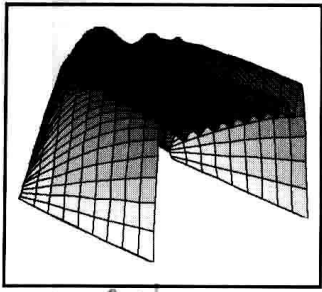


SIGNALS, SYSTEMS, AND TRANSFORMS

SECOND EDITION



CHARLES L. PHILLIPS
JOHN M. PARR



SIGNALS, SYSTEMS, AND TRANSFORMS

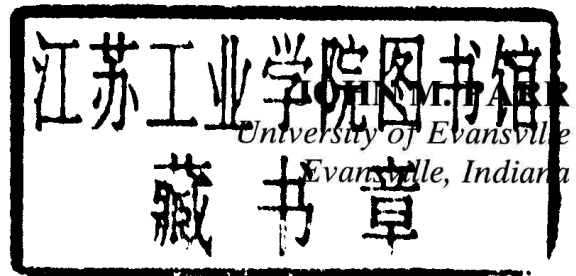
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A BRIEF TABLE OF LAPLACE TRANSFORMS

	$f(t)$	$F(s)$
1.	$u(t)$	$\frac{1}{s}$
2.	$tu(t)$	$\frac{1}{s^2}$
3.	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
4.	$e^{-\alpha t} u(t)$	$\frac{1}{s + \alpha}$
5.	$te^{-\alpha t} u(t)$	$\frac{1}{(s + \alpha)^2}$
6.	$t^n e^{-\alpha t} u(t)$	$\frac{n!}{(s + \alpha)^{n+1}}$
7.	$\sin(\omega_0 t) u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$
8.	$\cos(\omega_0 t) u(t)$	$\frac{s}{s^2 + \omega_0^2}$
9.	$2Ae^{-\alpha t} \cos(\omega_0 t + \phi) u(t)$	$\frac{Ae^{j\phi}}{s + \alpha - j\omega_0} + \frac{Ae^{-j\phi}}{s + \alpha + j\omega_0}$
10.	$\delta(t)$	1
11.	$p\delta(t)$	s
12.	$e^{-\alpha t} \cos(\omega_0 t) u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2}$
13.	$e^{-\alpha t} \sin(\omega_0 t) u(t)$	$\frac{\omega_0}{(s + \alpha)^2 + \omega_0^2}$

SOME SELECTED FOURIER TRANSFORMS PAIRS ($a > 0$ throughout)

	$f(t)$	$F(\omega)$
1.	$e^{-at}u(t)$	$\frac{1}{a + j\omega}$
2.	$te^{-at}u(t)$	$\left(\frac{1}{a + j\omega}\right)^2$
3.	$p_T(t) \equiv \begin{cases} 1, & t < T/2 \\ 0, & t > T/2 \end{cases}$	$T \operatorname{sinc} \frac{\omega T}{2}$
4.	$\Delta_T(t) \equiv \begin{cases} 1 - \frac{ t }{T}, & t < T \\ 0, & t > T \end{cases}$	$T \operatorname{sinc}^2 \frac{\omega T}{2}$
5.	$e^{-a t }$	$\frac{2a}{a^2 + \omega^2}$
6.	$e^{-at}(\sin \omega_0 t)u(t)$	$\frac{\omega_0}{(a + j\omega)^2 + \omega_0^2}$
7.	$e^{-at}(\cos \omega_0 t)u(t)$	$\frac{a + j\omega}{(a + j\omega)^2 + \omega_0^2}$
8.	e^{-at^2}	$\left(\frac{\pi}{a}\right)^{1/2} e^{-\omega^2/4a}$
9.	$\operatorname{sinc} \frac{Tt}{2}$	$\frac{2\pi}{T} p_T(\omega)$
10.	$\operatorname{sinc}^2 \frac{Tt}{2}$	$\frac{2\pi}{T} \Delta_T(\omega)$
11.	$\frac{1}{a^2 + t^2}$	$\frac{\pi}{a} e^{-a \omega }$
12.	$\delta(t)$	1
13.	1	$2\pi\delta(\omega)$
14.	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
15.	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
16.	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
17.	$\sin \omega_0 t$	$j\pi[\delta(\omega + \omega_0) - \delta(\omega - \omega_0)]$
18.	$\operatorname{sgn} t$	$\frac{2}{j\omega}$
19.	$(\cos \omega_0 t)u(t)$	$\frac{\pi}{2}[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)] + \frac{j\omega}{\omega_0^2 - \omega^2}$
20.	$(\sin \omega_0 t)u(t)$	$\frac{\pi}{2j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)] + \frac{\omega_0}{\omega_0^2 - \omega^2}$

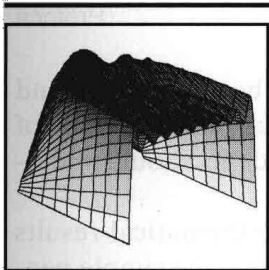
SIGNALS, SYSTEMS, AND TRANSFORMS

Second Edition

To

Taylor, Justin, Jackson, Rebecca, and Alex

Judith, Dara, Johna, and Duncan



PREFACE

The structure and philosophy of the previous edition of Signals, Systems, and Transforms remains unchanged in the second edition. However, the focus has been sharpened as a result of the experience of using the first edition and the reactions of colleagues who have taught from the book. Some of the more advanced topics have been omitted; some explanations have been enhanced. Where appropriate, a number of examples have been simplified and others have been replaced with examples that more clearly illustrate the theory. The majority of end-of-chapter problems have been either altered or replaced.

The MATLAB program SIMULINK is introduced to illustrate the simulation of both continuous and discrete systems. In addition, the symbolic mathematics of MATLAB is used in verifying the calculations of transforms, and in other appropriate applications.

New capabilities of MATLAB have been added and most examples now contain short MATLAB programs. In this second edition, the MATLAB programs given in the examples may be downloaded from the internet site: <ftp://ftp.mathworks.com/pub/books/phillips>. Students may then alter the data statements in these programs for the end-of-chapter problems, which eases debugging problems.

Our appreciation and thanks to reviewers of both the first and second editions.

PREFACE TO THE FIRST EDITION

This book is intended to be used primarily as a text for junior-level students in engineering curricula and for self-study by practicing engineers. It is assumed that the reader has had some introduction to signal models, system models, and differential equations (as in, for example, circuits courses and courses in mathematics), and some laboratory work with physical systems.

The authors have attempted to emphasize the difference between signal and system models and physical signals and systems. Although a true understanding of this difference can be acquired only through experience, the students should understand that a difference does exist.

In addition, the authors have attempted to relate the mathematical results to physical systems with which students are familiar (for example, the simple pendulum) or physical systems that students can visualize (for example, a picture in a picture for television). The descriptions of these physical systems, given in Chapter 1, are not complete in any sense of the word; these systems are introduced simply to illustrate practical applications of the mathematical procedures presented.

Generally, practicing engineers must, in some manner, validate their work. To introduce the topic of validation, the results of examples are verified, using different procedures, where practical. In addition, homework problems require verification of the results, where practical. Hence students become familiar with the process of validating their own work.

The software tool MATLAB is integrated into the text in two ways. First, in appropriate examples that require computations, a MATLAB program of usually two or three statements is given that will verify the computations. The results of running the program are also given. Then, in appropriate homework problems, the student is asked to verify the calculations using MATLAB. This verification is not difficult because the required programs are given in the examples. Hence, another procedure for verification is given. However, the book is written such that all references to MATLAB may be omitted, if desired. The MATLAB programs given in the examples are available at the Internet site <ftp://ftp.mathworks.com/pub/books/phillips> and may be downloaded. Students may then alter the data statements in these programs for the end-of-chapter problems, which eases debugging problems.

In this book, Laplace transforms are covered in Chapter 7 and z -transforms are covered in Chapter 11. At many universities, one or both transforms are introduced prior to the signals and systems courses. Chapters 7 and 11 are written such that the material can be covered anywhere in the signals and systems course, or it can be omitted entirely, except for required references.

The more advanced material has been placed toward the end of the chapters where possible. Hence, this material may be omitted if desired.

The material of this book is organized into two principal areas: continuous-time signals and systems, and discrete-time signals and systems. Some professors prefer to cover first one of these topics, followed by the second. Other professors prefer to cover continuous-time material and discrete-time material simultaneously. The authors have taken the first approach, with the continuous-time material covered in Chapters 2 through 8, and the discrete-time material covered in Chapters 9 through 13.

The material may also be arranged such that basic continuous-time material and discrete-time material are intermixed. Chapters 2 and 9 may be covered simultaneously and Chapters 3 and 10 may also be covered simultaneously.

In Chapter 1 we present a brief introduction to signals and systems, followed

by short descriptions of several physical continuous-time and discrete-time systems. In addition, some of the signals that appear in these systems are described. Then a very brief introduction to MATLAB is given.

In Chapter 2 we present general material basic to continuous-time signals and systems; the same material for discrete-time signals and systems is presented in Chapter 9. However, as stated above, Chapter 9 can be covered simultaneously with Chapter 2. Chapter 3 extends this basic material to continuous-time linear time-invariant systems, while Chapter 10 does the same for discrete-time linear time-invariant systems.

Presented in Chapters 4, 5, and 6 are the Fourier series and the Fourier transform for continuous-time signals and systems. The Laplace transform is then developed in Chapter 7. State variables for continuous-time systems are covered in Chapter 8; this coverage utilizes the Laplace transform.

The z-transform is developed in Chapter 11, with the discrete-time Fourier transform and the discrete Fourier transform presented in Chapter 12. However, Chapter 12 may be covered prior to Chapter 11. State variables for discrete-time systems are given in Chapter 13. This material is independent of the state variables for continuous-time systems of Chapter 8.

In Appendix A we give some useful integrals and trigonometric identities. In general, the table of integrals is used in the book, rather than taking the longer approach of integration by parts. Leibnitz's rule for the differentiation of an integral and L'Hospital's rule for indeterminate forms are given in Appendix B and are referenced in the text where needed. Appendix C covers the closed forms for certain geometric series; this material is useful in discrete-time signals and systems.

In Appendix D we review complex numbers and introduce Euler's relation, in Appendix E the solution of linear differential equations with constant coefficients, and in Appendix F partial-fraction expansions. Relevant MATLAB programs are given in Appendix F. Matrices are reviewed in Appendix G; this appendix is required for the state-variable coverage of Chapters 8 and 13. As each matrix operation is defined, MATLAB statements that perform the operation are given.

This book may be covered in its entirety in two 3-semester-hour courses, or in quarter courses of approximately the equivalent of 6 semester hours. With the omission of appropriate material, the remaining parts of the book may be covered with fewer credits. For example, much of the material of Chapters 2, 9, 3, 10, 4, and 5 (in that order) has been covered in one 3-quarter-hour course; the students were already familiar with some linear-system analysis (including feedback systems) and the Laplace transform. A second 3-quarter-hour course then covered much of the material of Chapters 6, 11, and 12, with supplemental material on filters in the second course. No state variables were covered in either course.

We wish to acknowledge the many colleagues and students at both Auburn University and the University of Evansville who have contributed to the development of this book. In particular, the first author wishes to express thanks to Professors Charles M. Gross, Martial A. Honnell, and Charles L. Rogers of Auburn University for many stimulating discussions on the topics in this book, and to Professor Roger Webb, director of the School of Electrical Engineering at the Georgia Institute of

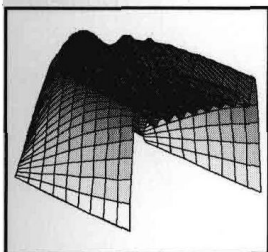
Technology, for the opportunity to teach the signal and system courses at Georgia Tech. The second author wishes to thank Dean John Tooley and Professors Dick Blandford and William Thayer for their encouragement and support for this effort, and Professor David Mitchell for his enthusiastic discussions of the subject matter. Finally, we express our gratitude and love of our families, without whom this undertaking would not have been possible.

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John M. Parr

SIGNALS, SYSTEMS, AND TRANSFORMS

Second Edition



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