MECHANICS OF STRUCTURAL SYSTEMS

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NONLINEAR MECHANICS OF STRUCTURES

Introduction

The aim of this book is to provide a unified presentation of modern mechanics of structures in a form which is suitable for graduate students as well as for engineers and scientists working in the field of applied mechanics. Traditionally, students at technical universities have been taught subjects such as continuum mechanics, elasticity, plates and shells, frames or finite element techniques in an entirely separate manner. The authors' teaching experience clearly suggests that this situation frequently tends to create in students' minds an incomplete and inconsistent picture of the contemporary structural mechanics. Thus, it is very common that the fundamental laws of physics appear to students hardly related to simplified equations of different "technical" theories of structures, numerical solution techniques are studied independently of the essence of mechanical models they describe, and so on.

The book is intended to combine in a reasonably connected and unified manner all these problems starting with the very fundamental postulates of nonlinear continuum mechanics via different structural models of "engineering" accuracy to numerical solution methods which can effectively be used for solving boundary-value problems of technological importance.

The authors have tried to restrict the mathematical background required to that which is normally familiar to a mathematically minded engineering graduate.

Of course, no single book can fully embrace the broad field of nonlinear structural mechanics. Holding the book to reasonable length did not permit to include many interesting, and certainly important issues. In making our choice we have primarily aimed at logical presentation of the selected material and its anticipated usefulness in the long run. In particular, it is emphasized that the book is not intended to provide the reader with a great number of sophisticated nonlinear theories applicable to the refined analysis of specific structural configurations. Quite to the contrary, after giving a thorough continuum mechanics background we consistently develop merely relatively simple, or conventional, theories and apply them to solve numerous practical problems of engineering significance. It is our strong belief, however, that the general framework laid down in the present book will turn out to be a proper tool to develop more sophisticated theories in the near future.

A brief outline of the main portions of the book is as follows. The first part devoted to theoretical foundations of nonlinear mechanics of structures starts with the short presentation of the fundamental notions and laws of nonlinear continuum mechanics. We introduce the notation and get the reader acquainted with the description of motion of the material continuum, the states of strain and stress and finally with the conservation laws. The temperature effects are included into the formulation.

In Chapter 2 we outline the theory of constitutive equations and thoroughly discuss loadings and boundary constraints typical of structural configurations. Specific constitutive laws are given for materials known as linear elastic, visco-elastic and thermo-elastic-plastic.

Chapter 3 is of fundamental significance to the approach consequently followed in the book since it deals with the key concept of internal constraints imposed on the kinematics and/or state of stress of the continuum. By specifying the constraints we are able to systematically derive classical and non-classical equations governing different "engineering" theories which describe the nonlinear behaviour of beam-, plate- and shell-type configurations. The considerations of this chapter are based on a number of variational principles. Emphasis is placed on demonstrating the power of the principles in deriving the governing equations in a systematic way. Next, the method of constraints is used to introduce a discrete description of structural configurations. By this we mean setting up approximate models of structures described in terms of algebraic rather than differential equations. An interesting feature of the approach to discretization as presented in this book is that it is again based upon the internal constraint concept thus making the discussion fully compatible with that given previously in Sections 3.2, 3.3 of this chapter. Apart from some attractive error estimates offered by the way we look at these problems this portion of the book (Secs. 3.4, 3.5) essentially discusses the foundations and applications of the well-known and powerful finite element method in its displacement and mixed-type versions. Discretization with respect to space and time variables is discussed and final sets of nonlinear algebraic equations are derived.

Some useful extensions and specifications of the general theory proposed in Chapter 3 are dealt with in the next Chapter 4. We consider in it such problems as the domain discretization, assembly procedures to be used in summing up the element matrices and vectors, specific forms of typical element matrices as well as an explicit form of equations describing thermo-elastic-plastic materials. The theoretical background for the so-called shake-down analysis of structures is also presented to be used later while discussing specific boundary-value solutions to inelastic problems under variable loads.

The solution methods for the discretized problems are described in Chapter 5. Two basic sections in this chapter deal with numerical algorithms for static and dynamic nonlinear problems, respectively. The algorithms are based on repetitive solutions of sets of linear algebraic equations describing the instantaneous equilibrium of the discretized body and are additionally endowed with some Newton–Raphson iterative corrections to improve the accuracy of the results. Part I of the book is completed by a short discussion of software concepts which are typically employed while developing computer programs.

The contents of Part I of the book is made more specific in Part II, in which we consider different discretized models for beams, plates and shells. A number of effective finite elements is described in detail and test calculations are given to illustrate their performance in modelling different aspects of nonlinear structural behaviour. Trusses, frames, grids and shell-like structures made of regularly distributed beams are dealt with in Chapter 6. Chapter 7 is devoted to the analysis of thin plates loaded in-plane whereas Chapter 8 concentrates on shell-type problems. A short discussion of heat conduction and thermal stress problems given in Chapter 9 completes the book.

Problems for independent studies are provided at the end of most chapters so that students can check their understanding of the subjects. At places, problems serve also the purpose of extending the scope of the book into areas which could not have been covered for the sake of compactness of the presentation.

The course, as outlined here, is designed for a two-semester sequence at the graduate level. On the whole, it is believed that the treatment used in this book is in harmony with current trends toward a more fundamental approach in engineering education.

Conventions and notations

Throughout the book we shall use such mathematical objects as scalars, vectors, scalar and vector functions, vector and tensor components, coordinates of points, etc. We shall also perform certain operations on these objects such as scalar or vector products. We assume that the reader knows the elementary vector and tensor calculus both in the absolute and the component form. As far as possible we shall use the following conventions:

1. Sub- and superscripts i, j, k, l, m, n run over the sequence 1, 2, 3 and are related to a cartesian coordinate system (so-called spatial coordinate system) in the physical space identified with the Euclidean 3-space $\boldsymbol{R}^3$ of spatial points. Sub- and superscripts $\alpha, \beta, \gamma, \delta$ run also over the sequence 1, 2, 3 (unless a different range is clearly stated) and are related to a coordinate system introduced exclusively in a region Ω occupied by a body in a certain reference configuration (so-called material coordinate system). Ranges of any other indices appearing in the text will always be explained prior to their use.

2. Summation convention holds with respect to the indices $\alpha, \beta, \ldots$ and $i, j, \ldots$, provided they are repeated twice in a given expression and situated in it on the upper and lower level, respectively. Ricci symbol and Kronecker delta are denoted by ε^{ijk}, δ^{ij} respectively (position of i, j, k is immaterial).

3. Spatial points are denoted by $\mathbf{z}, \mathbf{z} \in \boldsymbol{R}^3$, and their (spatial) coordinates by z^k; points of Ω are denoted by $\mathbf{X}, \mathbf{X} \in \Omega$, and their (material) coordinates by X^α (instead of k or α we may be using any other pertinent Latin or Greek superscript, respectively). The symbols τ and t stand for time instants with $\acute{t}$ used to indicate a specific rather than an arbitrary time.

4. Scalars are represented mostly by small light-faced Greek italics: ϱ, α, π. Scalar-valued functions are denoted by $\varrho(\cdot), \alpha(\cdot), \pi(\cdot)$ etc.; domains of these functions are specified in the text.

5. Vectors and vector-valued functions are represented by small bold face Latin or Greek Roman letters; examples: $\mathbf{f}$, $\boldsymbol{\chi}$, $\hat{\mathbf{f}}(\cdot)$, $\hat{\boldsymbol{\chi}}(\cdot)$, respectively; their components are represented by small face Latin or Greek italics: f^k, χ^k, $\hat{f}^k(\cdot)$, $\hat{\chi}^A(\cdot)$.

6. Second order tensors and tensor-valued functions are represented by capital bold face Latin letters: $\mathbf{T}$, $\mathbf{S}$ and $\hat{\mathbf{T}}(\cdot)$, $\hat{\mathbf{S}}(\cdot)$; their components are denoted by T^{ij}, $S^{\alpha\beta}$ and $\hat{T}^{ij}(\cdot)$, $\hat{S}^{\alpha\beta}(\cdot)$, respectively.

7. Higher order tensors and tensor functions are represented by German capitals: $\mathfrak{C}$, $\mathfrak{H}$ and $\hat{\mathfrak{C}}(\cdot)$, $\hat{\mathfrak{H}}(\cdot)$, respectively; their components are indicated by light-faced Roman letters: C^{ijkl}, $\mathrm{H}_{\alpha\beta\gamma\delta}(\cdot)$ etc. Symbols $C[\cdot]$, $H[\cdot]$ stand for linear mappings of the form $\mathrm{C}^{ijkl}D_{kl}$, $\mathrm{H}_{\alpha\beta\gamma\delta}S^{\gamma\delta}$, respectively.

8. Comma denotes partial differentiation with respect to the spatial coordinates (example: $r_{,i} \equiv \frac{\partial r}{\partial x^i}$) or to the material coordinates (example: $T_{,\alpha} \equiv \frac{\partial T}{\partial X^\alpha}$). Dot denotes substantial differentiation with respect to the time coordinate (hence $\dot{g} = \frac{\partial g}{\partial \tau}$ provided $g = g(\mathbf{X}, \tau)$ and $\mathbf{X} = (X^\alpha)$ are the material coordinates). The gradient with respect to the material coordinates $\mathbf{X} = (X^\alpha)$ is denoted by ∇ (example: $\nabla\pi = (\pi_{,1}\pi_{,2}\pi_{,3})$, $\pi_{,\alpha} \equiv \frac{\partial \pi}{\partial X^\alpha}$, where $\pi = \pi(\mathbf{X}, \tau)$).

9. The component forms of products $\mathbf{a}\cdot\mathbf{b}$, $\mathbf{a}\times\mathbf{b}$, $\mathbf{Tn}$, $\mathbf{SE}$ are $a^k b_k$, $\varepsilon^{kl}{}_m a_l b^m$, $T^{kl}n_l$, $S^{\alpha\beta}E_{\beta\gamma}$, respectively.

10. Symbols $\operatorname{tr}\mathbf{A}$, $\det\mathbf{A}$ stand for trace and determinant of an arbitrary second order tensor $\mathbf{A}$. Symbols $\mathbf{A}^T$, $\mathbf{A}^{-1}$ stand for the transpose and the inverse of $\mathbf{A}$, respectively, $\mathbf{1}$ is a unit tensor.

11. Sets of points or vectors are represented by capital light face Greek letters (examples: Ω, Φ); sets of mappings are denoted by a light face script (examples: $\mathscr{P}$, $\mathscr{T}$).

Contents

PART I

THEORETICAL FOUNDATIONS

1

General notions and principles

Purpose of the chapter

The purpose of the chapter is to introduce and explain all general concepts and principles of thermo-mechanics which are common to all problems we are going to study throughout the book. We use the term "general" in order to underline the fact that these notions and rules are independent of such characteristic features of specific structural mechanics problems as the shape of the structure, the form of external agencies acting on it, materials the structure is made of as well as constraints imposed on possible deformations of the structure. The notions and principles developed in this chapter lead to a system of resulting relations which constitute the physical basis for all the problems of structural mechanics to be subsequently considered.

1.1 Introductory concepts

1.1.1 Space and time

We shall assume that a set of all events constitutes the Galilean space-time in which there is fixed once and for all an inertial coordinate system. Such a system assigns to every event a fourtouple (z_1, z_2, z_3, τ) of real numbers, $(z_1, z_2, z_3, \tau) \in \boldsymbol{R}^4$. Any triplet $\mathbf{z} = (z_1, z_2, z_3)$, $\mathbf{z} \in \boldsymbol{R}$, will be called a *spatial point*, reals z_i, $i = 1, 2, 3$, will be called *spatial coordinates* and real τ will be referred to as a *time coordinate* or a *time instant*. We shall always tacitly assume that a spatial distance $\|\mathbf{z}-\bar{\mathbf{z}}\| \equiv \sqrt{(z_i-\bar{z}_i)(z_j-\bar{z}_j)\delta^{ij}}$ between any two spatial points $\mathbf{z}, \bar{\mathbf{z}}$ is determined in the fixed once for all unit length dimension $[L]$. Similarly, a time distance $|\tau-\bar{\tau}|$ between any two time instants $\tau, \bar{\tau}$ is assumed to be expressed in the fixed unit time dimension $[T]$. As a rule we shall use the International System of unit measures, setting $[L] = [\mathrm{m}]$, $[T] = [\mathrm{s}]$.

1.1.2 Motion

In order to describe a motion of a certain material (real) element of a solid or a structure under consideration, we introduce: