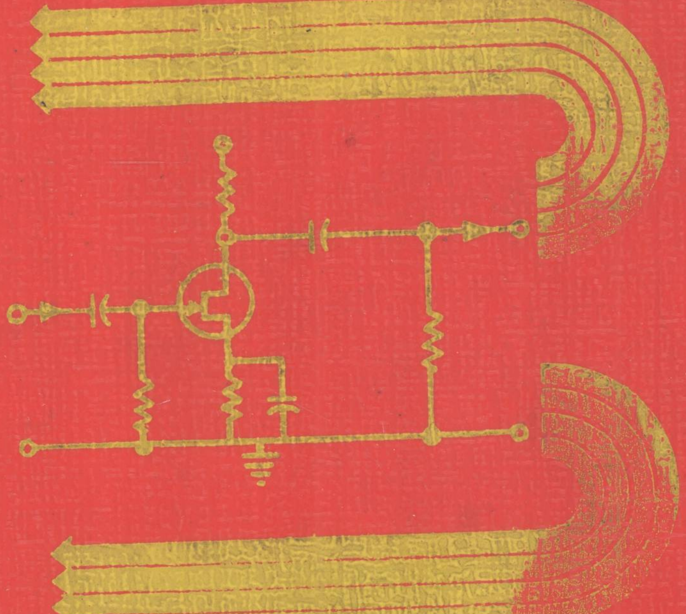
Control System Analysis and Designs

K.K. Aggarwal



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CONTROL SYSTEMS ANALYSIS AND DESIGNS

[*For A.M.I.E Part I (New Scheme) and Engineering Degree Students*]

Dr. K.K. AGGARWAL
Professor and Head
Department of Electronics and Communication Engineering,
Regional Engineering College, Kurukshetra

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PREFACE

An extraordinary fact about the human race has been that 'Man' was born as a 'Man-inventor and discoverer'. The history of the inventions done by man is no way less old than the history of man itself. Man has the ability to control his environments and their effect upon his life. Modern societies have developed efficient methods for heating and cooling to provide year-round comfort.

In the process of development, man produced machine and man himself was only necessary to control in detail the various operations that are required to complete any process. Slowly, the machines and the processes become more complicated. Also quick and accurate results were desired in most of the processes. Man became unable to perfectly control his own machine. It was in this context that it was tried to replace the human controller by same form of automatic controller which would precisely and speedily do what the man wants. The use of analog and digital computers has revolutionised the automatic control systems.

The present book is intended to provide a unified treatment of the classical as well as modern concepts of Control System Engineering. This book will serve as a textbook in the subject for the Undergraduate students of almost all Universities and Engineering Colleges. The subject matter can be fully covered in two Semester Courses which are now being offered at most of the leading Universities. However, the distribution of the contents of the various chapters is such that the book can be used for one Semester course also and relevant chapters can be chosen according to the syllabus of the particular University.

The book will also be helpful to the practising engineers who may like to up-date their knowledge in this most important area of the day. This book will also provide the essential beckground for most of the postgraduate courses. The book contains about 250 solved examples and many more unsolved problems at the end of each chapter, with answers to most of the problems, to give the students a better confidence in the subject.

The first chapter summarises the important results in the Laplace transform theory which are of direct relevance to the Control System Engineering. The next chapter gives a brief introduction and historical background of the subject. Chapters 3 and 4 discuss in detail the modelling of systems based on classical concepts such as transfer function, block diagrams and signal flow graphs.

Chapters 5 to 8 give a detailed treatment of most of the components used in the Control Systems. These components have not been described in most of the existing books and in my long experience of teaching the subject I always felt the necessity of introducing the students to the components actually used in such systems. This will enable the students to grasp the subject better.

Chapters 9 and 10 discuss the mathematical modelling of the systems and present an elaborate treatment on the Specifications of Systems and system stability. Chapters 11 to 13 give a detailed treatment of the more important classical methods of analysis and dasign of Control Systems. The methods are discussed in detail with several solved examples. The use of the methods in the design of systems is illustrated in each chapter. Chapter 14 presents some other important techniques which can be used for the design of Control Systems.

Chapter 15 presents, in a reasonable length, the modelling of Control Systems based on state variables and the analysis and design of systems using the important concepts of state space approach.

The last five chapters give brief discussions on Non-linear Control Systems, Discrete Time Systems, Optimal Control Theory, Stochastic Control Systems and Self-Adaptive Control Systems. The study of these chapters is recommended depending upon the requirement of a particular course and also to prepare the students for Postgraduate studies in the subject.

Although, every care has been taken to avoid the mistakes in the book yet the possibility of some mistakes still being present can not be ruled out. The author and the publishers will be grateful for any comments or suggestions which may help to improve the book in its following editions.

The material contained in this book has been mainly organised based on the long experience of the author. In this duration, many students, teachers and colleagues have influenced the author's approach towards the subject and it is impossible to individually acknowledge all concerned. However, I shall mention the help of Dr. Krishna Gopal and Shri I.M.N. Soi of my Institution in this regard. It is equally impossible to reference adequately all sources of information. A list of references is, however, given at the end of the book, which may also be found useful for further study in the subject.

I also wish to thank the authorities of the Regional Engineering College, Kurukshetra, for providing an academic environment necessary for such an important task. Finally, I wish to express my thanks to my family for their continued interest and encouragement throughout this long period of writing the book.

Kurukshetra K.K. AGGARWAL

April 1981

CONTENTS

1

Laplace Transforms

The Laplace transforms are so widely used in the study of control systems that this chapter is devoted to the discussion and review of Laplace transforms. These transforms have become popular because of the following advantages :

(*i*) This transformation transforms the transcendental and exponential functions to simple algebraic functions.

(*ii*) This transformtion transforms the operations of differentiation and integration to multiplication and division respectively.

(*iii*) In the solution of differential equations, arbitrary constants do not occur.

(*iv*) We can effectively make use of step and impulse response which is very relevant in control systems.

1·1. Definition

The Laplace transform of a function $f(t)$ is denoted by $Lf(t)$ and is a function of s normally written as

$$F(s)=L\,f(t) \qquad \ldots(1)$$

The correspondence between $f(t)$ and $F(s)$ is unique and is established by the following relation :

$$F(s)=\int_0^\infty f(t)\, e^{-st}\, dt \qquad \ldots(2)$$

Example 1·1. *Find the Laplace transforms of :*

(*a*) *a costant, K*

(*b*) $f(t)=t^2$

(*c*) $f(t)=e^{-\alpha t}$

Solution. (*a*) $f(t)=K$

$$F(s)=\int_0^\infty K\, e^{-st}\, dt$$

$$=\left|\frac{K}{-s}\, e^{-st}\right|_0^\infty=\frac{K}{s}$$

(*b*) $f(t)=t^2$

$$F(s)=\int_0^\infty t^2\, e^{-st}\, dt$$

By repeated application of the rules of integration by parts,

$$F(s)=t^2\left|\frac{e^{-st}}{-s}\right|_0^\infty -(2t)\left|\frac{e^{-st}}{s^2}\right|_0^\infty +(2)\left|\frac{e^{-st}}{-s^3}\right|_0^\infty$$

Making use of l'Hospital's rule,

$$F(s)=\frac{2}{s^3}$$

As a matter of fact, it can be shown that

$$L(t^n)=\frac{n!}{s^{n+1}}$$

(c) $f(t)=e^{-\alpha t}$

$$F(s)=\int_0^\infty e^{-\alpha t}\, e^{-st}\, dt=\left.\frac{e^{-(s+\alpha)}}{-(s+\alpha)}\right|_0^\infty=\frac{1}{s+\alpha}$$

Example 1·2. *Find the Laplace transform for*

$$f(t)=\cos \omega t.$$

Solution. Evaluation of Laplace transform, by direct integration, in this case is quite tedious and an easy way to find this transform is to recognize :

$$\cos \omega t=Re[e^{j\omega t}]$$

$$\therefore \quad L[\cos \omega t]=Re[L(e^{j\omega t})]$$

Now, $L(e^{j\omega t})=\dfrac{1}{s-j\omega}$ [example 1·1(c)]

Hence, $L[\cos \omega t]=Re\left[\dfrac{1}{s-j\omega}\right]=\dfrac{s}{s^2+\omega^2}$.

It is not always convenient to derive Laplace transforms by definition. Mostly we make use of same of the properties of Laplace transforms. These properties are listed below.

1·2. Properties of Laplace Transforms

1·2·1. Laplace transform of the sum of two functions is equal the sum of the Laplace transforms of the two functions

$$L[f_1(t)\pm f_2(t)]=Lf_1(t)\pm Lf_2(t) \quad \ldots(3)$$

1·2·2. Laplace transform of a function multiplied by any constant is constant times the Laplace transform of the function.

$$L[cf(t)]=cL\,f(t) \quad \ldots(4)$$

1·2·3. Laplace transform of the derivative of a function is

$$L\left[\frac{d}{dt}\right]f(t)=s\,F(s)-f'(0) \quad \ldots(5)$$