Applied Probability and Stochastic Processes

In Engineering and Physical Sciences

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Preface

Application of probability and stochastic process theory is playing an ever increasing role in a number of diverse fields in engineering and the physical sciences. This is due, in part, to the growing realization that many random phenomena observed in physics and engineering are described with reasonable accuracy following recent comprehensive advances in stochastic prediction methodologies. As a consequence, the probabilistic approach has become an important component of practical reasoning in physical sciences as well as an integrated part of modern design technology in engineering.

This book is designed to give senior undergraduate and graduate students and researchers in engineering and the physical sciences a thorough understanding of the modern concepts of stochastic process theory and its application for predicting statistical characteristics of random phenomena. Toward this end, emphasis is placed on clarification of basic principles supporting current prediction techniques and practical application of prediction methods.

No advanced knowledge of probability theory on the part of the reader is assumed. However, a sound knowledge of advanced calculus and functional analysis is essential in order to comprehend the mathematical analysis. For the readers' convenience a brief review of certain subjects such as the Fourier transform, the Hilbert transform, the unit impulse function, etc., which are useful in understanding the prediction techniques, are summarized in the appendices.

This book consists of two parts, although no such formal division is designated in the text. The first part consists of Chapters 1 through 8, which present probability theory relevant to probabilistic analysis of stochastic processes. Effort in these chapters is devoted to selecting subjects pertinent to predictions appearing in later chapters (the second part of the text), rather than to introducing general topics in probability theory. Needless to say, probability theory is a prerequisite for predicting the statistical as well as the quantitative properties of random phenomena.

Chapter 9 through 17 discuss principles and advanced techniques in the various subjects in stochastic processes and their application in the analysis of random phenomena observed in engineering and the physical sciences. In particular, the principles and procedures of spectral analysis and development of the probability density function derived therefrom are discussed in detail, since these provide the basis for modern probabilistic prediction techniques. Included also is material found in the recent literature but which has not been incorporated in textbooks such as higher order spectral analysis, the joint probability distribution of amplitudes and periods, and non-Gaussian random processes. Many examples are provided in order to facilitate understanding of the material.

This book is a direct result of my teaching and research in stochastic processes, and I am grateful to the College of Engineering, University of Florida, for granting me sabbatical leave during which significant progress in this undertaking was achieved.

I wish to acknowledge the encouragement and suggestions received from a number of learned scholars in the diverse fields of mathematical statistics, physics, and engineering. I am especially indebted to Professor Longuet-Higgins, University of Cambridge, and Professor Emeritus St. Denis, University of Hawaii, who inspired me to apply in depth the stochastic process approach to engineering problems. I would like to express my sincere appreciation to Mrs. Cathy Freeman and Ms. Amanda Graham for typing the manuscript, and to Ms. Lillean Pieter for drawing the illustrations. Assistance in editorial work rendered by my wife Margaret for the final preparation of the manuscript is deeply appreciated.

MICHEL K. OCHI

Gainesville, Florida
October 1989

Contents

CHAPTER 1

Elements of Probability

1.1 BASIC CONCEPT

The theory of probability deals with the mathematical analysis of quantities obtained from observations of random phenomena. Here, the term *random phenomena* is defined as phenomena that in repeated observations under identical circumstances do not nearly yield the same outcomes. There is no deterministic regularity in the occurrence of outcomes; instead, there is a statistical regularity in the sense that the relative frequency of occurrence of the outcome may be evaluated. That is, the relative frequency of occurrence of the event fluctuates, but the degree of the fluctuation decreases, in general, with the increase in the number of observations and therefore the frequency settles to a certain value.

To elaborate on the above statement, let us consider, as an example, the magnitude of peak-to-trough excursions of wind-generated waves in the ocean. As shown in Figure 1.1(a), the magnitude of the excursion, denoted by X in the figure, varies in random fashion from one wave to another, and hence it may be said that there is no deterministic regularity. If the observed data of X are classified in 1/2-m intervals, for example, and the relative frequency of occurrence of X is calculated for each interval, then we can obtain the relative frequencies as a function of X which is called the histogram. The shape of the histogram is inconsistent when the number of observations is small. However, the degree of inconsistency is reduced and converges to a certain shape as shown in Figure 1.1(b) with the increase in the number of observations, for example, on the order of 200. This may be called statistical regularity.

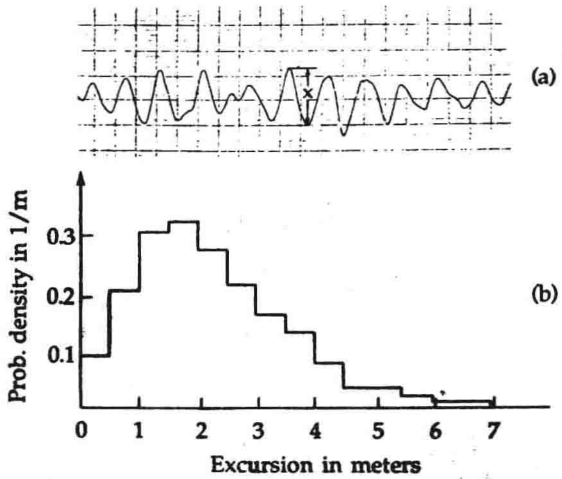

Figure 1.1 Time history of ocean waves and histogram of peak-to-trough excursions.

Prior to continuing further discussion on the fundamental concept of probability, it may be well to give the definition of sample space and a random event.

Definition 1.1. In the observation of a random phenomenon or random experiment, a set consisting of all possible outcomes that could occur is called the *sample space* and is denoted by Ω. A set belonging to the sample space for which the probability can be defined is called a *random event*.

The relative frequency of occurrence of a random event fluctuates even though observations (or experiments) are repeated under the same environment. However, it approaches a stable limit value as the number of observations becomes large, and this limit value is called the *probability* of the random event.

Example 1.1. Let us consider the simple random experiment of tossing a fair coin. The outcome of this experiment is either a head, H, or a tail, T. Hence, the sample space contains two elements, $\Omega = \{H, T\}$. Suppose we are interested in the occurrence of a head, then $\{H\}$ is a random event. Although we cannot predetermine the result of any particular toss, the frequency of occurrence of a head will converge to a certain limit value, 0.5, after many tosses. This limit value is called the probability of occurrence of a head. ■

Example 1.2. Let us consider the launching of a missile from a submarine. The outcome of this random experiment is either a success, S, or a failure, F, and hence the sample space for this example is given by $\Omega = \{S, F\}$. This situation is exactly the same as shown in Example 1.1. The relative frequency of the random event, $\{S\}$, will converge to a certain limit value after many trials, but the value may not necessarily be 0.5. This is because, unlike the case of a fair coin, a success significantly depends on various factors such as performance of the launching device and the control mechanism of the missile. ■

Example 1.3. Let us consider the launching of a missile three times from a submarine. Although the outcome is either a success, S, or a failure, F, the sample space for this case does not consist of only two elements. Note that the sample space is a set consisting of all possible outcomes; hence, for this example, we have a set consisting of eight outcomes:

$$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7, \omega_8\}$$

where
$$\begin{aligned} \omega_1 &= \mathrm{S\ S\ S} & \omega_5 &= \mathrm{F\ S\ S} \\ \omega_2 &= \mathrm{S\ S\ F} & \omega_6 &= \mathrm{F\ S\ F} \\ \omega_3 &= \mathrm{S\ F\ S} & \omega_7 &= \mathrm{F\ F\ S} \\ \omega_4 &= \mathrm{S\ F\ F} & \omega_8 &= \mathrm{F\ F\ F} \end{aligned}$$

Suppose we are interested in the possibility of hitting the target (even one hit is acceptable), then the random event is a set $\{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$. If we want to know the possibility of hitting the target at least twice, then the random event is a set $\{\omega_1, \omega_2, \omega_3, \omega_5\}$. The relative frequencies of occurrence of the random events for this example will be shown later. ■

Example 1.4. The wind-generated wave profile (the deviation from the still water level) is observed at a location where the water depth is 5 m. The sample space of this example consists of elements that take any value between -5 and ∞. Suppose we are interested in the possibility that the wave profile will exceed ± 2 m, then a set of continuous ranges $\{(-5, -2)$ and $(2, \infty)\}$ is the random event. ■

The discussion thus far briefly outlines the fundamental concept of probability in a heuristic sense. Modern probability theory, however, has been developed based on a rigorous mathematical foundation that provides a precise definition of probability, random variables, probability functions, and so on, so that the outcome of random events or experiments can be mathematically described. To discuss fundamental probability theory, it is necessary to use several definitions and terminologies from fundamental set theory. These are summarized in the following section.

1.2 ALGEBRA OF SETS AND FIELDS

A *set* is a collection of objectives. Each member, x, of a set A is called an *element* of set A, and is denoted by $x \in A$.

Definition 1.2. If every element of a set A_2 is also an element of set A_1, then A_2 is the *subset* of the set A_1 and is denoted by $A_2 \subset A_1$.

We may write the definition of a subset as follows:

$$A_2 \subset A_1 = \{x \in A_2;\ x \in A_2 \text{ implies } x \in A_1\}$$

It may be very convenient to illustrate various definitions concerning the algebra of sets by a pictorial sketch called a *Venn diagram*. For example, Figure 1.2 is a Venn diagram indicating the definition of a subset.

Definition 1.3. Two sets A_1 and A_2 are said to be *equal*, denoted by $A_1 = A_2$, if $A_2 \subset A_1$ and $A_1 \subset A_2$.

Definition 1.4. A set that contains no elements is called the *empty set* or *null set*, and is denoted by $A = 0$.

Definition 1.5. The set of all elements that belong to at least one of the sets $A_1, A_2, \ldots, A_n$ is called the *union* of the sets A_i, $i = 1, 2, \ldots, n$, and is denoted by $\bigcup_{i=1}^{n} A_i$.

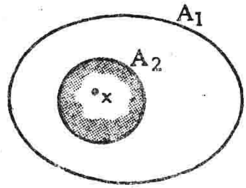

Figure 1.2 Subset $A_2 \subset A_1$.

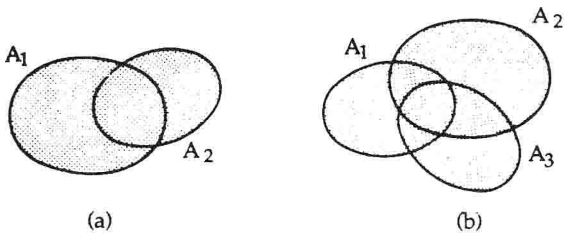

Figure 1.3 Unions (a) $A_1 \cup A_2$ and (b) $A_1 \cup A_2 \cup A_3$.

We may write the definition as

$$\bigcup_{i=1}^{n} A_i = \{x \in A_i \text{ for at least one } i = 1, 2, \ldots, n\}$$

Figures 1.3(a) and (b) show the unions of $A_1 \cup A_2$ and $A_1 \cup A_2 \cup A_3$, respectively.

Definition 1.6. The set of all elements that belong to each of the sets $A_1, A_2, \ldots, A_n$ is called the *intersection* of the sets $A_1, A_2, \ldots, A_n$, and is denoted by $\bigcap_{i=1}^{n} A_i$.

We may write

$$\bigcap_{i=1}^{n} A_i = \{x \in A_i \text{ for all } i = 1, 2, \ldots, n\}$$

Figures 1.4(a) and (b) show the intersections $A_1 \cap A_2$ and $A_1 \cap A_2 \cap A_3$, respectively.

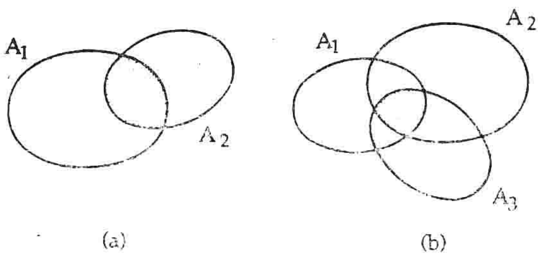

Figure 1.4 Intersections (a) $A_1 \cap A_2$ and (b) $A_1 \cap A_2 \cap A_3$.