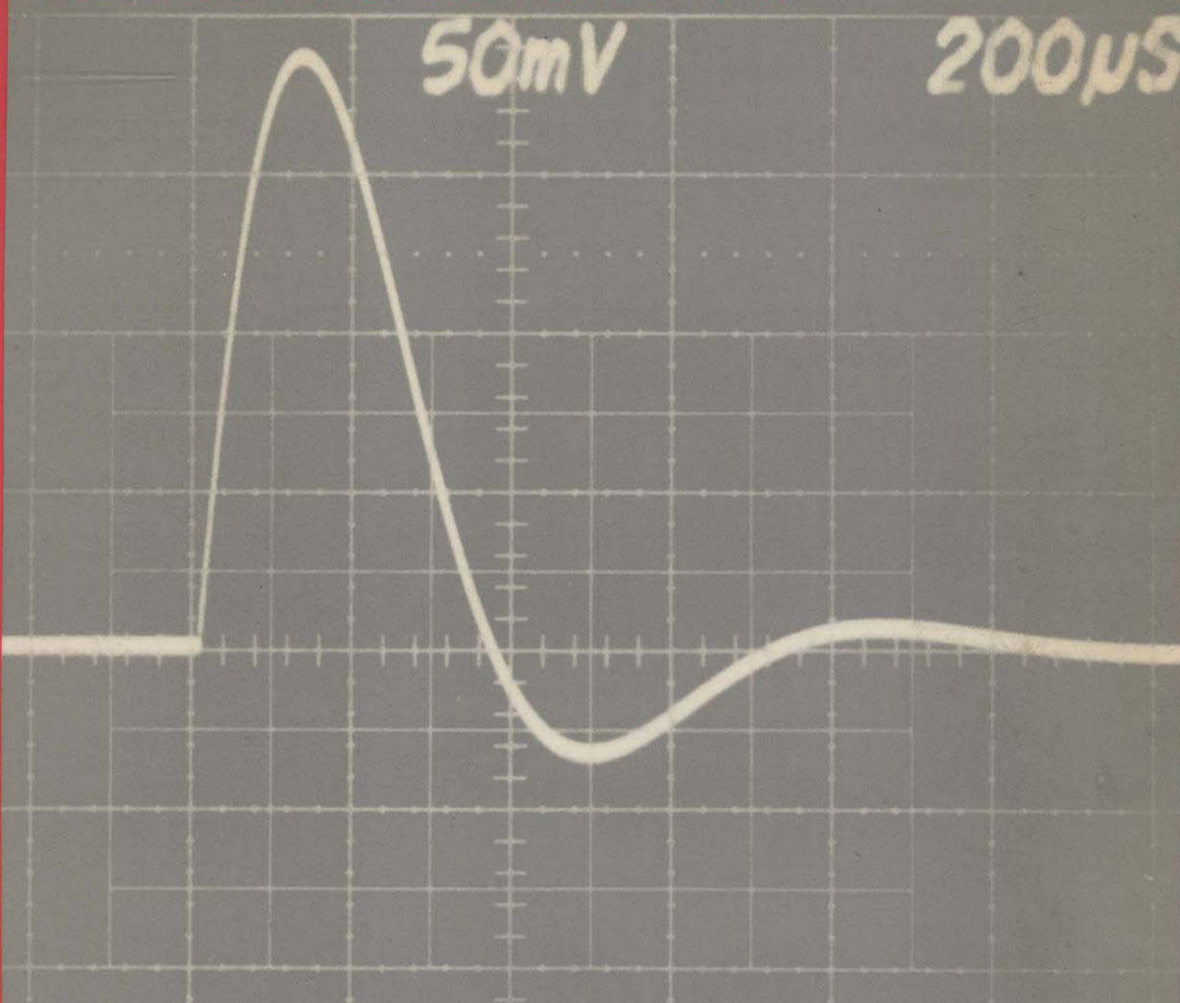
BASIC CONCEPTS IN LINEAR SYSTEMS

THEORY AND EXPERIMENTS



THEODORE F.
BOGART

BASIC CONCEPTS IN LINEAR SYSTEMS
THEORY AND EXPERIMENTS

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PREFACE

BASIC CONCEPTS IN LINEAR SYSTEMS: THEORY AND EXPERIMENTS is intended to introduce students to linear systems analysis through study and experimental verification of many fundamental linear concepts. Topics covered include transient analysis, frequency response, summation and integration of signals, feedback and oscillation, filtering, and modulation. Stress is placed on mathematical formulations that encourage students to generalize their perception of linear behavior and thus improve their ability to understand, design, and predict the response of linear devices. For example, a simple and familiar RC network is generalized and studied from the viewpoints of its transient response, its transfer function, its frequency response, its use as a filter, and its behavior as a lag network and as an electronic integrator.

Students are expected to have a knowledge of basic DC and AC network analysis and of electronic (semiconductor) devices. The mathematical level required does not include calculus, though a practical knowledge of integration would be helpful for the material on electronic integrators. An ability to manipulate complex numbers and to use phasors in AC circuit analysis is assumed.

The material is appropriate for a second year course in a two-year associate degree program in electronics technology, or for a second-or third-year course in a four-year BS degree program. All experiments have been student tested and used successfully in an ABET accredited BS program, where it is a prerequisite for courses in transform analysis of networks and control systems theory.

Many of the experimental procedures have intentionally been designed to be too long for a typical scheduled laboratory period. The intent is to provide instructors with a degree of flexibility in their choice of assignments and to enable the scheduling of multiple or open lab sessions if desired. Generally speaking, the procedures toward the end of each experiment explore more advanced applications and require somewhat more complex computations. These can be pruned at the instructor's discretion.

The questions at the end of each experiment generally require students to compare their experimental results with those predicted by the theory. Since many instructors prefer that students perform a theoretical analysis of each network before they construct and test it in the laboratory, each experiment contains a set of exercises that require this analysis. In many cases these exercises duplicate some of the questions that appear after the experimental procedure.

This volume is the fourth in a series of teaching-laboratory manuals prepared by the author. Some of the experiments herein have been taken from earlier volumes and modified as necessary to suit the context of the present volume. For a more exhaustive treatment of transients and linear network analysis using transform methods, see LAPLACE TRANSFORMS: THEORY AND EXPERIMENTS (Wiley, 1983). For a comprehensive coverage of operational amplifiers and linear integrated circuits, see LINEAR INTEGRATED CIRCUITS: THEORY AND EXPERIMENTS (Wiley, 1983). For a study of analog and digital computer techniques in linear systems analysis, see COMPUTER SIMULATION OF LINEAR CIRCUITS AND SYSTEMS (Wiley, 1983).

In this, the last volume of the series, I want to express a very deep gratitude to my wife, Becky, who encouraged and sustained me throughout the many months these volumes were in preparation. I am also grateful to the many students who labored diligently through the first drafts of the experiments and made many valuable suggestions on how to improve them. Reviewers engaged by John Wiley & Sons, Inc. also provided many valuable suggestions and contributed greatly to the final product. Finally, thanks to Dr. Howard Heiden, University of Southern Mississippi, for the departmental support and encouragement that was given to me.

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CONTENTS

INTRODUCTION

When describing electronic devices or systems, it is customary to use the word linear in a somewhat loose sense to mean, simply, nondigital. Examples include audio amplifiers, servomotors, tachometers, operational amplifiers, piezoelectric transducers, filters, position control systems, in short, any device or combination of devices whose output is directly proportional to its input. The equations that describe the behavior of these devices, that is, how outputs are functionally related to inputs, are therefore linear equations. The word continuous is also used to describe such components, though this broader category can include devices whose equations are nonlinear. Examples are diodes, AM and FM modulators, electronic multipliers, and function generators. In either case, the adjectives linear and/or continuous are used when we wish to emphasize that we are referring to nondigital devices or systems. We thus exclude pulse code modulators, logic gates, flip-flops, shift registers, digital computers, and so forth.

In this book we will study some fundamental concepts that are extremely useful for describing and predicting the behavior of all linear devices and systems. We use the word "system" in a very broad sense, as is customary, to include any collection of linear devices, from simple combinations of resistive and reactive components to complex combinations of amplifiers, filters, and the like. "Linear systems analysis" traditionally means the study of the properties of linear systems with an emphasis on mathematical characterizations rather than on the physical characteristics of devices themselves. That is the perspective we will adopt in this book. We will learn how the use of mathematical equations can help us to generalize and thus better understand such concepts as transients, frequency response, transfer functions, feedback, and filters. At the same time, we will apply these concepts to the design, construction and experimental verification of the properties of linear electronic devices, many of whose physical characteristics may already be familiar.

A sound knowledge of complex number theory and the use of phasors in ac circuit analysis is required to obtain maximum benefit from the material in this book. A review of these topics may be found in Bogart, Laplace Transforms and Control Systems Theory for Technology (Wiley, 1982), which also contains supplemental reading on other topics that will be covered in this book.

EXPERIMENT 1

Transients in RC and RL Networks

I. OBJECTIVES

1. To learn how voltages and currents in RC and RL circuits change with time when dc voltage sources are switched into them.

2. To verify experimentally the equations that describe voltage and current changes in RC and RL circuits.

3. To learn how to identify and distinguish between initial, transient and steady-state currents and voltages.

4. To observe the effect of changes in circuit time-constants on the behavior of transient voltages.

5. To learn how to use the Thevenin equivalent circuit to predict transient and steady-state conditions in RC and RL networks.

II. DISCUSSION

When a dc voltage source is switched into a series RC circuit, current immediately flows from the source and begins to charge the capacitor. See Figure 1.1.

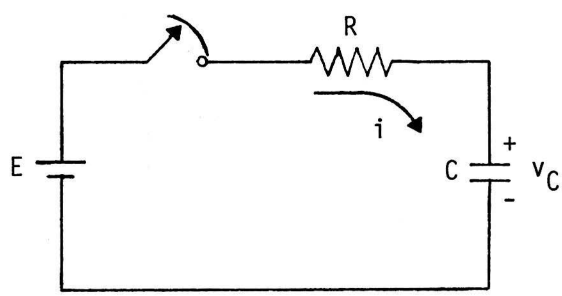


Figure 1.1 The transient current i causes a voltage v_C to be developed on the capacitor after the switch is closed.

The capacitor voltage v_C builds up as charge accumulates on the plates of the capacitor. The polarity of v_C is such that it opposes the applied voltage E, as shown in Figure 1.1. As more time passes, the voltage v_C increases, so the current i decreases, until such time as C is fully charged (v_C = E volts). At that time, current ceases to flow. During the time that the capacitor is charging, the current and voltage are said to be transient, because only during that time are they changing. When the capacitor is fully charged, the current and voltage are said to have reached their steady-state values. Thus, the steady-state value of v_C is E volts and the steady-state value of i is zero.

The equations that describe how the voltage v_C and current i change with time t after the switch is closed at t = 0 are

$$v_C(t) = E(1 - e^{-t/RC})V \qquad (1)$$

$$i(t) = \frac{E}{R} e^{-t/RC} \text{ A} \qquad (2)$$

where e is the base of the natural logarithm (e ≃ 2.7183). By substituting a specific value of t into (1) or (2), we can determine the instantaneous value of v_C or i at that time. For example, suppose C = 1.0 μF and R = 500 K, and we wish to know the value of v_C and i after the switch in Figure 1.1 has been closed for 0.25 seconds. Assume E = 10 volts. Substituting these values into (1) and (2), we find

$$v_C(0.25) = 10 \left[1 - e^{-\frac{0.25}{(5 \times 10^5)(10^{-6})}}\right]$$

$$= 10 \left[1 - e^{-\frac{0.25}{0.5}}\right] = 10(1 - e^{-0.5})$$

$$= 10(1 - .6065) = 3.94 \text{ V}$$

$$i(0.25) = \frac{10}{5 \times 10^5} e^{-\frac{0.25}{0.5}} = 2 \times 10^{-5} e^{-0.5}$$

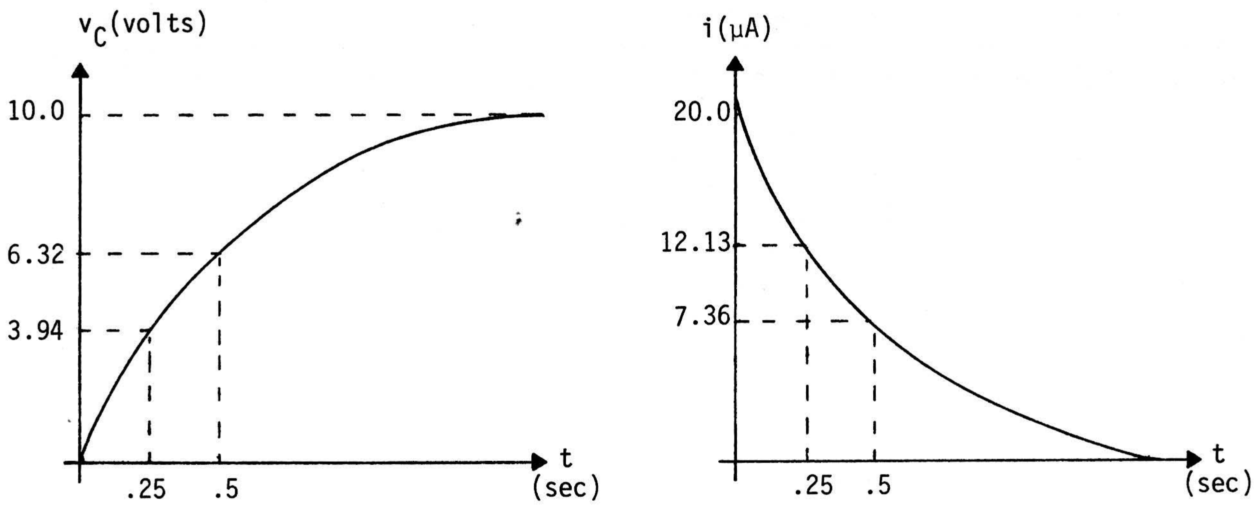
$$= (2 \times 10^{-5})(.6065) = 12.13 \ \mu\text{A}$$

If we were to calculate values of v_C and i at numerous different times t and then carefully plot $v_C(t)$ and i(t) versus t, we would obtain the plots shown in Figure 1.2. Note how these plots reveal the steady-state values of $v_C(t)$ and i(t). At large values of time t, $v_C(t)$ is seen to approach E = 10 volts, while i approaches zero. The instant that the switch is closed, i.e. at t = 0, we see that $v_C(0)$ = 0 and i(0) = 20 μA. These are called the initial values of $v_C(t)$ and i(t). We can verify the initial values by substituting t = 0 in equations (1) and (2):

$$v_C(0) = E(1 - e^{-0/RC}) = E(1 - 1) = 0 \text{ v}$$

$$i(0) = \frac{E}{R} e^{-0/RC} = \frac{E}{R}(1) = \frac{10 \text{ V}}{5 \times 10^5 \ \Omega} = 20 \ \mu\text{A}$$

These initial values confirm our intuitive understanding of the behavior of the circuit in Figure 1.1. At the instant the switch is closed, there is no voltage on the capacitor because no charge has yet accumulated on the capacitor. It is not possible to change the voltage across a capacitor instantaneously. Since there is no capacitor voltage present



(a)
The rise of capacitor voltage in Figure 1.1

(b)
The decay of current in Figure 1.1.

Figure 1.2

to oppose the source voltage E, the current at t = 0 must be E/R amperes.

The quantity RC in equations (1) and (2) is called the <u>time-constant</u> of the circuit and has the units of time, in seconds, when R is in ohms and C is in farads. The conventional symbol for a time-constant is the Greek letter tau (τ). In the previous example, the time constant is $\tau = RC = 5 \times 10^5(10^{-6}) = 0.5$ seconds. When equations (1) and (2) are written in terms of τ they become

$$v_C(t) = E(1-e^{-t/\tau}) \tag{3}$$

$$i(t) = \frac{E}{R}e^{-t/\tau} \tag{4}$$

The significance of the time-constant is that when $t = \tau$ seconds, i.e. at τ seconds after the switch is closed, we have, from (3) and (4),

$$v_C = E(1-e^{-t/\tau}) = E(1-e^{-1}) = .632\ E$$

$$i = \frac{E}{R}e^{-t/\tau} = \frac{E}{R}e^{-1} = .368\ E/R$$

Thus, one time-constant after the application of the source voltage E, the capacitor voltage has risen to 63.2% of its steady-state value, and i has decayed to 36.8% of its initial value. In a similar way, we can show that at $t = 2\tau$, v_C = 86.5% of its steady-state value and i = 13.5% of its initial value. Further, at $t = 3\tau$ we find v_C = 95% of its final value and i = 5% of its initial value. At $t = 5\tau$ the capacitor is essentially fully charged ($v_C \simeq E$) and $i \simeq 0$, so we say that steady-state conditions have been reached, for all practical purposes, after an elapse of time equal to 5 time-constants. The student should verify this fact, and the percentages given above, by using equations (3) and (4).

Suppose now that the voltage source in Figure 1.1 is suddenly replaced by a short-circuit, after the capacitor has been allowed to charge fully, i.e. after steady-state conditions have been reached. When this happens, the charge on the capacitor flows from one plate through the resistor R to the other plate, and thus discharges. See Figure 1.3.

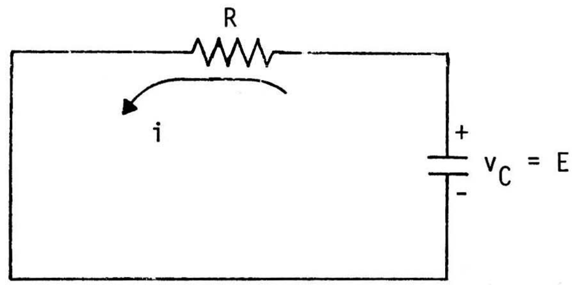


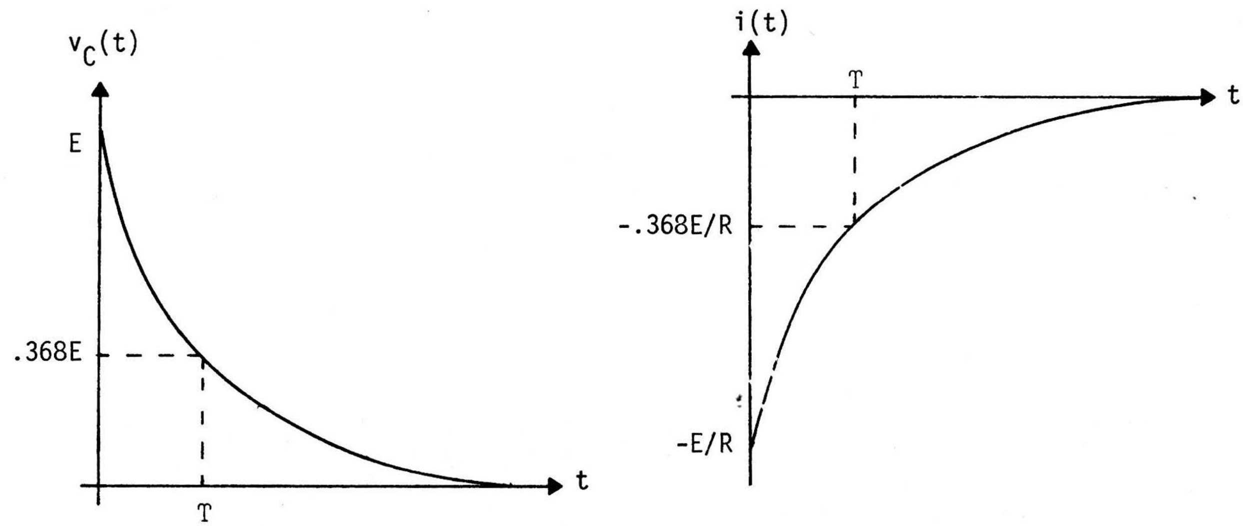
Figure 1.3 Current flow when the capacitor discharges.

Note in Figure 1.3 that the discharge current i flows in the circuit in the opposite direction from that which it flowed when the capacitor was charging. If we regard the direction of charging current as positive, then the discharge current is negative. Of course, as the capacitor discharges the voltage v_C decays toward zero volts and the current decays toward zero amperes, their ultimate steady-state values. The equations that describe the decay of voltage and current are

$$v_C(t) = Ee^{-t/RC} = Ee^{-t/T} \tag{5}$$

$$i(t) = -\frac{E}{R}e^{-t/RC} = -\frac{E}{R}e^{-t/T} \tag{6}$$

These equations are plotted in Figure 1.4.



(a)
The decay of capacitor voltage during discharge.

(b)
The decay of capacitor current during discharge.

Figure 1.4

The significance of the time-constant $T = RC$ in the discharge equations (5) and (6) is the same as before, except of course in this case both $v_C(t)$ and $i(t)$ are decaying with time. Both decay to 36.8% of their initial values (E and -E/R) after one time-constant, and both reach steady-state after 5 time-constants.

Since we have an equation for i(t) when the capacitor is charging and one for i(t) when it is discharging, we can easily derive an equation for the voltage v_R across the resistor in each of these cases. Of course the same current exists everywhere in a series circuit, and since $v_R(t) = Ri(t)$, we find that the resistor voltage when the capacitor is charging is

$$v_R(t) = R\frac{E}{R}e^{-t/RC} = Ee^{-t/RC} \tag{7}$$

and when the capacitor is discharging

$$v_R(t) = -R\frac{E}{R}e^{-t/RC} = -Ee^{-t/RC} \tag{8}$$

The plots of these equations have the same general shapes as those shown in Figure 1.4. By Kirchhoff's voltage law, we know that the sum of the voltage drops around the circuit of Figure 1.1 must equal the applied voltage E. <u>This is true at every instant of time t.</u> We can verify this result mathematically using equations (1) and (7):

$$\begin{aligned} E &= v_R(t) + v_C(t) \\ &= Ee^{-t/RC} + E(1-e^{-t/RC}) \\ &= Ee^{-t/RC} + E - Ee^{-t/RC} \\ &= E \end{aligned}$$

As an exercise, verify Kirchhoff's voltage law around the circuit of Figure 1.3, when the capacitor is discharging.

Consider now the series RL circuit shown in Figure 1.5. The switch is thrown at t = 0, thus connecting the voltage source E to the circuit.

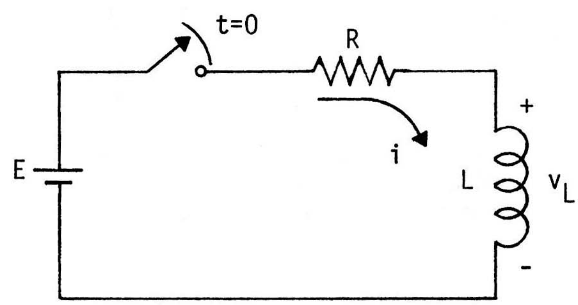


Figure 1.5 The transient current in an RL circuit after the switch is closed.

It is not possible to change the current through an inductor instantaneously, that is, it is not possible for the current to jump from its initial value of zero to some new value the instant the switch is closed. Consequently, the current in the circuit builds up gradually.

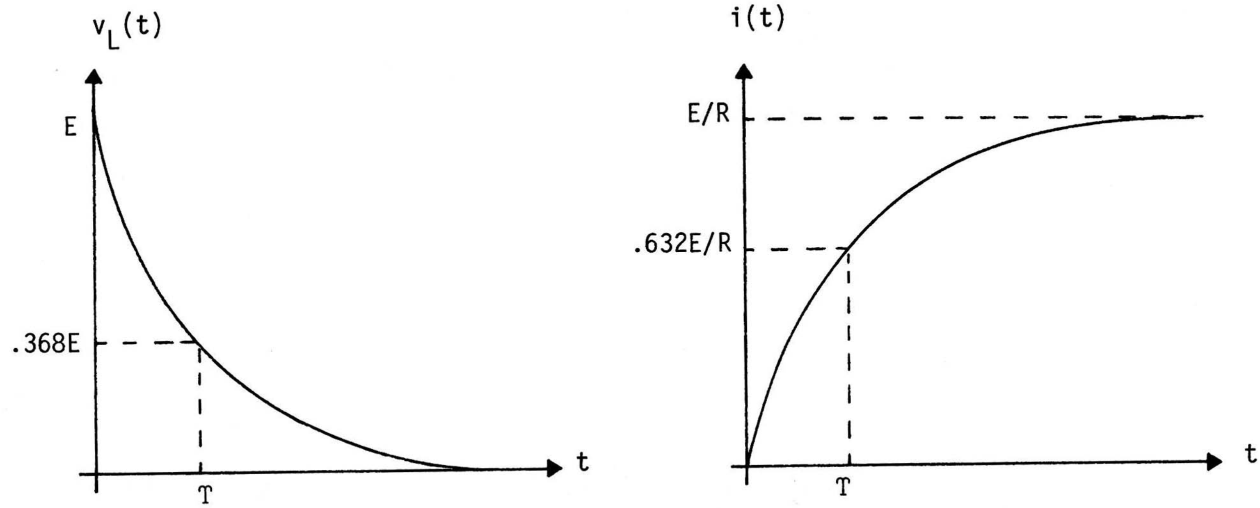
On the other hand, the voltage v_L across the inductor immediately jumps to E volts when the switch is thrown, and the polarity of this voltage opposes that of the applied voltage source. As the current builds up, the voltage v_L decays towards zero. The steady-state values of v_L and i are 0 volts and E/R amperes, respectively. Under steady-state conditions the inductor behaves exactly as a short-circuit. The equations that describe v_L and i as functions of time after the switch is closed are

$$v_L(t) = Ee^{-t/L/R} = Ee^{-t/T} \tag{9}$$

$$i(t) = \frac{E}{R}(1-e^{-t/L/R}) = \frac{E}{R}(1-e^{-t/T}) \tag{10}$$

where T = L/R seconds is the time-constant for the RL circuit.

We note that equations (9) and (10) are of the same general form as those we discussed earlier in connection with the RC circuit. Equations (9) and (10) are plotted in Figure 1.6.



(a)
The decay of inductor voltage in Figure 1.5.

(b)
The rise of inductor current in Figure 1.5.

Figure 1.6

Note that the shape of the plot of inductor voltage resembles that of the plot of capacitor current (see Figure 1.2). Also, the inductor current behaves like capacitor voltage. These similarities are to be expected when we compare the forms of equations (9) and (10) with those of (1) and (2).

The significance of the time-constant T in the RL circuit is the same as we have previously discussed for the RC circuit, as can be seen in Figure 1.6. The voltage and current transients again have a duration which for all practical purposes equals 5 time-constants. Note that large values of R reduce the time-constant in an RL circuit (T = L/R), while large values of R increase the time-constant of an RC circuit (T = RC). We frequently encounter exponential equations for voltages and currents containing a term written in the form e^{-at}, as for example, $e^{-.1t}$. Note that the time constant in these cases would be found from T = 1/a, since

$$e^{-at} = e^{-t/(1/a)} = e^{-t/T}$$

Thus a term such as $e^{-.1t}$ implies that the time-constant is 1/.1 = 10 seconds, while e^{-10^3t} implies that the time-constant is 1 ms.

If the voltage source E in Figure 1.5 were suddenly replaced by a short-circuit, then the voltage v_L across L would again change instantaneously, this time to -E volts, thus maintaining the instantaneous magnitude and direction of current. During the next 5 time-constants the voltage v_L and current i would decay exponentially towards zero. These results are evident from the decay equations for the RL circuit:

$$v_L(t) = -Ee^{-t/T} \tag{11}$$

$$i(t) = \frac{E}{R} e^{-t/T} \tag{12}$$

Note again the similarity of the forms of inductor voltage to capacitor current, and of capacitor voltage to inductor current (see equations 5 and 6, and Figure 1.4). As an exercise, the student should derive expressions for the resistor voltage v_R in Figure 1.5, and in the case when E is replaced by a short, and verify that Kirchhoff's voltage law holds in each case.

We often encounter RL and RC networks that contain series-parallel combinations of resistors, rather than consisting simply of a single resistor in series with a capacitor or inductor. In these situations, we can determine the circuit time-constant and predict voltage and current transients by finding the Thevenin equivalent circuit of the network to which the capacitor or inductor is connected. Recall that a Thevenin equivalent circuit is found by the following process.

1. Open-circuit the terminals with respect to which the equivalent circuit is to be found. In our case, this means simply remove the capacitor or inductor from the circuit and leave an open in its place.

2. Find the Thevenin equivalent resistance R_{TH} by computing the resistance seen when looking into the open-circuited terminals with all voltage sources replaced by short-circuits and all current sources replaced by open circuits.

3. Find the Thevenin equivalent voltage E_{TH} by computing the voltage that appears at the open-circuited terminals when all voltage and current sources have been restored.

When the Thevenin equivalent circuit has been determined, the capacitor or inductor can be restored and the circuit analyzed as a series RL or series RC circuit. Figure 1.7 shows these circuits as they appear after a series/parallel network is replaced by its Thevenin equivalent.

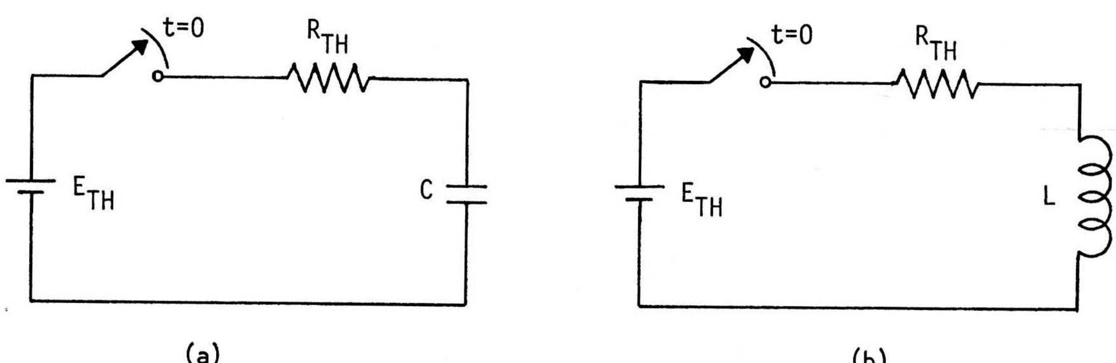


Figure 1.7 RC and RL networks containing series/parallel resistor combinations can be analyzed as series circuits by using Thevenin equivalent components.

Note that the time-constants of the circuits in Figures 1.7 (a) and (b) are $\tau = R_{TH}C$ and $\tau = L/R_{TH}$, respectively. Also, the steady-state capacitor voltage in 1.7 (a) is E_{TH}, while the steady-state current in 1.7 (b) is E_{TH}/R_{TH}.

To illustrate the use of a Thevenin equivalent circuit in the analysis of an RC network, we will determine the equations for the capacitor voltage and current after the switch is closed in the network of Figure 1.8.

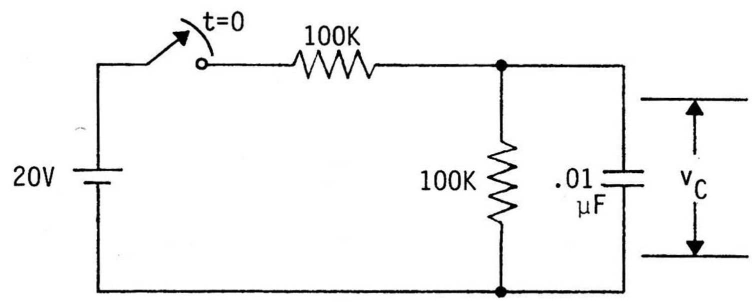


Figure 1.8 An RC network that can be analyzed using a Thevenin equivalent circuit.

When the capacitor in Figure 1.8 is removed and the 20 V source replaced by a short-circuit, we obtain the circuit shown in Figure 1.9.

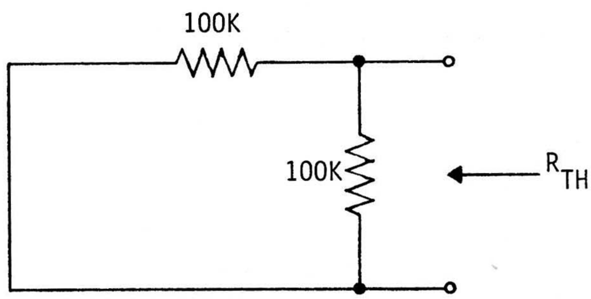


Figure 1.9 The Thevenin equivalent resistance seen by the capacitor in Figure 1.8 is 100K||100K = 50K.

From Figure 1.9 it is clear that the two 100 kΩ resistors are in parallel, and so R_{TH} = 50 kΩ. Figure 1.10 shows the circuit with the 20 V source restored.

From Figure 1.10 it is easy to see that E_{TH} = 10 V. Therefore, the Thevenin equivalent circuit with the capacitor restored appears as shown in Figure 1.11.

The time-constant of the circuit is $\tau = R_{TH}C = (5 \times 10^4)(10^{-8}) = 5 \times 10^{-4}$ sec = 0.5 ms. From equations (3) and (4), we find

$$v_C(t) = 10(1-e^{-t/.5 \times 10^{-3}})\,V$$

$$i(t) = 0.2e^{-t/.5 \times 10^{-3}}\ mA$$

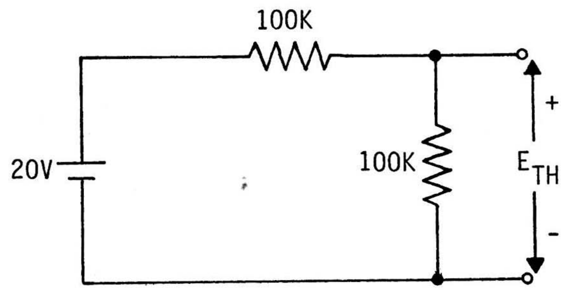


Figure 1.10 The Thevenin equivalent voltage is (100K/200K) 20 V = 10 V.

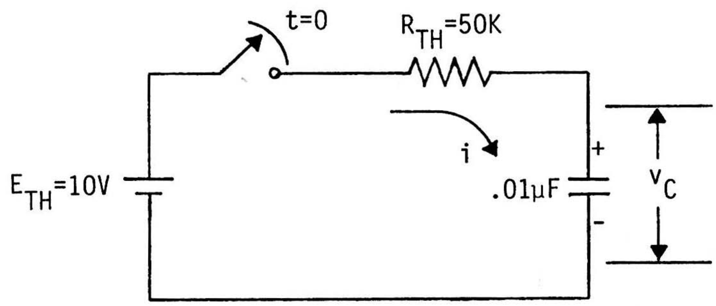


Figure 1.11 The Thevenin equivalent circuit of Figure 1.8 with the capacitor restored.

The steady-state values of v_C and i are 10 V and zero, respectively.

In this experiment we will investigate transients in RC and RL networks by connecting a square wave generator to the networks. The square wave will alternate between a positive voltage and 0 volts, thus simulating the repeated closing of a switch that alternately connects a positive voltage to the network and then grounds it. In this way we can examine the transient build-up and decay of voltages on an oscilloscope. The frequency of the square wave must be set low enough to allow steady-state conditions to be reached in between each application of the positive voltage and ground. Thus the period of the square wave will be at least 10 times the time-constant of the circuit.

III. EXERCISES

1. Write the equations for $v_C(t)$ and i(t) in the circuit of Figure 1.12 when a 5 volt dc level is switched in at t = 0 (instead of the square wave generator shown). What is the time constant τ of this circuit? Evaluate $v_C(t)$ and i(t) at t = 0, t = T, t = 0.5 ms and t = 5 ms. Sketch $v_C(t)$ and i(t) versus time t.

2. Assume that the capacitor in Figure 1.12 is fully charged and that the input is switched to zero volts at t = 0. Write the equations for $v_C(t)$ and i(t) in this case. What is the time-constant T? Evaluate $v_C(t)$ and i(t) at t = 0, t = T, t = 0.5 ms, and t = 5 ms. Sketch $v_C(t)$ and i(t) versus time t.

3. Repeat exercises 1 and 2 for the voltage $v_R(t)$ across the resistor in Figure 1.12.