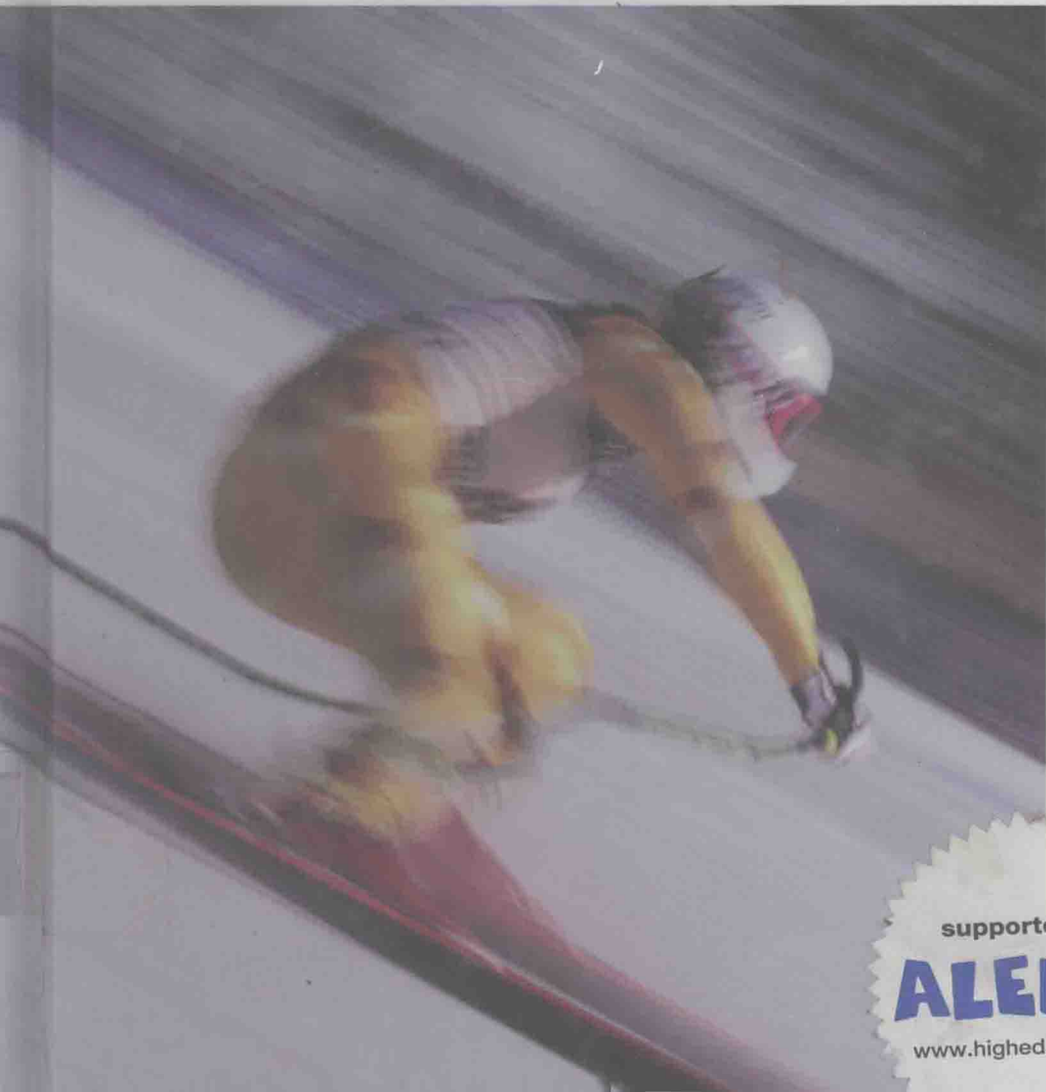
MARK DUGOPOLSKI

# INTERMEDIATE Algebra



3RD EDITION

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# Intermediate Algebra

EDITION 3

**Mark Dugopolski**

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*To my wife and daughters,*
*Cheryl, Sarah, and Alisha*

# PREFACE

*Intermediate Algebra* is designed to provide students with the algebra background needed for further college-level mathematics courses. The unifying theme of this text is the development of the skills necessary for solving equations and inequalities, followed by the application of those skills to solving applied problems. My primary goal in writing the third edition of *Intermediate Algebra* has been to retain the features that made the second edition so successful while incorporating the comments and suggestions of second-edition users. In addition, I have provided many new features that will help instructors to reach the goals that they have set for their students. As always, I endeavor to write texts that students can read, understand, and enjoy, and at the same time gain confidence in their ability to use mathematics. Although a complete development of each topic is provided in *Intermediate Algebra,* the text *Elementary Algebra* in this series would be more appropriate for students with no prior experience in algebra.

## Content Changes

While the essence of the text remains, the topics have been rearranged and new features added to reflect the current needs of instructors who are teaching intermediate algebra courses.

- Graphing is covered earlier in the text. Graphing linear equations in two variables is now in Chapter 3 immediately following linear equations in one variable. An introduction to functions and their graphs is also included in Chapter 3.
- Functions are also covered earlier in the text. The phrase, "is a function of" can be found in Section 3.3. The definition of a function as an object is then given in Section 3.5.
- Systems of equations and inequalities (Chapter 4) is also covered earlier in the third edition.
- More emphasis is given to reading and understanding graphs. Exercises that involve graphs have been added to many sections of the text along with conceptual questions relating to the graphs.

In addition to these changes, the text and exercise sets have been carefully revised where necessary. Many new, applied examples have been added to the text and a large number of new, applied exercises included in the exercise sets. Particular care has been given to achieving an appropriate balance of problems that progressively increase in difficulty from routine exercises in the beginning of the set to more challenging exercises at the end of the set. As in earlier editions, fractions and decimals are used in the exercises and throughout the text discussions to help reinforce the basic arithmetic skills that are required for success in algebra.

## Features

- Each chapter begins with a Chapter Opener that discusses a real application of algebra. The discussion is accompanied by a photograph, and in most cases by a real-data application graph that helps students to visualize algebra and more fully to understand the concepts discussed in the chapter. In addition, each chapter contains a Math at Work feature, which profiles a real person and the mathematics that he or she uses on the job. These two features have corresponding real-data exercises.

- **NEW!** An increased emphasis on real-data applications that involve graphs is a focus for the third edition. Applications have been added throughout the text to help demonstrate concepts, to motivate students, and to give students practice using new skills. Many of the real-data exercises contain data obtained from the Internet. Internet addresses are provided as a resource for both students and teachers. Because Internet addresses frequently change, a list of addresses will also be available on the web site. An Index of Applications listing applications by subject matter is included at the front of the text.
- Every section begins with In This Section, a list of topics that shows the student what will be covered. Because the topics correspond to the headings within each section, students will find it easy to locate and study specific concepts.
- Important ideas, such as definitions, rules, summaries, and strategies, are set apart in boxes for quick reference. Color is used to highlight these boxes as well as other significant points in the text.
- **NEW!** The third edition contains three new margin features that appear throughout the text:

  Calculator Close-ups give students an idea of how and when to use a graphing calculator. Some Calculator Close-ups simply introduce the features of a graphing calculator, whereas others enhance the understanding of algebraic concepts. For this reason many of the Calculator Close-ups will benefit even those students who do not use a graphing calculator. A graphing calculator is not required for studying from this text.

  Study Tips are included in the margins throughout the text. These short tips are meant to reinforce continually good study habits and to keep reminding students that it is never too late to make improvements in the manner in which they study.

  Helpful Hints are short comments that enhance the material in the text, provide another way of approaching a problem, or clear up misconceptions.
- At the end of every section there are Warm-up exercises, a set of ten simple statements that are to be answered true or false. These exercises are designed to provide a smooth transition between the ideas and the exercise sets. They help students to understand that every statement in mathematics is either true or false. They are also good for discussion or group work.
- **NEW!** Every section-ending exercise set in the third edition generally begins with six simple writing exercises. These exercises are designed to get students to review the definitions and rules of the section before doing more traditional exercises. For example, the student might be simply asked what properties of equality were discussed in this section.
- The end-of-section Exercises follow the same order as the textual material and contain exercises that are keyed to examples, as well as numerous exercises that are not keyed to examples. This organization allows the instructor to deal with only part of a section if necessary and to easily determine which exercises are appropriate to assign. The keyed exercises give the student a place to start practicing and

building confidence, whereas the nonkeyed exercises are designed to "wean" the student from following examples in a step-by-step manner. Getting More Involved exercises are designed to encourage writing, discussion, exploration, and cooperative learning. Graphing Calculator Exercises require a graphing calculator and are identified with a graphing calculator logo. Exercises for which a scientific calculator would be helpful are identified with a scientific calculator logo.

- Every chapter ends with a four-part Wrap-up, which includes the following:

  The Chapter Summary lists important concepts along with brief illustrative examples.

  NEW! Enriching Your Mathematical Word Power appears at the end of each chapter and consists of multiple-choice questions in which the important terms are to be matched with their meanings. This feature emphasizes the importance of proper terminology.

  The Review Exercises contain problems that are keyed to the sections of the chapter as well as numerous miscellaneous exercises.

  The Chapter Test is designed to help the student assess his or her readiness for a test. The Chapter Test has no keyed exercises, thus enabling the student to work independently of the sections and examples.

- At the end of each chapter there is a Collaborative Activities feature that is designed to encourage interaction and learning in groups. Instructions and suggestions for using these activities and answers to all problems can be found in the Instructor's Solutions Manual.
- The Making Connections exercises at the end of Chapters 2–12 are designed to help students review and synthesize the new material with ideas from previous chapters, and in some cases to review material necessary for success in the upcoming chapter. Every Making Connections exercise set includes at least one applied exercise that requires ideas from one or more of the previous chapters.

## Coverage

For those who wish to cover more on functions, Chapter 9 can be covered after functions are introduced in Chapter 3. For those who wish to cover less on functions, Sections 3.5 and 3.6 can be omitted. Some or all of Chapter 4 can be omitted for those who desire a less extensive treatment of systems of linear equations. However, if you have a graphing calculator to do the determinants, Cramer's rule with three variables is rather fun.

## Supplements for the Instructor

### ANNOTATED INSTRUCTOR'S EDITION

This ancillary includes answers to all exercises and tests. Each answer is printed next to each problem on the page where the problem appears. The answers are printed in a second color for ease of use by instructors.

### PRINT AND COMPUTERIZED TEST BANK

The testing materials provide an array of formats that allow the instructor to create tests using both algorithmically generated test questions and those from a standard test bank. This testing system enables the instructor to choose questions either manually or randomly by section, question type, difficulty level, and other criteria. Testing is available for IBM, IBM compatible, and Macintosh computers. Instructors can edit questions in the testing system as well if they seek a degree of customization. The print version of the test bank is softcover and provides questions found in the computerized version along with answer keys. Each chapter of the print version contains three different tests. Additionally, the print test bank contains four different, comprehensive final exams.

#### INSTRUCTOR'S SOLUTIONS MANUAL

Prepared by Mark Dugopolski, this supplement contains detailed, worked solutions to all of the exercises in the text. The solutions are done by the techniques used in the text. Instructions and suggestions for using the Collaborative Activities feature in the text are also included in the Instructor's Solutions Manual.

### Supplements for the Student

#### STUDENT'S SOLUTIONS MANUAL

Prepared by Mark Dugopolski, the Student's Solutions Manual contains complete worked-out solutions to all of the odd-numbered exercises in the text. It also contains solutions for all exercises in the Chapter Tests. It may be purchased by your students from McGraw-Hill.

#### DUGOPOLSKI VIDEO SERIES

The video tape series contains instructional material and presents opportunities for students to work problems and to check their results. The tapes are text-specific and cover all chapters of the text. The tapes are facilitated by instructors who introduce topics and work through examples. Students are encouraged to work examples on their own and to check their results with those provided.

#### DUGOPOLSKI TUTORIAL CD-ROM

This interactive CD-ROM is a self-paced tutorial specifically linked to the text and reinforces topics through unlimited opportunities to review concepts and to practice problem solving. The CD-ROM contains text-, chapter-, and section-specific tutorials, multiple-choice questions with feedback, as well as algorithmically-generated questions. It requires virtually no computer training on the part of students and supports IBM and Macintosh computers.

In addition, a number of other technology and Web-based ancillaries are under development; they will support the ever-changing technology needs in developmental mathematics. For further information about these or any supplements, please contact your local McGraw-Hill sales representative.

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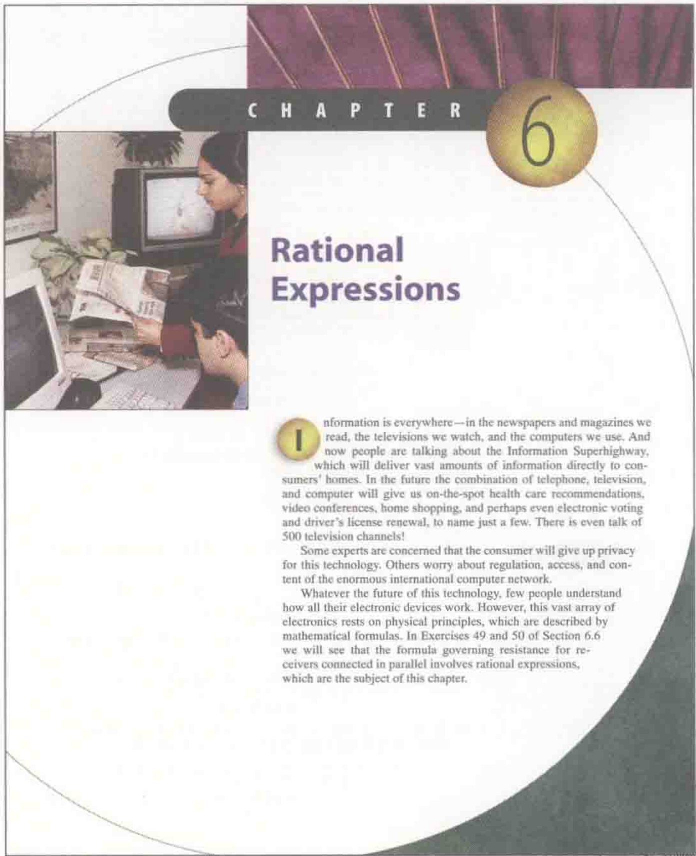
*Hammond, Louisiana* M.D.


CHAPTER 6

# Rational Expressions

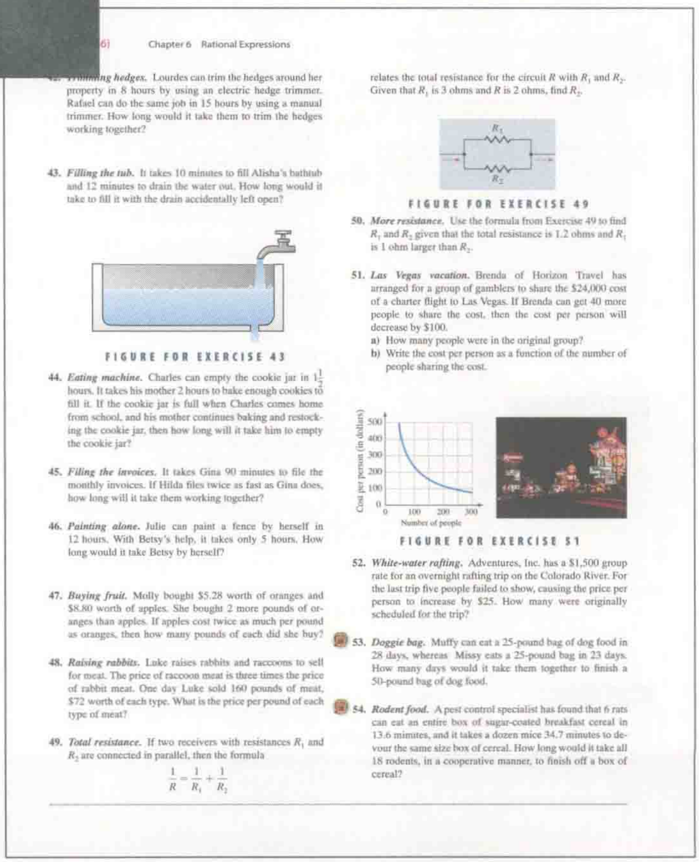
Information is everywhere—in the newspapers and magazines we read, the televisions we watch, and the computers we use. And now people are talking about the Information Superhighway, which will deliver vast amounts of information directly to consumers' homes. In the future the combination of telephone, television, and computer will give us on-the-spot health care recommendations, video conferences, home shopping, and perhaps even electronic voting and driver's license renewal, to name just a few. There is even talk of 500 television channels!

Some experts are concerned that the consumer will give up privacy for this technology. Others worry about regulation, access, and content of the enormous international computer network.

Whatever the future of this technology, few people understand how all their electronic devices work. However, this vast array of electronics rests on physical principles, which are described by mathematical formulas. In Exercises 49 and 50 of Section 6.6 we will see that the formula governing resistance for receivers connected in parallel involves rational expressions, which are the subject of this chapter.

## Chapter Opener

Each **chapter opener** features a real-world situation that can be modeled using mathematics. Each chapter contains exercises that relate back to the chapter opener.


6) Chapter 6 Rational Expressions

... *...ming hedges.* Lourdes can trim the hedges around her property in 8 hours by using an electric hedge trimmer. Rafael can do the same job in 15 hours by using a manual trimmer. How long would it take them to trim the hedges working together?

43. *Filling the tub.* It takes 10 minutes to fill Alisha's bathtub and 12 minutes to drain the water out. How long would it take to fill it with the drain accidentally left open?

FIGURE FOR EXERCISE 43

44. *Eating machine.* Charles can empty the cookie jar in $1\frac{1}{2}$ hours. It takes his mother 2 hours to bake enough cookies to fill it. If the cookie jar is full when Charles comes home from school, and his mother continues baking and restocking the cookie jar, then how long will it take him to empty the cookie jar?

45. *Filing the invoices.* It takes Gina 90 minutes to file the monthly invoices. If Hilda files twice as fast as Gina does, how long will it take them working together?

46. *Painting alone.* Julie can paint a fence by herself in 12 hours. With Betsy's help, it takes only 5 hours. How long would it take Betsy by herself?

47. *Buying fruit.* Molly bought \$5.28 worth of oranges and \$8.80 worth of apples. She bought 2 more pounds of oranges than apples. If apples cost twice as much per pound as oranges, then how many pounds of each did she buy?

48. *Raising rabbits.* Luke raises rabbits and raccoons to sell for meat. The price of raccoon meat is three times the price of rabbit meat. One day Luke sold 160 pounds of meat, \$72 worth of each type. What is the price per pound of each type of meat?

49. *Total resistance.* If two receivers with resistances $R_1$ and $R_2$ are connected in parallel, then the formula

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

relates the total resistance for the circuit $R$ with $R_1$ and $R_2$. Given that $R_1$ is 3 ohms and $R$ is 2 ohms, find $R_2$.

FIGURE FOR EXERCISE 49

50. *More resistance.* Use the formula from Exercise 49 to find $R_1$ and $R_2$ given that the total resistance is 1.2 ohms and $R_1$ is 1 ohm larger than $R_2$.

51. *Las Vegas vacation.* Brenda of Horizon Travel has arranged for a group of gamblers to share the \$24,000 cost of a charter flight to Las Vegas. If Brenda can get 40 more people to share the cost, then the cost per person will decrease by \$100.
a) How many people were in the original group?
b) Write the cost per person as a function of the number of people sharing the cost.

FIGURE FOR EXERCISE 51

52. *White-water rafting.* Adventures, Inc. has a \$1,500 group rate for an overnight rafting trip on the Colorado River. For the last trip five people failed to show, causing the price per person to increase by \$25. How many were originally scheduled for the trip?

53. *Doggie bag.* Muffy can eat a 25-pound bag of dog food in 28 days, whereas Missy eats a 25-pound bag in 23 days. How many days would it take them together to finish a 50-pound bag of dog food.

54. *Rodent food.* A pest control specialist has found that 6 rats can eat an entire box of sugar-coated breakfast cereal in 13.6 minutes, and it takes a dozen mice 34.7 minutes to devour the same size box of cereal. How long would it take all 18 rodents, in a cooperative manner, to finish off a box of cereal?

## Margin Notes

Margin notes include **Helpful Hints, Study Tips,** and **Calculator Close-ups.** The *Helpful Hints* point out common errors or reminders. The *Study Tips* provide practical suggestions for improving study habits. The optional *Calculator Close-ups* provide tips on using a graphing calculator to aid in your understanding of the material. They also include insightful suggestions for increasing calculator proficiency.



**calculator close-up**

These Calculator Close-ups are designed to help reinforce the concepts of algebra, not replace them. Do not rely too heavily on your calculator or use it to replace the algebraic methods taught in this course.

### Graphing on the Number Line

To construct a number line, we draw a straight line and label any convenient point with the number 0. Now we choose any convenient length and use it to locate points to the right of 0 as points corresponding to the positive integers and points to the left of 0 as points corresponding to the negative integers. See Fig. 1.4. The numbers corresponding to the points on the line are called the **coordinates** of the points. The distance between two consecutive integers is called a **unit,** and it is the same for any two consecutive integers. The point with coordinate 0 is called the **origin.** The numbers on the number line increase in size from left to right. When we compare the size of any two numbers, the larger number lies to the right of the smaller one on the number line.

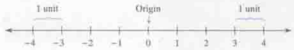



**FIGURE 1.4**

It is often convenient to illustrate sets of numbers on a number line. The set of integers, $J$, is illustrated or **graphed** as in Fig. 1.5. The three dots to the right and left on the number line indicate that the integers go on indefinitely in both directions.


**FIGURE 1.5**

**EXAMPLE 2** **Graphing on the number line**

List the elements of each set and graph each set on a number line.

**a)** $\{x \mid x$ is a whole number less than $4\}$

**b)** $\{a \mid a$ is an integer between 3 and $9\}$

**c)** $\{y \mid y$ is an integer greater than $-3\}$

**Solution**

**study tip**

Take notes in class. Write down everything you can. As soon as possible after class, rewrite your notes. Fill in details and make corrections. Make a note of examples and exercises in the text that are similar to examples in your notes. If your instructor takes the time to work an example in class, it is a "good bet" that your instructor expects you to understand the concepts involved.

**a)** The whole numbers less than 4 are 0, 1, 2, and 3. Figure 1.6 shows the graph of this set.


**FIGURE 1.6**

**b)** The integers between 3 and 9 are 4, 5, 6, 7, and 8. The graph is shown in Fig. 1.7.


**FIGURE 1.7**

**c)** The integers greater than $-3$ are $-2$, $-1$, 0, 1, and so on. To indicate the continuing pattern, we use a series of dots on the graph in Fig. 1.8.


**FIGURE 1.8**



**38.** ***Three-digit number.*** The sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the old number. If the hundreds digit plus twice the tens digit is equal to the units digit, then what is the number?

**39.** ***Working overtime.*** To make ends meet, Ms. Farnsby works three jobs. Her total income last year was $48,000. Her income from teaching was just $6,000 more than her income from house painting. Royalties from her textbook sales were one-seventh of the total money she received from teaching and house painting. How much did she make from each source last year?

**40.** ***Pocket change.*** Harry has $2.25 in nickels, dimes, and quarters. If he had twice as many nickels, half as many dimes, and the same number of quarters, he would have $2.50. If he has 27 coins altogether, then how many of each does he have?

#### GETTING MORE INVOLVED

**41.** ***Exploration.*** Draw diagrams showing the possible ways to position three planes in three-dimensional space.

**42.** ***Discussion.*** Make up a system of three linear equations in three variables for which the solution set is $\{(0, 0, 0)\}$. A system with this solution set is called a *homogeneous* system. Why do you think it is given that name?

**43.** ***Cooperative learning.*** Working in groups, do parts (a)–(d) below. Then write a report on your findings.

**a)** Find values of $a$, $b$, and $c$ so that the graph of $y = ax^2 + bx + c$ goes through the points $(-1, -2)$, $(1, 0)$, and $(2, 7)$.

**b)** Arbitrarily select three ordered pairs and find the equation of the parabola that goes through the three points.

**c)** Could more than one parabola pass through three given points? Give reasons for your answer.

**d)** Explain how to pick three points for which no parabola passes through all of them.

## 4.4 SOLVING LINEAR SYSTEMS USING MATRICES

**In this section**

- Matrices
- The Augmented Matrix
- The Gaussian Elimination Method
- Inconsistent and Dependent Equations

You solved linear systems in two variables by substitution and addition in Sections 4.1 and 4.2. Those methods are done differently on each system. In this section you will learn the Gaussian elimination method, which is related to the addition method. The Gaussian elimination method is performed in the same way on every system. We first need to introduce some new terminology.

### Matrices

A **matrix** is a rectangular array of numbers. The **rows** of a matrix run horizontally, and the **columns** of a matrix run vertically. A matrix with $m$ rows and $n$ columns has **order** $m \times n$ (read "$m$ by $n$"). Each number in a matrix is called an **element** or **entry** of the matrix.

**EXAMPLE 1** **Order of a matrix**

Determine the order of each matrix.

**a)** $\begin{bmatrix} -1 & 2 \\ 5 & \sqrt{2} \\ 0 & 3 \end{bmatrix}$ **b)** $\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix}$ **c)** $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 0 & 2 \end{bmatrix}$ **d)** $[1 \quad 3 \quad 6]$

**Solution**

Because matrix (a) has 3 rows and 2 columns, its order is $3 \times 2$. Matrix (b) is a $2 \times 2$ matrix, matrix (c) is a $3 \times 3$ matrix, and matrix (d) is a $1 \times 3$ matrix.

**study tip**

As soon as possible after class, find a quiet place and work on your homework. The longer you wait, the harder it is to remember what happened in class.

### The Augmented Matrix

The solution to a system of linear equations such as

$$x - 2y = -5$$
$$3x + y = 6$$

## 8.1 FACTORING AND COMPLETING THE SQUARE

**In this section**

- Review of Factoring
- Review of the Even-Root Property
- Completing the Square
- Miscellaneous Equations
- Imaginary Solutions

Factoring and the even-root property were used to solve quadratic equations in Chapters 5, 6, and 7. In this section we first review those methods. Then you will learn the method of completing the square, which can be used to solve any quadratic equation.

### Review of Factoring

A quadratic equation is a second-degree polynomial equation of the form

$$ax^2 + bx + c = 0,$$

where $a$, $b$, and $c$ are real numbers with $a \neq 0$. If the second-degree polynomial on the left-hand side can be factored, then we can solve the equation by breaking it into two first-degree polynomial equations (linear equations) using the following strategy.

**Strategy for Solving Quadratic Equations by Factoring**

1. Write the equation with 0 on the right-hand side.
2. Factor the left-hand side.
3. Use the zero factor property to set each factor equal to zero.
4. Solve the simpler equations.
5. Check the answers in the original equation.

# Strategy Boxes

The **strategy boxes** generally provide a numbered list of concepts from a section or a set of steps to follow in problem solving. They can be used by students who prefer a more structured approach to problem solving or used as a study tool to review important points within sections.

**EXAMPLE 1** **Solving a quadratic equation by factoring**

Solve $3x^2 - 4x = 15$ by factoring.

**Solution**

Subtract 15 from each side to get 0 on the right-hand side:

$$3x^2 - 4x - 15 = 0$$

$$(3x + 5)(x - 3) = 0 \quad \text{Factor the left-hand side.}$$

$$3x + 5 = 0 \quad \text{or} \quad x - 3 = 0 \quad \text{Zero factor property}$$

$$3x = -5 \quad \text{or} \quad x = 3$$

$$x = -\frac{5}{3}$$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$. Check the solutions in the original equation.

**helpful hint**

After you have factored the quadratic polynomial, use FOIL to check that you have factored correctly before proceeding to the next step.

### Review of the Even-Root Property

In Chapter 7 we solved quadratic equations by using the even-root property.



**MATH AT WORK** $x^2+(x+1)^2=5^2$

**MARINE INSURANCE AGENT**

Cargo has been lost, or the hull of a ship has been damaged. What is the amount of money that should be paid to the insured party? Lisa M. Paccione, Ocean Marine Claim Representative for the St. Paul Insurance Company, investigates, evaluates, resolves, and pays these types of claims. Ms. Paccione does this by gathering data, occasionally doing a visual inspection, interviewing witnesses, and negotiating with attorneys.

Decisions about losses are based on the insured party's individual policy as well as traditional marine practices and maritime law. When consignees suffer a cargo loss, they not only are compensated for the actual amount of the damaged goods, but also receive an additional "advance" in the settlement. Customarily, the advance is 10% over the value of the goods. The amount that St. Paul pays the insured party for a valid claim is computed by using a proportion. In Exercises 59 and 60 of this section you will solve problems involving this proportion.

# Math at Work

The **Math at Work** feature that appears in each chapter explores the careers of individuals who use the mathematics presented in the chapter in their work. Students are referred to exercises that directly relate to the occupation highlighted in Math at Work.

**EXAMPLE 8** **Ratios and proportions**

The ratio of men to women at a football game was 4 to 3. If there were 12,000 more men than women in attendance, then how many men and how many women were in attendance?

**Solution**

Let $x$ represent the number of men in attendance and $x - 12{,}000$ represent the number of women in attendance. Because the ratio of men to women was 4 to 3, we can write the following proportion:

$$\frac{4}{3} = \frac{x}{x - 12{,}000}$$

$$4x - 48{,}000 = 3x$$

$$x = 48{,}000$$

So there were 48,000 men and 36,000 women at the game.

**WARM-UPS**

**True or false? Explain.**

1. In solving an equation involving rational expressions, multiply each side by the LCD for all of the denominators.
2. To solve $\frac{1}{x} + \frac{1}{2x} = \frac{1}{3}$, first change each rational expression to an equivalent rational expression with a denominator of $6x$.
3. Extraneous roots are not real numbers.
4. To solve $\frac{1}{x-2} + 3 = \frac{1}{x+2}$, multiply each side by $x^2 - 4$.
5. The solution set to $\frac{x}{3x+4} - \frac{6}{2x+1} = \frac{7}{5}$ is $\left\{-\frac{4}{3}, -\frac{1}{2}\right\}$.

## Warm-ups

**Warm-ups** appear before each set of exercises at the end of every section. They are true or false statements that can be used to check conceptual understanding of material within each section.



Because there are no real even roots of negative numbers, the expressions

$$a^{1/2}, \quad x^{-3/4}, \quad \text{and} \quad y^{1/6}$$

are not real numbers if the variables have negative values. To simplify matters, we sometimes assume the variables represent only positive numbers when we are working with expressions involving variables with rational exponents. That way we do not have to be concerned with undefined expressions and absolute value.

**EXAMPLE 8** **Expressions involving variables with rational exponents**

Use the rules of exponents to simplify the following. Write your answers with positive exponents. Assume all variables represent *positive* real numbers.

a) $x^{2/3}x^{4/3}$ b) $\frac{a^{1/2}}{a^{1/4}}$

c) $(x^{1/2}y^{-3})^{1/2}$ d) $\left(\frac{x^2}{y^{1/3}}\right)^{-1/2}$

**Solution**

a) $x^{2/3}x^{4/3} = x^{6/3}$ Use the product rule to add the exponents.

$= x^2$ Reduce the exponent.

b) $\frac{a^{1/2}}{a^{1/4}} = a^{1/2-1/4}$ Use the quotient rule to subtract the exponents.

$= a^{1/4}$ Simplify.

c) $(x^{1/2}y^{-3})^{1/2} = (x^{1/2})^{1/2}(y^{-3})^{1/2}$ Power of a product rule

$= x^{1/4}y^{-3/2}$ Power of a power rule

$= \frac{x^{1/4}}{y^{3/2}}$ Definition of negative exponent

d) Because this expression is a negative power of a quotient, we can first find the reciprocal of the quotient, then apply the power of a power rule:

$$\left(\frac{x^2}{y^{1/3}}\right)^{-1/2} = \left(\frac{y^{1/3}}{x^2}\right)^{1/2} = \frac{y^{1/6}}{x} \qquad \frac{1}{3}\cdot\frac{1}{2} = \frac{1}{6}$$

**WARM-UPS**

**True or false? Explain.**

1. $4^{-1/2} = \frac{1}{2}$
2. $16^{1/2} = 8$
3. $(3^{2/3})^3 = 9$
4. $8^{-2/3} = -4$
5. $2^{1/2} \cdot 2^{1/2} = 2$
6. $\left(\frac{1}{4}\right)^{1/2} = \frac{1}{2}$
7. $\frac{3}{3^{1/2}} = 3^{1/2}$
8. $(2^9)^{1/2} = 2^3$
9. $3^{1/3} \cdot 6^{1/3} = 18^{2/3}$
10. $2^{3/4} \cdot 2^{1/4} = 4$



**COLLABORATIVE ACTIVITIES**

**Beorg's Business**

In manufacturing or other businesses in which time is money and tasks are easily shared, problems involving work appear. An owner or manager who wants to know how to bid a job often develops a table of times needed to complete the job as determined by how much work is required and who could be assigned to the job.

Beorg owns a kaleidoscope-manufacturing company with two employees, Scott and Salina. It takes Scott one hour to make one kaleidoscope, and it takes Salina $\frac{1}{2}$ hour to make one kaleidoscope. Beorg wants to know how long it would take to complete a certain number of kaleidoscopes. Using the information given and answering the questions below, fill in the following table for Beorg.

| Name of Employee | Time for one kaleidoscope | Time for 20 kaleidoscopes |
|---|---|---|
| Scott | 1 hr | |
| Salina | $\frac{1}{2}$ hr | |
| Scott & Salina | | |
| Sammy | | |
| Scott & Sammy | $\frac{3}{4}$ hr | |
| Salina & Sammy | | |
| Scott, Salina, & Sammy | | |

*Grouping:* Four students per group
*Topic:* Applications of work problems

1. How long will it take Scott and Salina working together to make one kaleidoscope?
2. Beorg hires a third person, Sammy, and has him and Scott make one kaleidoscope. Working together, it takes them $\frac{3}{4}$ hour to make one kaleidoscope. How long would it take Sammy by himself to make one kaleidoscope?
3. How long would it take Salina and Sammy working together to make one kaleidoscope? How long would it take for all three working together?

   Now Beorg wants to finish his time table. He would like to have 20 kaleidoscopes completed each day.
4. Finish the preceding table, and find the best combination or combinations of employees to use to have 20 kaleidoscopes at the end of an 8-hour day.

**Extension:** Is Sammy in the combination or combinations you found in the last question? Is it worth having Sammy work? Remember that when someone is starting a new job, he or she may work more slowly until he or she learns how to do the job more efficiently. Find out how fast Sammy would need to work for production to double (40 kaleidoscopes in an 8-hour day).

**WRAP-UP CHAPTER 6**

**SUMMARY**

| **Rational Expressions** | | **Examples** |
|---|---|---|
| Rational expression | The ratio of two polynomials with the denominator not equal to zero | $\frac{x^2-1}{2x-3}$ |
| Domain of a rational expression | The set of all possible numbers that can be used as replacements for the variable | $D = \left\{x \mid x \neq \frac{3}{2}\right\}$ |

## Collaborative Activities

**Collaborative Activities** appear at the end of each chapter. The activities are designed to encourage interaction and learning in a group setting.

# Exercises

The theme of mathematics in everyday situations is carried over to the exercise sets. Applications based on real-world data are included in each set. The **Index of Applications** can help students to quickly identify exercises that associate the mathematics that may be used in their areas of interest.

64 (2-12) Chapter 2 Linear Equations and Inequalities in One Variable

*Solve each equation. Practice combining some steps. Look for more efficient ways to solve each equation. See Example 8.*

**81.** $3x - 9 = 0$
**82.** $5x + 1 = 0$
**83.** $7 - z = -9$
**84.** $-3 - z = 3$
**85.** $\frac{2}{3}x = \frac{1}{2}$
**86.** $\frac{3}{2}x = -\frac{9}{5}$
**87.** $-\frac{3}{5}y = 9$
**88.** $-\frac{2}{7}w = 4$
**89.** $3y + 5 = 4y - 1$
**90.** $2y - 7 = 3y + 1$
**91.** $5x + 10(x + 2) = 110$
**92.** $1 - 3(x - 2) = 4(x - 1) - 3$

*Solve each equation.*

**93.** $\frac{P+7}{3} - \frac{P-2}{5} = \frac{7}{3} - \frac{P}{15}$
**94.** $\frac{w-3}{8} - \frac{5-w}{4} = \frac{4w-1}{8} - 1$
**95.** $x - 0.06x = 50{,}000$
**96.** $x - 0.05x = 800$
**97.** $2.365x + 3.694 = 14.8095$
**98.** $-3.48x + 6.981 = 4.329x - 6.851$

*Solve each problem. See Example 9.*

**99.** ***Public school enrollment.*** The expression

$$0.45x + 39.05$$

can be used to approximate in millions the total enrollment in public elementary and secondary schools in the year $1985 + x$ (National Center for Education Statistics, www.nces.ed.gov).

a) What was the public school enrollment in 1992?
b) In which year will enrollment reach 50 million students?
c) Judging from the accompanying graph, is enrollment increasing or decreasing?


FIGURE FOR EXERCISE 99

**100.** ***Teacher's average salary.*** The expression

$$553.7x + 27{,}966$$

can be used to approximate the average annual salary in dollars of public school teachers in the year $1985 + x$ (National Center for Education Statistics, www.nces.ed.gov).

a) What was the average teacher's salary in 1993?
b) In which year will the average salary reach $40,000?

**101.** ***Solid waste recovery.*** In 1960 the United States generated 87.1 million tons of municipal solid waste and recovered (or recycled) only 4.3% of it (U.S. Department of Energy, www.doe.gov). The amount of solid waste generated in the United States in the year $1960 + n$ is given by

$$w = 3.14n + 87.1,$$

whereas the amount recovered is given by

$$w = 0.576n + 3.78,$$

where $w$ is in millions of tons.

a) Use the accompanying graph to estimate the first year in which the United States generated over 100 million tons of municipal solid waste.
b) Find the year in which 13% of the municipal solid waste will be recovered by solving

$$0.576n + 3.78 = 0.13(3.14n + 87.1).$$

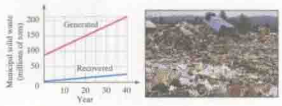



FIGURE FOR EXERCISE 101

**102.** ***Recycling progress.*** Find the year in which 14% of the municipal solid waste will be recovered by solving

$$0.576n + 3.78 = 0.14(3.14n + 87.1).$$

See the previous exercise.

## GETTING MORE INVOLVED

**103.** ***Exploration.*** If you solved the equations in the two previous exercises, then you found the years in which recovery of municipal solid waste will reach 13% and 14%.

a) Write equations corresponding to recovery rates of 15%, 16%, 17%, 18%, and 19%.
b) Solve your equations to find the years in which those recovery rates will be achieved.
c) Use your results to judge whether we are making real progress in recovery of municipal solid waste.

**104.** ***Writing.*** Explain how to eliminate the decimals in an equation that involves numbers with decimal points. Would you use the same technique when using a calculator?

**105.** ***Discussion.*** Explain why the multiplication property of equality does not allow us to multiply each side of an equation by zero.

456 (8-10) Chapter 8 Quadratic Equations and Inequalities

*Find the complex solutions to each equation. See Example 10.*

**67.** $x^2 + 2x + 5 = 0$
**68.** $x^2 + 4x + 5 = 0$
**69.** $x^2 + 12 = 0$
**70.** $-3x^2 - 21 = 0$
**71.** $5z^2 - 4z + 1 = 0$
**72.** $2w^2 - 3w + 2 = 0$

*Find all real or imaginary solutions to each equation. Use the method of your choice.*

**73.** $4x^2 + 25 = 0$
**74.** $5w^2 - 3 = 0$
**75.** $\left(p + \frac{1}{2}\right)^2 = \frac{9}{4}$
**76.** $\left(y - \frac{2}{3}\right)^2 = \frac{4}{9}$
**77.** $5t^2 + 4t - 3 = 0$
**78.** $3v^2 + 4v - 1 = 0$
**79.** $m^2 + 2m - 24 = 0$
**80.** $q^2 + 6q - 7 = 0$
**81.** $\left(a + \frac{2}{3}\right)^2 = -\frac{32}{9}$
**82.** $\left(w + \frac{1}{2}\right)^2 = -6$
**83.** $-x^2 + x + 6 = 0$
**84.** $-x^2 + x + 12 = 0$
**85.** $x^2 - 6x + 10 = 0$
**86.** $x^2 - 8x + 17 = 0$
**87.** $2x - 5 = \sqrt{7x + 7}$
**88.** $\sqrt{7x + 29} = x + 3$
**89.** $\frac{1}{x} + \frac{1}{x-1} = \frac{1}{4}$
**90.** $\frac{1}{x} - \frac{2}{1-x} = \frac{1}{2}$

*If the solution to an equation is imaginary or irrational, it takes a bit more effort to check. Replace x by each given number to verify each statement.*

**91.** Both $2 + \sqrt{3}$ and $2 - \sqrt{3}$ satisfy $x^2 - 4x + 1 = 0$.
**92.** Both $1 + \sqrt{2}$ and $1 - \sqrt{2}$ satisfy $x^2 - 2x - 1 = 0$.
**93.** Both $1 + i$ and $1 - i$ satisfy $x^2 - 2x + 2 = 0$.
**94.** Both $2 + 3i$ and $2 - 3i$ satisfy $x^2 - 4x + 13 = 0$.

*Solve each problem.*

**95.** ***Approach speed.*** The formula $1211.1L = CA^2S$ is used to determine the approach speed for landing an aircraft, where $L$ is the gross weight of the aircraft in pounds, $C$ is the coefficient of lift, $S$ is the surface area of the wings in square feet ($ft^2$), and $A$ is approach speed in feet per second. Find $A$ for the Piper Cheyenne, which has a gross weight of 8,700 lb, a coefficient of lift of 2.81, and wing surface area of 200 $ft^2$.

**96.** ***Time to swing.*** The period $T$ (time in seconds for one complete cycle) of a simple pendulum is related to the length $L$ (in feet) of the pendulum by the formula $8T^2 = \pi^2L$. If a child is on a swing with a 10-foot chain, then how long does it take to complete one cycle of the swing?

**97.** ***Time for a swim.*** Tropical Pools figures that its monthly revenue in dollars on the sale of $x$ above-ground pools is given by $R = 1500x - 3x^2$, where $x$ is less than 25. What number of pools sold would provide a revenue of $17,568?

**98.** ***Pole vaulting.*** In 1981 Vladimir Poliakov (USSR) set a world record of 19 ft $\frac{3}{4}$ in. for the pole vault (Doubleday Almanac). To reach that height, Poliakov obtained a speed of approximately 36 feet per second on the runway. The function $h = -16t^2 + 36t$ gives his height $t$ seconds after leaving the ground.

a) Use the formula to find the exact values of $t$ for which his height was 18 feet.
b) Use the accompanying graph to estimate the value of $t$ for which he was at his maximum height.
c) Approximately how long was he in the air?

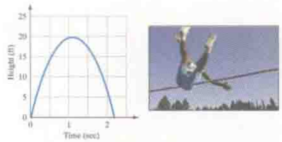



FIGURE FOR EXERCISE 98

## GETTING MORE INVOLVED

**99.** ***Discussion.*** Which of the following equations is not a quadratic equation?

a) $\pi x^2 - \sqrt{5}x - 1 = 0$
b) $3x^2 - 1 = 0$
c) $4x + 5 = 0$
d) $0.009x^2 = 0$

**100.** ***Exploration.*** Solve $x^2 - 4x + k = 0$ for $k = 0, 4, 5,$ and 10.

a) When does the equation have only one solution?
b) For what values of $k$ are the solutions real?
c) For what values of $k$ are the solutions imaginary?

**Getting More Involved** appears within selected exercise sets. This feature may contain

*Writing,*

*Cooperative Learning,*

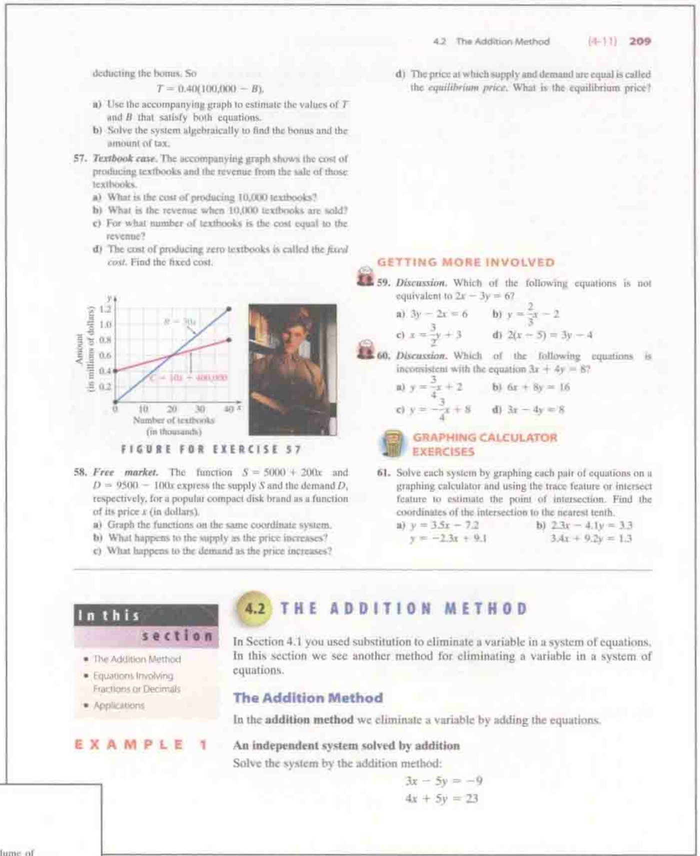
*Exploration,* and/or

*Discussion* exercises. Each of these components is designed to give students an opportunity to improve and develop the ways in which they express mathematical ideas.

The exercise sets contain exercises that are keyed to examples, as well as exercises that are not keyed to examples.

## Calculator Exercises

**Calculator exercises** are optional. They provide an opportunity for students to learn how a scientific or graphing calculator may be useful in solving various problems.





deducting the bonus. So

$$T = 0.40(100{,}000 - B).$$

a) Use the accompanying graph to estimate the values of $T$ and $B$ that satisfy both equations.
b) Solve the system algebraically to find the bonus and the amount of tax.

57. ***Textbook case.*** The accompanying graph shows the cost of producing textbooks and the revenue from the sale of those textbooks.
a) What is the cost of producing 10,000 textbooks?
b) What is the revenue when 10,000 textbooks are sold?
c) For what number of textbooks is the cost equal to the revenue?
d) The cost of producing zero textbooks is called the *fixed cost.* Find the fixed cost.



FIGURE FOR EXERCISE 57

58. ***Free market.*** The function $S = 5000 + 200x$ and $D = 9500 - 100x$ express the supply $S$ and the demand $D$, respectively, for a popular compact disk brand as a function of its price $x$ (in dollars).
a) Graph the functions on the same coordinate system.
b) What happens to the supply as the price increases?
c) What happens to the demand as the price increases?
d) The price at which supply and demand are equal is called the *equilibrium price.* What is the equilibrium price?

GETTING MORE INVOLVED

59. ***Discussion.*** Which of the following equations is not equivalent to $2x - 3y = 6$?
a) $3y - 2x = 6$ b) $y = \frac{2}{3}x - 2$
c) $x = \frac{3}{2}y + 3$ d) $2(x - 5) = 3y - 4$

60. ***Discussion.*** Which of the following equations is inconsistent with the equation $3x + 4y = 8$?
a) $y = \frac{3}{4}x + 2$ b) $6x + 8y = 16$
c) $y = -\frac{3}{4}x + 8$ d) $3x - 4y = 8$

GRAPHING CALCULATOR EXERCISES

61. Solve each system by graphing each pair of equations on a graphing calculator and using the trace feature or intersect feature to estimate the point of intersection. Find the coordinates of the intersection to the nearest tenth.
a) $y = 3.5x - 7.2$, $y = -2.3x + 9.1$ b) $2.3x - 4.1y = 3.3$, $3.4x + 9.2y = 1.3$

In this section
- The Addition Method
- Equations Involving Fractions or Decimals
- Applications

### 4.2 THE ADDITION METHOD

In Section 4.1 you used substitution to eliminate a variable in a system of equations. In this section we see another method for eliminating a variable in a system of equations.

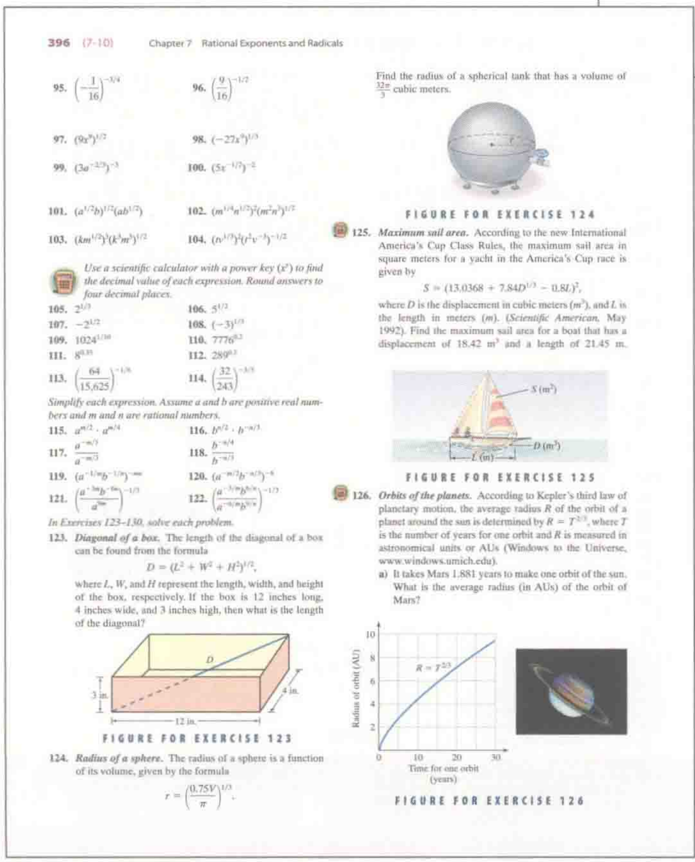
#### The Addition Method

In the **addition method** we eliminate a variable by adding the equations.

EXAMPLE 1 **An independent system solved by addition**

Solve the system by the addition method:

$$3x - 5y = -9$$
$$4x + 5y = 23$$





95. $\left(-\frac{1}{16}\right)^{-3/4}$ 96. $\left(\frac{9}{16}\right)^{-1/2}$
97. $(9x^9)^{1/2}$ 98. $(-27x^9)^{1/3}$
99. $(3a^{-2/3})^{-3}$ 100. $(5x^{-1/2})^{-2}$
101. $(a^{1/2}b)^{1/2}(ab^{1/2})$ 102. $(m^{1/4}n^{1/2})^2(m^2n^3)^{1/2}$
103. $(km^{1/2})^3(k^3m^5)^{1/2}$ 104. $(tv^{1/3})^2(t^2v^{-3})^{-1/2}$

*Use a scientific calculator with a power key ($x^y$) to find the decimal value of each expression. Round answers to four decimal places.*

105. $2^{1/3}$ 106. $5^{1/2}$
107. $-2^{1/2}$ 108. $(-3)^{1/3}$
109. $1024^{1/10}$ 110. $7776^{0.2}$
111. $8^{0.33}$ 112. $289^{0.5}$
113. $\left(\frac{64}{15{,}625}\right)^{-1/6}$ 114. $\left(\frac{32}{243}\right)^{-3/5}$

*Simplify each expression. Assume a and b are positive real numbers and m and n are rational numbers.*

115. $a^{m/2} \cdot a^{m/4}$ 116. $b^{n/2} \cdot b^{-n/3}$
117. $\frac{a^{-m/5}}{a^{-m/3}}$ 118. $\frac{b^{-n/4}}{b^{-n/3}}$
119. $(a^{-1/m}b^{-1/n})^{-mn}$ 120. $(a^{-m/2}b^{-n/3})^{-6}$
121. $\left(\frac{a^{-3m}b^{-6n}}{a^{9m}}\right)^{-1/3}$ 122. $\left(\frac{a^{-3/m}b^{6/n}}{a^{-6/m}b^{9/n}}\right)^{-1/3}$

*In Exercises 123–130, solve each problem.*

123. ***Diagonal of a box.*** The length of the diagonal of a box can be found from the formula
$$D = (L^2 + W^2 + H^2)^{1/2},$$
where $L$, $W$, and $H$ represent the length, width, and height of the box, respectively. If the box is 12 inches long, 4 inches wide, and 3 inches high, then what is the length of the diagonal?



FIGURE FOR EXERCISE 123

124. ***Radius of a sphere.*** The radius of a sphere is a function of its volume, given by the formula
$$r = \left(\frac{0.75V}{\pi}\right)^{1/3}.$$
Find the radius of a spherical tank that has a volume of $\frac{32\pi}{3}$ cubic meters.

FIGURE FOR EXERCISE 124

125. ***Maximum sail area.*** According to the new International America's Cup Class Rules, the maximum sail area in square meters for a yacht in the America's Cup race is given by
$$S = (13.0368 + 7.84D^{1/3} - 0.8L)^2,$$
where $D$ is the displacement in cubic meters ($m^3$), and $L$ is the length in meters ($m$). (*Scientific American,* May 1992). Find the maximum sail area for a boat that has a displacement of 18.42 $m^3$ and a length of 21.45 m.

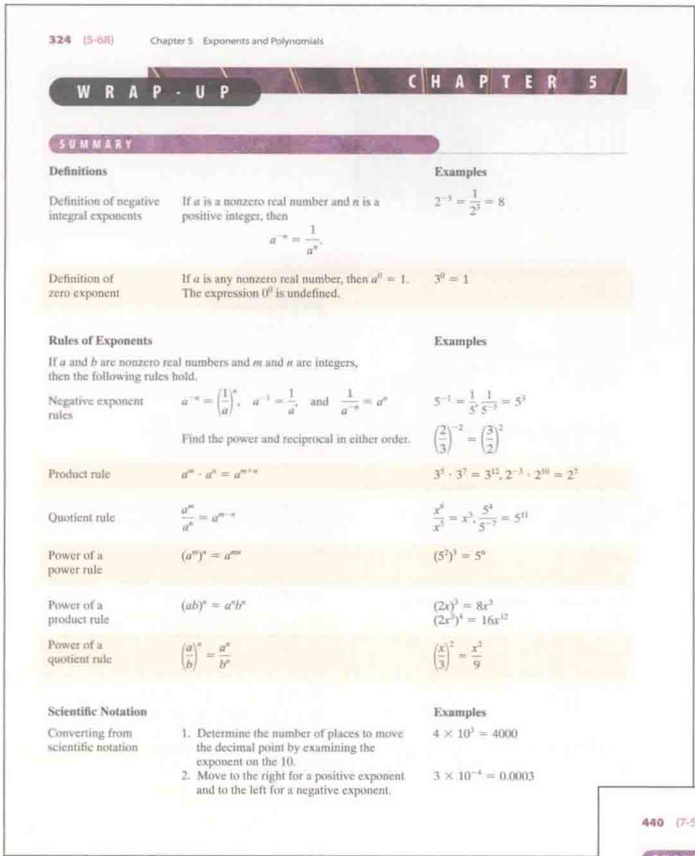


FIGURE FOR EXERCISE 125

126. ***Orbits of the planets.*** According to Kepler's third law of planetary motion, the average radius $R$ of the orbit of a planet around the sun is determined by $R = T^{2/3}$, where $T$ is the number of years for one orbit and $R$ is measured in astronomical units or AUs (Windows to the Universe, www.windows.umich.edu).
a) It takes Mars 1.881 years to make one orbit of the sun. What is the average radius (in AUs) of the orbit of Mars?



FIGURE FOR EXERCISE 126





WRAP-UP CHAPTER 5

SUMMARY

| Definitions | | Examples |
|---|---|---|
| Definition of negative integral exponents | If $a$ is a nonzero real number and $n$ is a positive integer, then $a^{-n} = \frac{1}{a^n}$. | $2^{-3} = \frac{1}{2^3} = 8$ |
| Definition of zero exponent | If $a$ is any nonzero real number, then $a^0 = 1$. The expression $0^0$ is undefined. | $3^0 = 1$ |

Rules of Exponents

If $a$ and $b$ are nonzero real numbers and $m$ and $n$ are integers, then the following rules hold.

| | | Examples |
|---|---|---|
| Negative exponent rules | $a^{-n} = \left(\frac{1}{a}\right)^n$, $a^{-1} = \frac{1}{a}$, and $\frac{1}{a^{-n}} = a^n$ | $5^{-1} = \frac{1}{5}, \frac{1}{5^{-3}} = 5^3$ |
| | Find the power and reciprocal in either order. | $\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2$ |
| Product rule | $a^m \cdot a^n = a^{m+n}$ | $3^5 \cdot 3^7 = 3^{12}, 2^{-3} \cdot 2^{10} = 2^7$ |
| Quotient rule | $\frac{a^m}{a^n} = a^{m-n}$ | $\frac{x^8}{x^5} = x^3, \frac{5^4}{5^{-7}} = 5^{11}$ |
| Power of a power rule | $(a^m)^n = a^{mn}$ | $(5^2)^3 = 5^6$ |
| Power of a product rule | $(ab)^n = a^n b^n$ | $(2x)^3 = 8x^3$<br>$(2x^3)^4 = 16x^{12}$ |
| Power of a quotient rule | $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ | $\left(\frac{x}{3}\right)^2 = \frac{x^2}{9}$ |

| Scientific Notation | | Examples |
|---|---|---|
| Converting from scientific notation | 1. Determine the number of places to move the decimal point by examining the exponent on the 10. | $4 \times 10^3 = 4000$ |
| | 2. Move to the right for a positive exponent and to the left for a negative exponent. | $3 \times 10^{-4} = 0.0003$ |

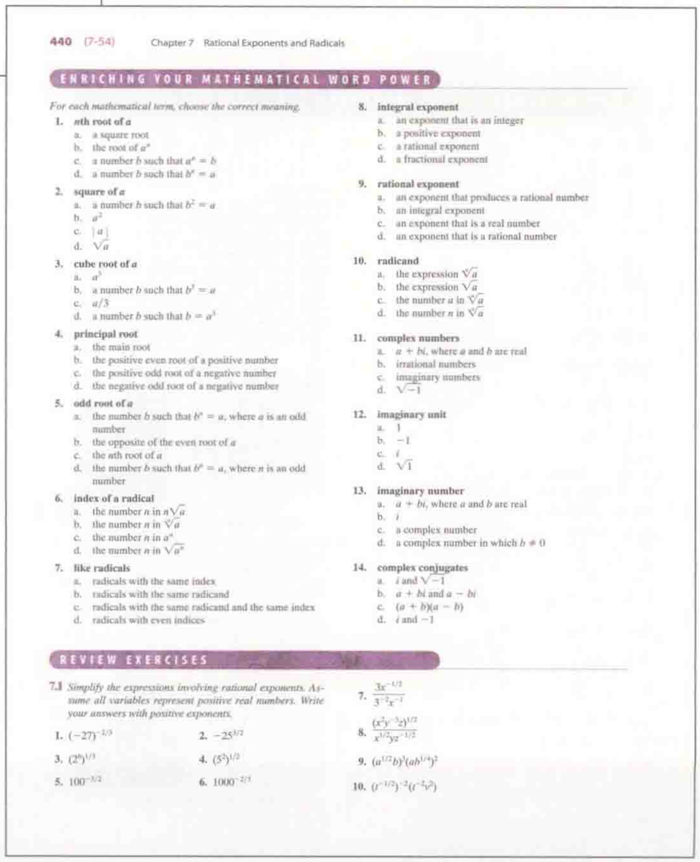
## Wrap-up

Every chapter ends with a four-part **Wrap-up:**

The **Summary** lists important concepts along with brief illustrative examples.

**Enriching Your Mathematical Word Power** enables students to review terms introduced in each chapter. It is intended to help reinforce students' command of mathematical terminology.

**Review Exercises** contain problems that are keyed to each section of the chapter as well as *miscellaneous exercises* that are not keyed to the sections. These *exercises* are designed to test the student's ability to synthesize various concepts.





ENRICHING YOUR MATHEMATICAL WORD POWER

*For each mathematical term, choose the correct meaning.*

1. **$n$th root of $a$**
   a. a square root
   b. the root of $a^n$
   c. a number $b$ such that $a^n = b$
   d. a number $b$ such that $b^n = a$
2. **square of $a$**
   a. a number $b$ such that $b^2 = a$
   b. $a^2$
   c. $|a|$
   d. $\sqrt{a}$
3. **cube root of $a$**
   a. $a^3$
   b. a number $b$ such that $b^3 = a$
   c. $a/3$
   d. a number $b$ such that $b = a^3$
4. **principal root**
   a. the main root
   b. the positive even root of a positive number
   c. the positive odd root of a negative number
   d. the negative odd root of a negative number
5. **odd root of $a$**
   a. the number $b$ such that $b^n = a$, where $a$ is an odd number
   b. the opposite of the even root of $a$
   c. the $n$th root of $a$
   d. the number $b$ such that $b^n = a$, where $n$ is an odd number
6. **index of a radical**
   a. the number $n$ in $n\sqrt{a}$
   b. the number $n$ in $\sqrt[n]{a}$
   c. the number $n$ in $a^n$
   d. the number $n$ in $\sqrt{a^n}$
7. **like radicals**
   a. radicals with the same index
   b. radicals with the same radicand
   c. radicals with the same radicand and the same index
   d. radicals with even indices
8. **integral exponent**
   a. an exponent that is an integer
   b. a positive exponent
   c. a rational exponent
   d. a fractional exponent
9. **rational exponent**
   a. an exponent that produces a rational number
   b. an integral exponent
   c. an exponent that is a real number
   d. an exponent that is a rational number
10. **radicand**
    a. the expression $\sqrt[n]{a}$
    b. the expression $\sqrt{a}$
    c. the number $a$ in $\sqrt[n]{a}$
    d. the number $n$ in $\sqrt[n]{a}$
11. **complex numbers**
    a. $a + bi$, where $a$ and $b$ are real
    b. irrational numbers
    c. imaginary numbers
    d. $\sqrt{-1}$
12. **imaginary unit**
    a. 1
    b. $-1$
    c. $i$
    d. $\sqrt{1}$
13. **imaginary number**
    a. $a + bi$, where $a$ and $b$ are real
    b. $i$
    c. a complex number
    d. a complex number in which $b \neq 0$
14. **complex conjugates**
    a. $i$ and $\sqrt{-1}$
    b. $a + bi$ and $a - bi$
    c. $(a + b)(a - b)$
    d. $i$ and $-1$

REVIEW EXERCISES

7.1 *Simplify the expressions involving rational exponents. Assume all variables represent positive real numbers. Write your answers with positive exponents.*

1. $(-27)^{-2/3}$
2. $-25^{3/2}$
3. $(2^6)^{1/3}$
4. $(5^2)^{1/2}$
5. $100^{-3/2}$
6. $1000^{-2/3}$
7. $\frac{3x^{-1/2}}{3^{-2}x^{-1}}$
8. $\frac{(x^2y^{-3}z)^{1/2}}{x^{1/2}yz^{-1/2}}$
9. $(a^{1/2}b)^3(ab^{1/4})^2$
10. $(t^{-1/2})^{-2}(t^{-2}v^2)$