

PLATO®

Interactive Mathematics
Prealgebra

Personal Academic Notebook

Interactive Mathematics **Prealgebra**



Prealgebra Personal Academic Notebook Contributors

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TOPIC 1 REAL NUMBERS

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LESSON F1.1 – WHOLE NUMBERS I

$$6 + 9 = 15$$

$$253$$

$$\$1.00$$

$$\begin{array}{r} 253 \\ - 64 \\ \hline \end{array}$$

$$189$$



Overview

In this lesson you will study how to add, subtract, multiply, and divide whole numbers. You will also learn how to round whole numbers and how to predict whether a whole number is divisible by 2, 3, 5, or 10.





Explain

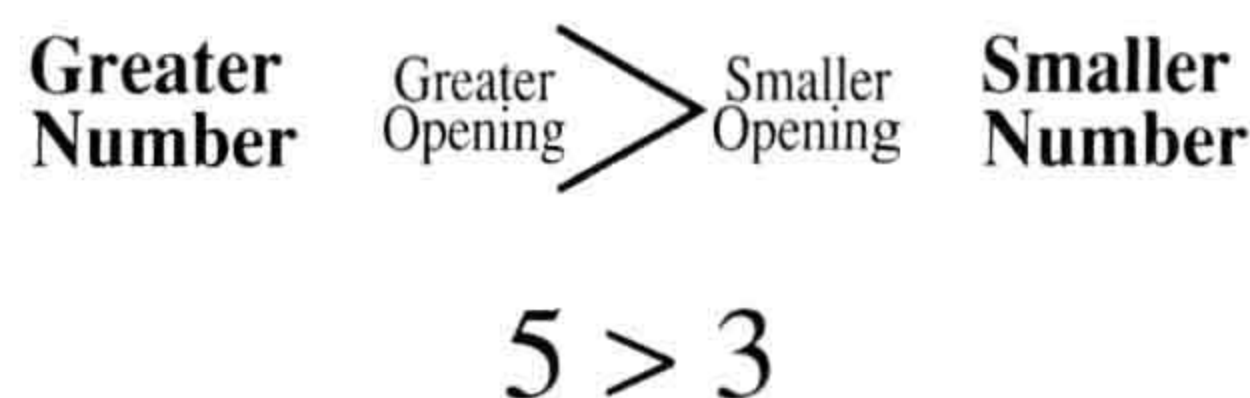
In Concept 1: Adding and Subtracting, you will find a section on each of the following:

- Using the Symbols $<$, $\leq$, $>$, or $\geq$ to Compare Two Numbers
- Finding the Place Value of a Digit in a Number
- Reading a “Large” Number
- Adding Whole Numbers
- Subtracting Whole Numbers
- Solving Some Equations That Contain Whole Numbers

These pictures of a hungry alligator may help you remember which symbol to use.



Here is another way to remember which symbol to use.



Concept 1: Adding and Subtracting

Comparing Whole Numbers

As you move from left to right along the number line in Figure 1, the numbers increase.



You can use this fact to compare two numbers.

For example, since 6 lies to the right of 2, you say 6 “is greater than” 2.

Or, using symbols, you can write $6 > 2$.

You can also say 6 “is greater than or equal to” 2.

Or you can also write $6 \geq 2$.

This table shows some comparison symbols and how to read them.

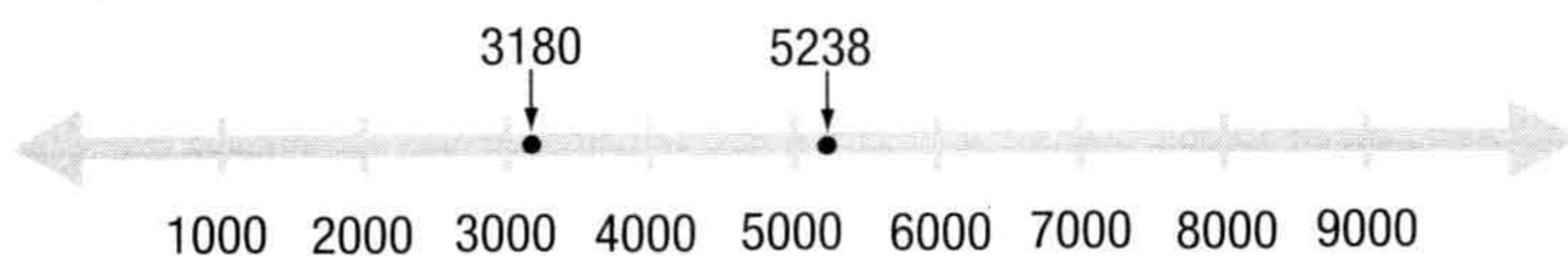
Symbol	Meaning	Example
$=$	is equal to	$9 = 9$
$\neq$	is not equal to	$9 \neq 4$
$<$	is less than	$4 < 9$
$>$	is greater than	$9 > 4$
$\leq$	is less than or equal to	$4 \leq 9$ $9 \leq 9$ $4 \leq 4$
$\geq$	is greater than or equal to	$9 \geq 4$ $9 \geq 9$ $4 \geq 4$

You know that the statement “ $9 = 9$ ” is true. But, why is the statement “ $9 \leq 9$ ” also true?

The statement “ $9 \leq 9$ ” means “9 is less than or equal to 9.” That’s a short way of saying either “9 is less than 9” or “9 is equal to 9.” This part is false. This part is true.

An “either-or” statement is true if at least one of its parts is true. And that’s the case here.

1. Compare the numbers 5238 and 3180.



Here's one way to compare 5238 and 3180:

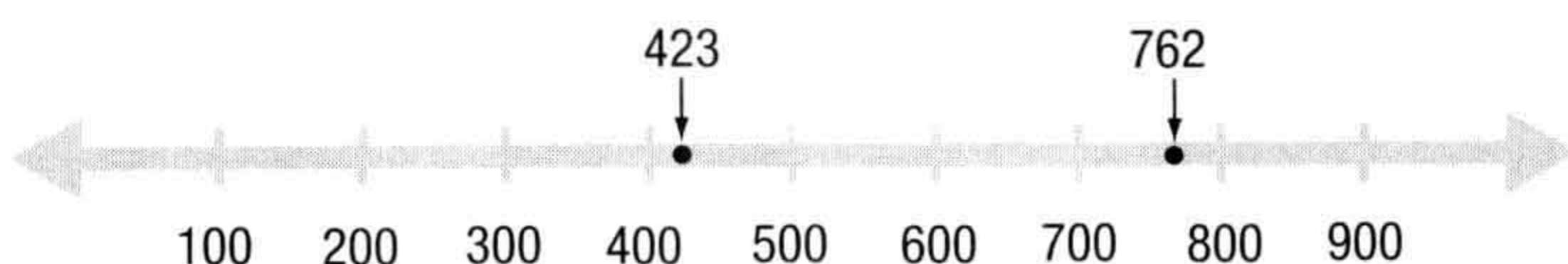
- 5238 lies to the right of 3180. Say "5238 is greater than 3180"
 - Using symbols: Write " $5238 > 3180$ "
- So, $5238 > 3180$.

Example 1

You may find these Examples useful while doing the homework for this section.

The statements " $5238 \geq 3180$ ", " $3180 < 5238$ ", and " $3180 \leq 5238$ " can also be used to compare the numbers 5238 and 3180.

2. Compare the numbers 762 and 423.



Here's a way to compare 762 and 423:

- 423 lies to the left of 762 Say 423 is less than 762
 - Using symbols: Write $423 < 762$
- So, $423 < 762$.

Example 2

The statements " $423 \leq 762$ ", " $762 > 423$ ", and " $762 \geq 423$ " can also be used to compare the numbers 762 and 423.

Place Value

Each digit in a whole number "holds" a place value. This allows a number to be represented in several equivalent ways.

For example, the number 2436 can be written as:

2 thousands + 4 hundreds + 3 tens + 6 ones

Or, it can be written this way:

$2000 + 400 + 30 + 6 = 2436$

Sometimes it is helpful to use a place value chart to answer questions about the digits of a number. Here's an example of a place value chart:

Place Value Chart			
2	4	3	6
thousands place	hundreds place	tens place	ones place

As you move to the left in the chart, the new place value is ten times the previous place value.

Since the digit 2 is in the thousands place, it represents 2 thousands or 2000. Similar statements can be made for each of the digits in 2436.