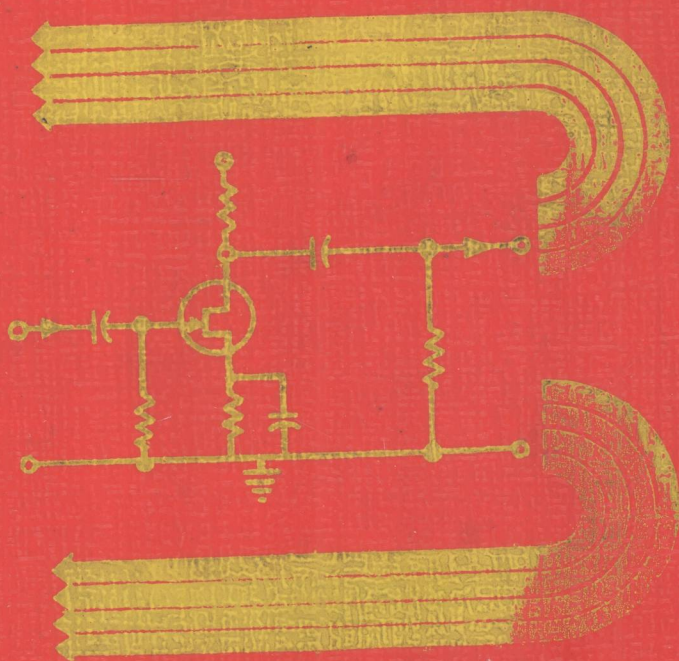


# Control System Analysis and Designs

**K.K. Aggarwal**



KHANNA PUBLISHERS DELHI-6

8263146

# CONTROL SYSTEMS ANALYSIS AND DESIGNS

[For A.M.I.E Part I (New Scheme) and Engineering Degree Students]



**Dr. K.K. AGGARWAL**

Professor and Head  
Department of Electronics and Communication Engineering,  
Regional Engineering College, Kurukshetra



E8263146

1981

**KHANNA PUBLISHERS**

2-B, NATH MARKET, NAI SARAK, DELHI-1100006

PHONE : 2 6 2 3 8 0

*Published by*  
Ramesh Chander Khanna  
KHANNA PUBLISHERS  
2-B, Nath Market, Nai Sarak  
Delhi-110006

© RESERVED

*This book or part thereof cannot be translated or reproduced in any form except for review or criticism without the written permission of Author and the Publishers*

*First Edition 1981*

PRICE : **Rs.** 27-50

*Printed through :*  
COMPOSERS  
at Rastravani Printers  
New Delhi-110064.

## PREFACE

An extraordinary fact about the human race has been that 'Man' was born as a 'Man-inventor and discoverer'. The history of the inventions done by man is no way less old than the history of man itself. Man has the ability to control his environments and their effect upon his life. Modern societies have developed efficient methods for heating and cooling to provide year-round comfort.

In the process of development, man produced machine and man himself was only necessary to control in detail the various operations that are required to complete any process. Slowly, the machines and the processes become more complicated. Also quick and accurate results were desired in most of the processes. Man became unable to perfectly control his own machine. It was in this context that it was tried to replace the human controller by same form of automatic controller which would precisely and speedily do what the man wants. The use of analog and digital computers has revolutionised the automatic control systems.

The present book is intended to provide a unified treatment of the classical as well as modern concepts of Control System Engineering. This book will serve as a textbook in the subject for the Undergraduate students of almost all Universities and Engineering Colleges. The subject matter can be fully covered in two Semester Courses which are now being offered at most of the leading Universities. However, the distribution of the contents of the various chapters is such that the book can be used for one Semester course also and relevant chapters can be chosen according to the syllabus of the particular University.

The book will also be helpful to the practising engineers who may like to up-date their knowledge in this most important area of the day. This book will also provide the essential background for most of the postgraduate courses. The book contains about 250 solved examples and many more unsolved problems at the end of each chapter, with answers to most of the problems, to give the students a better confidence in the subject.

The first chapter summarises the important results in the Laplace transform theory which are of direct relevance to the Control System Engineering. The next chapter gives a brief introduction and historical background of the subject. Chapters 3 and 4 discuss in detail the modelling of systems based on classical concepts such as transfer function, block diagrams and signal flow graphs.

Chapters 5 to 8 give a detailed treatment of most of the components used in the Control Systems. These components have not been described in most of the existing books and in my long experience of teaching the subject I always felt the necessity of intro-

ducing the students to the components actually used in such systems. This will enable the students to grasp the subject better.

Chapters 9 and 10 discuss the mathematical modelling of the systems and present an elaborate treatment on the Specifications of Systems and system stability. Chapters 11 to 13 give a detailed treatment of the more important classical methods of analysis and design of Control Systems. The methods are discussed in detail with several solved examples. The use of the methods in the design of systems is illustrated in each chapter. Chapter 14 presents some other important techniques which can be used for the design of Control Systems.

Chapter 15 presents, in a reasonable length, the modelling of Control Systems based on state variables and the analysis and design of systems using the important concepts of state space approach.

The last five chapters give brief discussions on Non-linear Control Systems, Discrete Time Systems, Optimal Control Theory, Stochastic Control Systems and Self-Adaptive Control Systems. The study of these chapters is recommended depending upon the requirement of a particular course and also to prepare the students for Postgraduate studies in the subject.

Although, every care has been taken to avoid the mistakes in the book yet the possibility of some mistakes still being present can not be ruled out. The author and the publishers will be grateful for any comments or suggestions which may help to improve the book in its following editions.

The material contained in this book has been mainly organised based on the long experience of the author. In this duration, many students, teachers and colleagues have influenced the author's approach towards the subject and it is impossible to individually acknowledge all concerned. However, I shall mention the help of Dr. Krishna Gopal and Shri I.M.N. Soi of my Institution in this regard. It is equally impossible to reference adequately all sources of information. A list of references is, however, given at the end of the book, which may also be found useful for further study in the subject.

I also wish to thank the authorities of the Regional Engineering College, Kurukshetra, for providing an academic environment necessary for such an important task. Finally, I wish to express my thanks to my family for their continued interest and encouragement throughout this long period of writing the book.

Kurukshetra

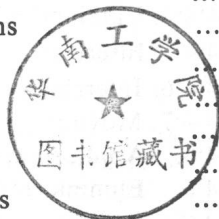
K.K. AGGARWAL

April 1981



## CONTENTS

Chap.		Page
<b>1. Laplace Transforms</b>	...	<b>1</b>
1'1. Definition	...	1
1'2. Properties of Laplace Transforms	...	2
1'2'1. Laplace Transform of the Sum	...	2
1'2'2. Laplace Transform of a Function	...	2
1'2'3. Laplace Transform of the Derivative	...	2
1'2'4. Laplace Transform of the Integral	...	3
1'2'5. Shifting Theorems	...	3
1'2'6. Scaling Theorem	...	3
1'2'7. Laplace Transform of a Function Multiplied by $t$	...	3
1'2'8. Laplace Transform of a Function Divided by $t$	...	3
1'2'9. Initial Value Theorem	...	3
1'2'10. Final Value Theorem	...	3
1'3. Laplace Transform for Waveforms	...	11
1'3'1. A Periodic Signals	...	11
1'3'2. Periodic Signals	...	13
1'4. Inverse Laplace Transforms	...	13
1'4'1. Convolution Theorem	...	15
1'5. Solution of Differential Equations	...	17
1'6. Application	...	18
<b>2. Introduction to Control Systems</b>	...	<b>26</b>
2'1. Function of a Control System	...	26
2'2. Open Loop and Closed Loop Systems	...	27
2'3. Servo Mechanisms and Regulators	...	29
2'4. Applications of Control Systems	...	33
2'4'1. Hero's Device for Opening Temple Doors	...	33
2'4'2. Automatic Machine Tool Control	...	34
2'4'3. Xylene Temperature Controlled System	...	35
2'4'4. Anti-aircraft Radar Tracking System	...	35
2'4'5. Application to Non-Engineering Fields	...	36
2'5. Elements of a Control System	...	36
<b>3. Transfer Function</b>	...	<b>40</b>
3'1. Definition	...	40



<i>Chap.</i>		<i>Page</i>
3.2.	Impulse Response of a Linear Systems	41
3.3.	Poles and Zeros	42
3.4.	Response to Excitations	44
3.5.	Relationship to Frequency Response	48
3.6.	Transfer Function of Electrical Networks	49
3.7.	Electronic Network	53
3.8.	Mechanical Systems	55
3.9.	Analogue Systems	58
3.10.	Electro-Mechanical Systems	64
3.11.	Transfer Function Matrices	66
<b>4</b>	<b>Block Diagrams and Signal Flow Graphs</b>	<b>73</b>
4.1.	Block Diagrams	73
4.2.	Mathematical Block Diagram	74
4.3.	Multiplicity of Block Diagram	76
4.4.	Reduction of Block Diagrams	77
4.4.1.	Combining Elements in Cascade	77
4.4.2.	Moving a Summing Point	78
4.4.3.	Eliminating a Feedback Loop	80
4.4.4.	Eliminating a Forward Path	80
4.4.5.	Insertion of a Feedback Loop	81
4.4.6.	Interchange of Take-off Points	81
4.4.7.	Moving a Take-off Point	81
4.5.	Application of Block Diagrams	87
4.6.	Eliminations of Block Diagrams	90
4.7.	Fundamentals of Signal Flow Graphs	91
4.8.	Signal Flow Graph Algebra	93
4.9.	Application of Signal Flow Graph to Network	96
4.10.	Mason's Gain Formula	98
4.11.	Application to Block Diagrams	101
4.12.	Application to Control Systems	102
<b>5.</b>	<b>Error Deductors and Transducers</b>	<b>109</b>
5.1.	Introduction	109
5.2.	Potentiometers	110
5.3.	Characteristics of Potentiometers	112
5.3.1.	Linearity	112
5.3.2.	Resolution	113
5.3.3.	Noise	114
5.3.4.	Inductance in Effects	115

<i>Chap.</i>		<i>Page</i>
5.3.5.	Mechanical Considerations	115
5.4.	Limitations of Potentiometers	115
5.5.	Differential Transformer	117
5.5.1.	Linear Variable Differential Transformer	118
5.6.	Synchros	119
5.7.	Gyroscope	121
5.8.	Digital Transducers	124
<b>6.</b>	<b>Servo Amplifiers</b>	<b>126</b>
6.1.	Special Features of Servo Amplifiers	126
6.2.	Electronic Amplifiers	127
6.3.	D.C. Amplifiers	131
6.3.1	The Chopper Stabilized D.C. Amplifier	134
6.4.	Feedback in Amplifiers	136
6.4.1.	Effect on Overall Gain	137
6.4.2.	Effect on Sensitivity	137
6.4.3.	Effect on Distortion	137
6.4.4.	Effect on Bandwidth	139
6.4.5.	Effect on Impedance	140
6.4.6.	Effect on Transient Response	141
6.5.	Magnetic Amplifiers	142
6.5.1.	Basic Principles	143
6.5.2.	Basic D.C. Amplifiers	145
6.6.	Series Connected Magnetic Amplifiers	146
6.6.1.	Feedback Compensation	152
6.7.	Rotating Amplifiers	153
6.7.1.	Amplidyne and Mecadyne	154
<b>7.</b>	<b>Servo Motors</b>	<b>159</b>
7.1.	Applications of Servo Motors	159
7.2.	D.C. Servo Motors	159
7.3.	Field Controlled D.C. Motor	161
7.4.	Armature Controlled D.C. Motor	163
7.5.	Series Connected D.C. Motor	165
7.6.	A.C. Servo Motor	168
7.7.	Constructional Features of Two Phase Motor	171
7.8.	Shaded Pole Induction Motor	173
7.9.	Tachometers	174
<b>8.</b>	<b>Miscellaneous Components</b>	<b>178</b>
8.1.	Demodulators and Modulators	178
8.2.	Types of Demodulators	180



<i>Chaps</i>	<i>Page</i>
8.3. Types of Modulators	... 183
8.4. Differentiators	... 186
8.5. Integrators	... 188
8.6. Attenuators	... 190
8.7. Compensators	... 191
8.7.1. Phase Lead Compensator	... 191
8.7.2. Phase Lag Compensator	... 193
8.7.3. Lag Lead Compensator	... 193
8.8. Resolver	... 194
8.9. Hydraulic Elements	... 196
8.9.1. The Pump Controlled Hydraulic System	... 197
8.9.2. The Valve Controlled Hydraulic System	... 199
8.10. Pneumatic Elements	... 206
8.10.1. A Pneumatic Power Servomechanism	... 209
<b>9. Specifications of a System</b>	<b>... 212</b>
9.1. Time Domain Specifications	... 212
9.2. First Order System	... 214
9.3. Second Order System	... 216
9.4. Steady State Error Constants	... 228
9.4.1. Step Displacement Input	... 229
9.4.2. Step Velocity Input	... 229
9.4.3. Step Acceleration Input	... 230
9.4.4. Error Constants for General System	... 232
9.5. Generalized Error Series	... 234
9.6. Frequency Domain Specifications	... 239
9.7. Resonance for a Second Order System	... 242
<b>10. System Stability</b>	<b>... 253</b>
10.1. General Notion of Stability	... 250
10.2. Stability in Linear Systems	... 351
10.3. Stability and the Roots of Characteristic Equation	... 253
10.4. Absolute and Relative Stability	... 259
10.5. Necessary Conditions for Stability	... 261
10.6. Hurwitz Stability Criterion	... 262
10.7. Routh Stability Criterion	... 264
10.7.1. Special Cases	... 268
10.8. Applications of Routh-Hurwitz Criterion	... 273
10.9. Limitations of the Routh-Hurwitz Method	... 278

<i>Chap.</i>		<i>Page</i>
<b>11. Nyquist Stability Method</b>	...	<b>281</b>
11.1. Direct Polar Plots	...	281
11.1.1. Type 0 Feedback Control Systems	...	282
11.1.2. Type 1 Feedback Control Systems	...	283
11.1.3. Dead Time	...	284
11.2. Inverse Polar Plots	...	185
11.3. Principle of Argument	...	287
11.4. Nyquist Criterion	...	289
11.5. Special Nyquist Criterion	...	294
11.6. Assessment of Relative Stability	...	297
11.6.1. Gain Margin and Phase Margin	...	298
11.6.2. Correlation Between Phase Margin and Damping Factor	...	304
11.7. Relation Between Closed Loop and Open Loop Response	...	305
11.7.1. Using the Inverse Polar Plots	...	309
11.8. Gain Adjustment	...	311
11.8.1. Gain Adjustment on Direct Plot	...	311
11.8.2. Gain Adjustment on Inverse Plot	...	313
11.9. Effect of the Addition of Poles and Zeros	...	316
11.9.1. Effect of Adding Finite Poles	...	316
11.9.2. Effect of Adding Poles at the Origin	...	317
11.9.3. Addition of Zeros	...	517
11.10. Effect of Compensators on the Polar Plots	...	320
11.10.1. Lead Compensation	...	320
11.10.2. Lag Compensation	...	323
11.11. Alternate $s$ -plane Path	...	325
<b>12. Bode Plot Method</b>	...	<b>331</b>
12.1. Introduction	...	331
12.2. Bode Plots of Elementary Function	...	333
12.2.1. A Constant Term, $K$	...	333
12.2.2. A Pole or Zero at the Origin	...	334
12.2.3. Simple Pole or Zero	...	335
12.2.4. Quadratic Pole or Zero	...	337
12.3. Complete Bode Plots	...	340
12.3.1. System Type and Log Magnitude Curves	...	345
12.3.2. All-pass and Minimum Phase System	...	347
12.4. Stability Studies from Bode Diagrams	...	348
12.4. Data for Other Plots	...	351

<i>Chap.</i>		<i>Page</i>
	12.6. Transfer Function from Experimental Data ...	353
	12.7. Design of Compensation Networks ...	355
	12.7.1. Design of Phase Lead Compensator ...	358
	12.7.2. Design of Phase Lag Network ...	361
	12.7.3. Use of Lag-lead Network ...	363
	12.8. Gain-Phase Plot ...	365
	12.9. The Nichols Chart ...	367
	12.9.1. Nichols Chart Design ...	370
<b>13.</b>	<b>Root Locus Technique</b> ...	<b>379</b>
	13.1. Introduction ...	379
	13.2. Basic Criterion ...	383
	13.3. The Construction of the Root Loci ...	385
	13.4. The Spirule ...	398
	13.5. Inverse Root Locus ...	399
	13.6. The Root Contours ...	401
	13.7. Effect of Adding Open Loop Poles or Zeros ...	403
	13.7.1. Addition of Poles ...	403
	13.7.2. Addition of Zeros ...	403
	13.8. Gain Factor Adjustment ...	404
	13.9. Phase Lead Compensation ...	405
	13.10. Phase Lag Compensation ...	408
<b>14.</b>	<b>Design Techniques</b> ...	<b>416</b>
	14.1. Design of Compensators by $s$ -plane Synthesis ...	416
	14.1.1. Steps for Design to Achieve a Desired Transient Response ...	418
	14.1.2. Improvement in Steady State Performance ...	419
	14.2. Integral Square Error Compensation ...	422
	14.2.1. Basic Mathematical Tools ...	422
	14.2.2. Evolution of Integral Square Values (ISV) ...	423
	14.2.3. Parameter Optimization ...	424
	14.3. Feedback Compensation ...	427
	14.3.1. Derivative Feedback ...	428
	14.3.2. Integral Feedback ...	431
	14.3.3. Techometer Feedback ...	431
	14.3.4. Feedback Compensation in Frequency Domain ...	435
	14.4. Feedforward Design ...	439
	14.5. Carrier Control Systems ...	443
	14.5.1. Realization of A.C. Compensation Networks ...	444

<i>Chap.</i>		<i>Page</i>
<b>15.</b>	<b>State Variable Characterization</b>	<b>451</b>
15.1.	Introduction	451
15.2.	Basic Definitions	452
15.3.	Review of Matrix Theory	456
15.3.1.	Matrix Inversion	458
15.3.2.	Eigen Values of a Matrix	460
15.3.3.	Diagonalization of a Matrix	461
15.4.	Decomposition of Transfer Functions	464
15.4.1.	Direct Decomposition	464
15.4.2.	Cascade Decomposition	467
15.4.3.	Parallel Decomposition	469
15.5.	State Equations and Higher Differential Equations	471
15.6.	Solution of the State Equations	475
15.6.1.	Infinite Series Solution	478
15.7.	Evaluation of STM	482
15.7.1.	Cayley-Hamilton Method	483
15.7.2.	Sylevester's Expansion Theorem	486
15.8.	Reduction to Canonial Forms	487
15.9.	Controllability and Observability	490
15.9.1.	Controllability	490
15.9.2.	Observability	493
15.9.3.	Relation with Transfer Function	495
15.10.	State Variable Feedback	495
<b>16.</b>	<b>Non-Linear Control Systems</b>	<b>505</b>
16.1.	Introduction	505
16.1.1.	Non-linearities in Servo Systems	505
16.1.2.	Properties of Non-linear Systems	508
16.2.	Describing Function Analysis	510
16.2.1.	Stability Analysis	519
16.3.	Phase Plane Analysis	525
16.3.1.	Analytical Method	527
16.3.2.	Isocline Method	528
16.3.3.	Lionard Construction	532
16.3.4.	Delta Method	536
16.3.5.	Interpretation of the Phase Portrait	538
16.3.6.	Singular Points	539
16.3.7.	Application to Control System	545

<i>Chap.</i>		<i>Page</i>
	16.4. Liapunov Stability Analysis	549
	16.4.1. Introduction	549
	16.4.2. The Concepts of Definiteness of Sign	550
	16.4.3. Outline of the Second Method	551
	16.5. Liapunov Method for Linear Systems	556
	16.6. Generation of Liapunov Function	557
	16.6.1. Krasovskii's Method	558
	16.6.2. Variable Gradient Method	559
	16.6.3. Lure's First Canonic Form Method	562
	16.7. Popov's Stability Criterion	566
<b>17.</b>	<b>Discrete Time Systems</b>	<b>576</b>
	17.1. Introduction	576
	17.1.1. Types of Controllers	578
	17.2. Laplace Transform of the Sampled Signal	579
	17.3. The Z-Transform	582
	17.3.1. The Residue Method	583
	17.3.2. Properties of Z-transforms	585
	17.4. Inverse Z-transform	587
	17.4.1. Partial Function Method	587
	17.4.2. Power Series Method	588
	17.4.3. Residue Method	588
	17.4.4. Data Hold	590
	17.5. Block Diagram and Signal Flow Algebra	595
	17.5.1. Cascaded Elements	595
	17.5.2. Block Diagrams	596
	17.5.3. Signal Flow Graph	598
	17.6. Transient Response	602
	17.6.1. Modified Routh Criterion	603
	17.6.2. The Root Locus Technique	605
	17.6.3. Modified Schur-Cohn Criterion	608
	17.6.4. Steady State Error	610
	17.7. Frequency Response	613
	17.8. State Space Representation of Discrete Time Systems	616
	17.8.1. State Space Representation of Difference Equations	616
	17.8.2. Decomposition of Discrete Transfer Function	620
	17.9. Solution Discrete Time State Equations	623

<i>Chap.</i>	<i>Page</i>
17·9·1. Evaluation of State Transition Matrix ...	624
17 10. Discretization of Continuous Time State Equations ...	627
<b>18. Optimal Control Theory</b> ...	<b>634</b>
18·1. Introduction ...	634
18·1·1. Optimal Control Problems ...	635
18·2. Calculus of Variations ...	636
18·2·1. Fixed End Problem ...	637
18·2·2. Variable End Point Problem ...	640
18·2·3. Optimization with Constraints ...	645
18·3. Pontryagin's Minimum/Maximum Principle ...	648
18·3·1. Minimum Time Problems ...	652
18·4. Hamilton Jacobi Approach ...	658
18·5. Dynamic Programming ...	664
18 6. Iterative Numerical Techniques ...	672
18 6·1. Method of Steepest Descent ...	672
<b>19. Stochastic Control System</b> ...	<b>681</b>
19·1. Introduction ...	681
19·2. Random Variables ...	683
19·2·1. Probability Distribution Function ...	684
19·2·2. Probability Density Function ...	685
19·2·3. Conditional and Joint Distributions ...	686
19·2·4. Functions of Random Variables ...	688
19·3. Statistical Averages and Moments ...	690
19·3·1. Two Functions of two Random Variables ...	691
19·4. Random Process ...	693
19·4·1. Statistics of a Random Process ...	695
19·4·2. Time Averages and Ergodicity ...	700
19·4·3. The Power Spectrum ...	700
19·5. Random Processes in Linear Systems ...	702
19·6. Mean Square Estimation ...	706
19·6·1. Estimating by a Constant ...	707
19·6·2. Estimating the Random Variable $X$ given $Y$ ...	707
19 6·3. Linear Estimation of $X$ given $Y$ ...	709
19·7. Estimation with an RC Filter ...	710
19·8. The Matched Filter ...	713



<i>Chap.</i>		<i>Page</i>
<b>20.</b>	<b>Self-Adaptive Control Syetems</b>	<b>722</b>
20'1.	Introduction	722
20'2.	Classification	724
20'3.	The Practical Problems of Adaptive Control	726
20'4.	The Essential Components	728
20'4'1.	On Line Identification	730
20'5.	Generalization into Multidimensions	732
20'6.	Stability Problem	736
20'7.	Learning in Adaptive Systems	736
	Answers to Selected Problems	740
	References	752

# Laplace Transforms

The Laplace transforms are so widely used in the study of control systems that this chapter is devoted to the discussion and review of Laplace transforms. These transforms have become popular because of the following advantages :

(i) This transformation transforms the transcendental and exponential functions to simple algebraic functions.

(ii) This transformation transforms the operations of differentiation and integration to multiplication and division respectively.

(iii) In the solution of differential equations, arbitrary constants do not occur.

(iv) We can effectively make use of step and impulse response which is very relevant in control systems.

## 1.1. Definition

The Laplace transform of a function  $f(t)$  is denoted by  $Lf(t)$  and is a function of  $s$  normally written as

$$F(s) = L f(t) \quad \dots(1)$$

The correspondence between  $f(t)$  and  $F(s)$  is unique and is established by the following relation :

$$F(s) = \int_0^{\infty} f(t) e^{-st} dt \quad \dots(2)$$

**Example 1.1.** Find the Laplace transforms of :

(a) a constant,  $K$

(b)  $f(t) = t^2$

(c)  $f(t) = e^{-\alpha t}$

**Solution.** (a)  $f(t) = K$

$$\begin{aligned} F(s) &= \int_0^{\infty} K e^{-st} dt \\ &= \left[ \frac{K}{-s} e^{-st} \right]_0^{\infty} = \frac{K}{s} \end{aligned}$$

(b)

$$\begin{aligned} f(t) &= t^2 \\ F(s) &= \int_0^{\infty} t^2 e^{-st} dt \end{aligned}$$

By repeated application of the rules of integration by parts,

$$F(s) = t^2 \left| \frac{e^{-st}}{-s} \right|_0^\infty - (2t) \left| \frac{e^{-st}}{s^2} \right|_0^\infty + (2) \left| \frac{e^{-st}}{-s^3} \right|_0^\infty$$

Making use of l'Hospital's rule,

$$F(s) = \frac{2}{s^3}$$

As a matter of fact, it can be shown that

$$L(t^n) = \frac{n!}{s^{n+1}}$$

$$(c) \quad f(t) = e^{-\alpha t}$$

$$F(s) = \int_0^\infty e^{-\alpha t} e^{-st} dt = \left| \frac{e^{-(s+\alpha)t}}{-(s+\alpha)} \right|_0^\infty = \frac{1}{s+\alpha}$$

**Example 1.2.** Find the Laplace transform for

$$f(t) = \cos \omega t.$$

**Solution.** Evaluation of Laplace transform, by direct integration, in this case is quite tedious and an easy way to find this transform is to recognize :

$$\cos \omega t = \operatorname{Re}[e^{j\omega t}]$$

$$\therefore L[\cos \omega t] = \operatorname{Re}[L(e^{j\omega t})]$$

$$\text{Now, } L(e^{j\omega t}) = \frac{1}{s-j\omega} \text{ [example 1.1(c)]}$$

$$\text{Hence, } L[\cos \omega t] = \operatorname{Re} \left[ \frac{1}{s-j\omega} \right] = \frac{s}{s^2 + \omega^2}.$$

It is not always convenient to derive Laplace transforms by definition. Mostly we make use of some of the properties of Laplace transforms. These properties are listed below.

## 1.2. Properties of Laplace Transforms

**1.2.1.** Laplace transform of the sum of two functions is equal to the sum of the Laplace transforms of the two functions

$$L[f_1(t) \pm f_2(t)] = Lf_1(t) \pm Lf_2(t) \quad \dots(3)$$

**1.2.2.** Laplace transform of a function multiplied by any constant is constant times the Laplace transform of the function.

$$L[cf(t)] = cL f(t) \quad \dots(4)$$

**1.2.3.** Laplace transform of the derivative of a function is

$$L \left[ \frac{d}{dt} \right] f(t) = s F(s) - f'(0) \quad \dots(5)$$