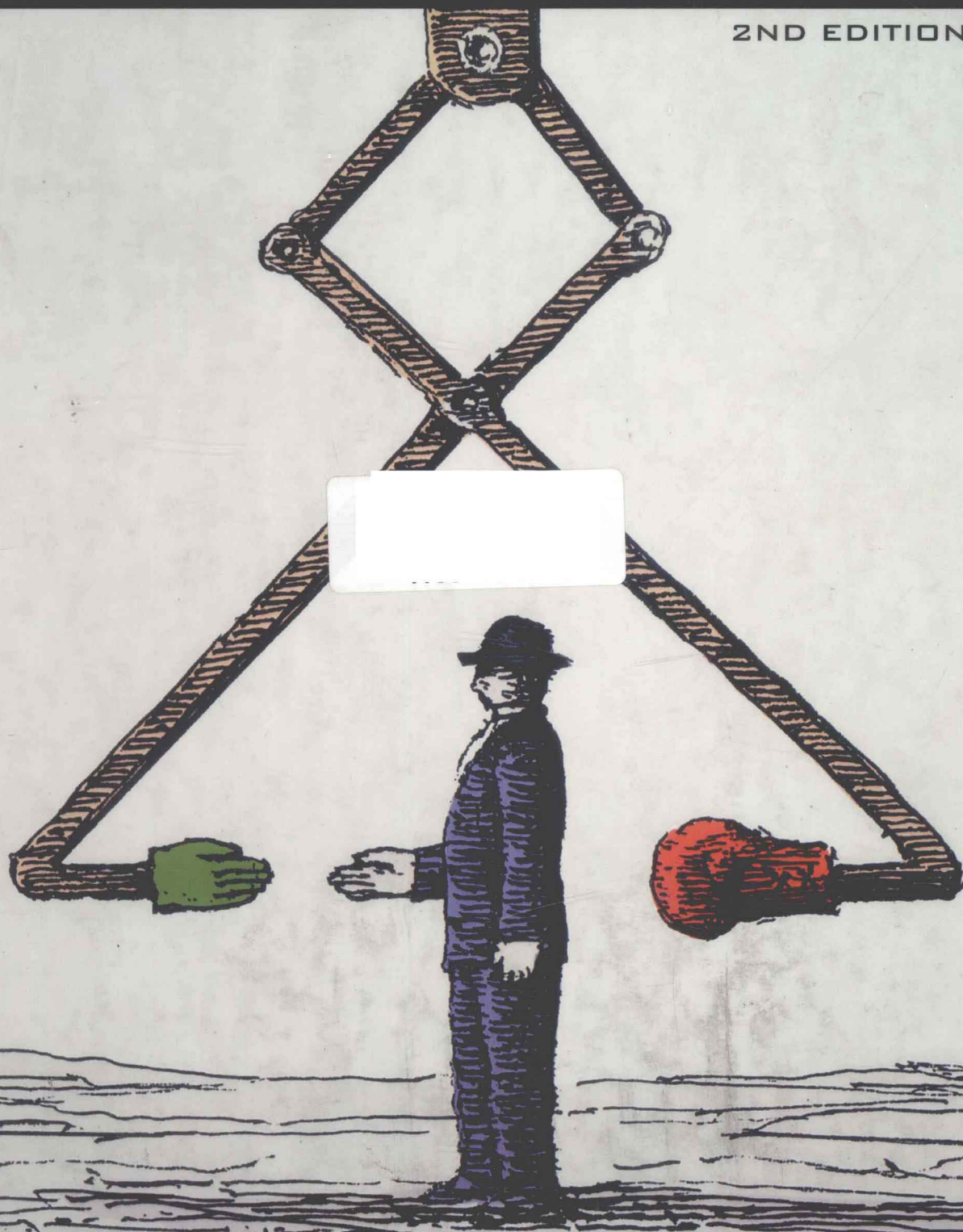


AVINASH DIXIT & SUSAN SKEATH

GAMES OF STRATEGY

2ND EDITION



GAMES OF STRATEGY

SECOND EDITION



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To the memories of our fathers,

Kamalakar Ramachandra Dixit

and

James Edward Skeath



Preface to the Second Edition

WE wrote this book to make it possible to teach game theory to first- or second-year college students at an introductory or “principles” level, without requiring prior knowledge of economics or political science or biology, and requiring only minimal high-school mathematics. We have been very pleased by the response. Hardly any courses of this kind existed a few years ago; now many teachers have developed new courses based on our book, and these have proved popular among students. The book is also used in some advanced summer programs for high school students. In the last few years, game theory has continued to find uses in old and new areas and has gained popularity following Sylvia Nasar’s biography of John Nash, *A Beautiful Mind*, and the award-winning movie adaptation starring Russell Crowe. Therefore, the appeal of the subject, and the popularity of introductory courses, should continue to grow.

We received valuable feedback from many teachers and students who used the First Edition. This, and our own experience of teaching from the book, suggested to us many improvements of substance and exposition. The intervening years also brought some advances in the theory and many new and interesting applications. This Second Edition takes advantage of all of these suggestions and new developments.

The most substantive changes are to be found in Part Two, which comprises Chapters 3 to 8 (3 to 7 in the First Edition). We rewrote these chapters, which expound the basic theory of sequential- and simultaneous-move games, almost completely. We reordered the material to improve its flow, deleted or replaced

some sections that proved too difficult or less directly relevant, and changed or improved most of the examples through which we develop the theoretical concepts. We hope that teachers will find the new order easier to use and to select from. These are the major changes to this set of chapters:

[1] The treatment of Nash equilibrium in pure strategies is now divided into two chapters: 4, which deals only with games having a finite number of available strategies; and 5, which treats strategies that are continuous variables (and other matters like a general discussion of criticisms of the Nash equilibrium concept and evidence from laboratory experiments and observation of real-life games). Both chapters now incorporate discussions of Nash equilibrium as a system of beliefs and choices. For teachers of simpler or shorter courses, it will be possible to omit parts of chapter 5.

[2] Chapter 5 discusses one prominent alternative to Nash equilibrium, namely rationalizability, at greater length than was done in chapter 7 of the First Edition. This discussion and the pricing game earlier in the chapter constitute a fuller introduction to applications in industrial economics, which many teachers like to include in their courses.

[3] The treatment of Nash equilibrium in mixed strategies is likewise split into two chapters: 7, which treats zero-sum games; and 8, which covers non-zero-sum games and further topics. The separation emphasizes the different roles that mixed strategies play in these two kinds of games. It also enables those teachers who do not like mixed-strategy equilibria in non-zero-sum games to omit that material entirely.

[4] The development of the whole class of coordination games with multiple Nash equilibria—pure coordination, assurance, and battle of the sexes—is now done using a unified example of “boy meets girl” that runs through many chapters. We hope that this example will appeal to college students.

[5] Some teachers wanted us to reverse the order in which we treat sequential- and simultaneous-move games. We have resisted this suggestion. Our experience is that for students lacking prior training in game theory, the sequential concept of strategy comes more naturally than the simultaneous one: thinking “if I do this, how will the other react, in turn thinking about my next reaction” is more natural than “I have to think about what the other is thinking about what I am thinking.” Therefore, we like to start by building on this natural inclination, show how it can be formalized into a method of analysis (rollback), and then go on to the new way of thinking and formalizing what is needed with simultaneous moves. However, teachers who like to do things the other way round will find that the new chapters 4 and 5 can be taught before chapter 3; only a few inconsequential references are made to sequential moves, trees, and rollback in chapters 4 and 5.

The chapters in Parts Three and Four are less thoroughly overhauled, but we offer new or improved examples everywhere and new evidence from observations and experiments where possible. In all chapters, we have developed several new exercises and revised and reordered old ones, so as to present a more complete and better graduated sequence. And instead of merely listing the key terms at the end of each chapter, we now give a full glossary at the end of the book where these terms are defined more formally. The most notable changes to these parts are as follows:

[1] Part Three has been reordered so that the chapter on uncertainty and information comes first; this should make it easier for teachers who want to cover that material early. This chapter, which includes a new section on “cheap talk,” is now followed by strategic moves and then the chapters on repeated games (the prisoners’ dilemma), coordination games, and evolutionary games. The last three share examples and methods of analysis that interconnect them; the new order allows us to make these connections more explicitly and should provide better flow through the material in courses covering all three topics. As in the First Edition, however, the chapters in this part of the book can be taught in almost any order as there is relatively little cross-referencing of material.

[2] In Part Four, we revised the chapters on voting and auctions most significantly. There is a broader analysis of the implications of Arrow’s Impossibility Theorem in the voting chapter and a new discussion of strategic voting with imperfect information. In the auction chapter, we replaced the section on the FCC spectrum auctions with one on the increasingly ubiquitous Internet auctions.

In the First Edition, we adopted the convention that generic pronouns were male in chapters for which Dixit was primarily responsible and female in the chapters for which Skeath was primarily responsible. The reordering of material has jumbled authorship of parts of chapters and made this distinction less clear. However, we have retained a uniform gender in each chapter and divided the book roughly equally into male and female chapters, to keep the book politically correct without its being linguistically ugly.

Since the First Edition was printed, a Web site for teachers has been developed, with an instructors’ manual and a discussion forum. The home page is: **www.wwnorton.com/college/titles/econ/games2/**. We encourage instructors to use this resource and to share their own teaching tips or comments on the discussion forum.

Finally, special thanks to those who have helped us in producing this Second Edition. Vincent Crawford (University of California, San Diego), David H. Reiley, Jr. (University of Arizona), Thomas Prusa (Rutgers University), and Greg Trandel (University of Georgia) all provided comments and suggestions on improving the

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A.D., Princeton, NJ

S.S., Wellesley, MA

September 2003



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