Stephen L. Snover & Mark A. Spikell jects to moreose your Ogramming Skill A collection of challenging problems, with A college program listings, hints, and solutions

MATHEMATICAL PROBLEM-SOLVING

WITH THE

MICROCOMPUTER

Projects to Increase Your BASIC
Programming Skill

Stephen L. Snover Mark A. Spikell



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MATHEMATICAL PROBLEM-SOLVING WITH THE MICROCOMPUTER

Dr. Stephen L. Snover, an assistant professor of mathematics and computer science at the University of Hartford in West Hartford, Connecticut, was an author for the Boston University Mathematics Project and has written articles on topics in both mathematics and computer science.

Dr. Mark A. Spikell, a mathematics educator and chairman of the Department of Education at George Mason University in Fairfax, Virginia, has co-written the books *Problem Solving in the Mathematics Laboratory* and *Multibase Activities*.

Jointly they have written three other books published by Prentice-Hall, How to Program Your Programmable Calculator, Brain Ticklers, and Programming the TI-55 Slide Rule Calculator.

PREFACE

For all practical purposes, microcomputers were first available commercially in 1977. By the early 1980s there were more than 500,000 of these computing machines in homes, schools, and offices. Some experts predict that by the year 2000 virtually every person will have his or her own personal computer (or *microcomputer*, as they are often called).

Whether the experts' predictions come true or not, there is little doubt that the microcomputer has already had a significant impact on our lives. These machines make it possible for virtually anyone to have, at an ever-decreasing cost, the powerful computing and problem-solving capacity usually thought to be available only to the scientific or business communities.

One can only imagine the possible implications of having computer technology available to millions of people rather than thousands. Come what may, it seems reasonable to infer that increasing numbers of people need and will want to know more about computers in general and what they can and cannot do.

This book gives readers an idea or feeling for some of the kinds of numerical problems that can be solved with a microcomputer. It also provides ample opportunity for individuals to develop or enhance programming and problem-solving skills. Finally, it permits the reader to gain some insight into both the power and the limitations of computer technology.

WHO IS THIS BOOK FOR?

This book is written for anyone who has access to or who owns a microcomputer or computer, including the following:

- beginning computer programmers
- · computer hobbyists
- · microcomputer owners
- · teachers of computer programming
- puzzle enthusiasts
- · computer science teachers
- · mathematics and science students
- · gift-giving friends and relatives

WHAT IS THE PURPOSE OF THE BOOK?

The purpose of this book is to enable readers to:

- · experience mathematical problem-solving using a computer
- · learn how to adapt a working program to solve a related problem
- get a feeling for the range of numerical problems that can be solved by the computer
- gain insight into the power and limitations of the computer in computational problems
- have a resource collection of interesting numerical problems for computer solution

WHICH COMPUTER SHOULD I USE WITH THIS BOOK?

This book has been written for use with any computer with BASIC and at least 1K of memory. For example, you can use any of these microcomputers:

- · Apple II or Apple III
- · TRS-80 Model I, II, or III
- . ATARI 400 or ATARI 800
- . Commodore PET
- Ohio Scientific Challenger
- · Radio Shack hand-held computer
- TI 99/4
- · Sinclair ZX80

- · Compucolor II
- · Exidy Sorcerer

and essentially all other small and large machines.

HOW MUCH MATHEMATICS OR COMPUTER PROGRAMMING DO I NEED TO KNOW?

Most of the problems in this book require no more mathematics than is customarily taught in first-year algebra and no more computer programming than a simple working knowledge of beginning BASIC. However,

- If any more advanced mathematics is needed, it is explained in the text and presented in the accompanying program.
- If more advanced BASIC programming techniques are used, they are carefully explained in the text.

ACKNOWLEDGMENTS

We wish to thank several people for their help in preparing this book. First, thanks to Judith Campbell-Reed for her patience and creative artwork. Her artistic flair does much to make this a visually appealing book. Second, thanks to John E. Hunger and his colleagues for useful brainstorming on the packaging and titling of this book. John's editorial support for this and other joint projects has always been appreciated, as has the able assistance of his assistant, Lou-Ann E. Leahy-O'Rourke. Third, thanks to Mark Schmidt for his help in preparing the program listings that appear in this book. We also extend special thanks to Laurie and J.B. for their continued support.

MATHEMATICAL PROBLEM-SOLVING WITH THE MICROCOMPUTER

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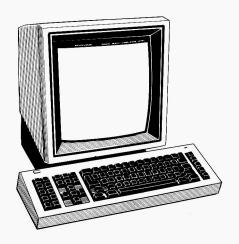
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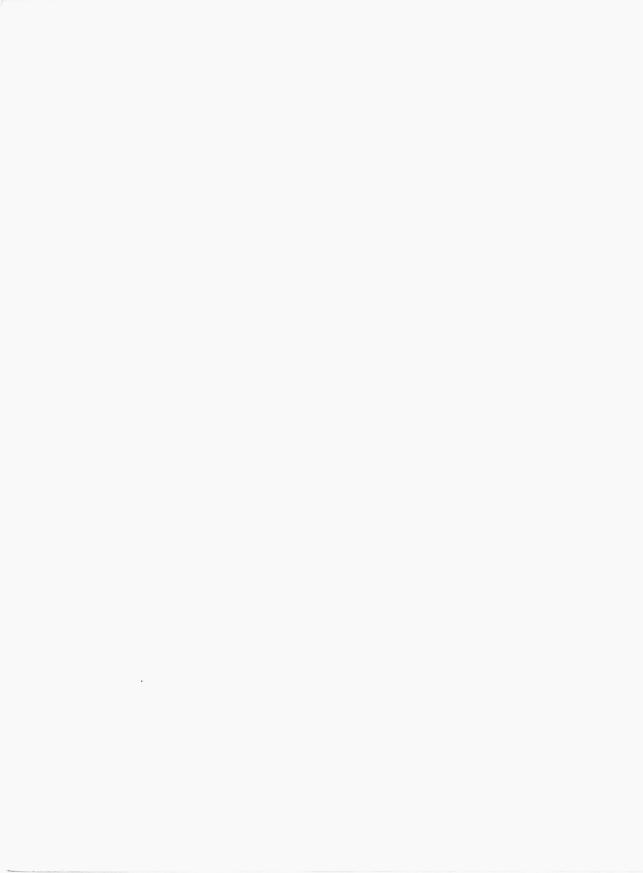
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HOW TO USE THIS BOOK

I



INTRODUCTION

Computers! Everywhere you turn, your life is increasingly affected by these electronic marvels. Airlines rely on computers to handle reservations; banks rely on computers to keep track of deposits and withdrawals; hospitals rely on computers to store medical records; large corporations rely on computers to issue employee paychecks; the list is endless.

Because computers are so pervasive in society, knowing something about how they work and how they can be used to solve problems is important information for the educated citizen. But how best does one learn about computers—by reading about them? Not likely, in our opinion. We believe that one learns the potential and power of computers by using them. And we are convinced that one learns best by actively participating in the learning process (doing) rather than passively participating (reading or listening).

To help you obtain a feeling for how computers solve various problems, we want you to actually write computer programs. That is why we designed this book around a collection of interesting mathematical (numerical) problems that you can solve with a computer. By using this book and exploring the problems, you will learn, we are confident, a great deal about computers, programming, and problem solving. Furthermore, you will experience our philosophy that people learn by doing and learn even more by doing and having fun at the same time.

PURPOSE OF THE BOOK

We have written this book so that you can use it for a number of different purposes. On the one hand, we want you to gain pleasure out of solving these problems with your computer. Each of the problems is designed to be solved in twenty to thirty minutes. And for each problem, there are several suggested extensions that you might explore. In the process, we hope you will gain insight into how to utilize your computer to help you solve problems. On the other hand, we hope you will be able to review and practice your BASIC computer programming skills by writing programs of your own and by seeing and running programs we present. Moreover, we think you will pick up new BASIC skills, such as ones related to updating or adapting BASIC programs to get them to perform related tasks. In particular, we hope you will gain a better feeling for the power and limitations of your computer as a mathematical problem-solving tool while having fun, too.

USING THE BOOK WITH A VARIETY OF MACHINES

We selected and designed problems for this book so that any computer having the BASIC computer language could be used. If you have access to or own a microcomputer (e.g., the Apple, ATARI, PET, TRS-80, etc.), you will find that these problems can be readily solved on these machines. Even small hand-held computers with only 1K of memory can be used.

The program listings presented either as core programs or problem solutions are written with BASIC instructions that are general enough to work on virtually every computer that uses a form of the BASIC language. We avoided instructions that work on only one or some computers so that you could use this book with any machine. Although many other books claim that you can *easily* modify any of these programs to run on your machine, we have discovered that it is usually far from easy. Thus, we really designed our book so that you do not need to make any modifications at all to get these programs to work on your machine. We trust that the extra effort on our part will mean less frustration and more benefit for you.

FORMAT OF THE BOOK

This book is designed in a six-part format so that you will have maximum flexibility to use it for educational or recreational purposes. The six parts are:

Part I: How to Use This Book

Part II: Problems, Core Programs, Modification Suggestions, and

Extensions

Part III: BASIC Program Solutions to the Problems

Part IV: Numerical Answers to the Problems

Part V: BASIC Program Solutions to the Extensions

Part VI: Numerical Answers to the Extensions

Although many books are available that give problems to solve or computer programs to run, we know of no other book quite like this one. The collection of problems for computer solution in Part II is presented in a unique way. We give each problem, with a cartoon for aesthetic appeal. Then we present a short BASIC program that focuses on a specific idea fundamental to the solution of that problem. Next, we present suggestions on how you can use the short program as the core of a larger program in BASIC that will actually give the solution. Also, we provide several exten-

sions to each problem so that you can have more practice at modifying programs to solve related tasks and to provide more experience with the use of the computer as a problem-solving tool. Finally, we include a blank page so that you can have a space to write your program, record extensions of your own, or make other notes. This unusual presentation should be flexible enough to enable you to use this book in a variety of ways.

In Parts III through VI, we present BASIC programs and numerical solutions to all the problems and extensions from Part II. We have separated the programs from the corresponding numerical solutions for an important reason—to provide even more flexibility of use. If you enjoy the challenge of writing your own programs, you can consult just the numerical answers in Parts IV or VI to verify the accuracy of your work. Or, if you wish to consult or copy the programs in Parts III and V, the numerical solutions will not be given away before you have a chance to enter and run these programs.

If you plan to use any of our core programs or modification suggestions, we encourage you to read the rest of this introductory chapter. In the next section, we proceed step by step through the solution of a sample problem. This will help you discover by "doing" how you can use our core programs and suggestions in designing a BASIC program to solve any of the problems in this book.

A SAMPLE PROBLEM

On the next page is a statement of Mad Mary's Problem, with a cartoon. We hope you will find the cartoon accompanying each problem an attractive feature to stimulate your interest in solving the problem.