability 411

- Preliminary Problem 411
- 8-1** How Probabilities Are Determined 412
- 8-2** Multistage Experiments with Tree Diagrams and Geometric Probabilities 424
- 8-3** Using Simulations in Probability 438
- 8-4** Odds and Expected Value 448
- 8-5** Methods of Counting 455
- Solution to the Preliminary Problem 465
- Questions from the Classroom 466
- Chapter Outline 467
- Chapter Review 467
- Selected Bibliography 469

9 Statistics: An Introduction 471

- Preliminary Problem 471
- 9-1** Statistical Graphs 473
- 9-2** Measures of Central Tendency and Variation 492
- *9-3** Normal Distributions 511
- 9-4** Abuses of Statistics 522
- Solution to the Preliminary Problem 530
- Questions from the Classroom 531
- Chapter Outline 532
- Chapter Review 533
- Selected Bibliography 535

10 Introductory Geometry 537

- Preliminary Problem 537
- 10-1 Basic Notions 538
- 10-2 Polygonal Curves 554
- 10-3 Linear Measure 565
- 10-4 More about Angles 578
- 10-5 Geometry in Three Dimensions 590
- *10-6 Networks 602
- Solution to the Preliminary Problem 608
- Questions from the Classroom 609
- Chapter Outline 610
- Chapter Review 611
- Selected Bibliography 613

11 Constructions and Similarity 615

- Preliminary Problem 615
- 11-1 Congruence through Constructions 616
- 11-2 Other Congruence Properties 628
- 11-3 Other Constructions 634
- 11-4 Similar Triangles and Similar Figures 644
- *11-5 Trigonometry Ratios via Similarity 661
- Solution to the Preliminary Problem 671
- Questions from the Classroom 672
- Chapter Outline 673
- Chapter Review 674
- Selected Bibliography 676

12 More Concepts of Measurement 677

- Preliminary Problem 677
- 12-1 Areas of Polygons and Circles 678
- 12-2 The Pythagorean Theorem 697
- 12-3 Surface Areas 711
- 12-4 Volumes 721
- *12-5 Mass and Temperature 735
- Solution to the Preliminary Problem 742
- Questions from the Classroom 743
- Chapter Outline 744
- Chapter Review 745
- Selected Bibliography 747