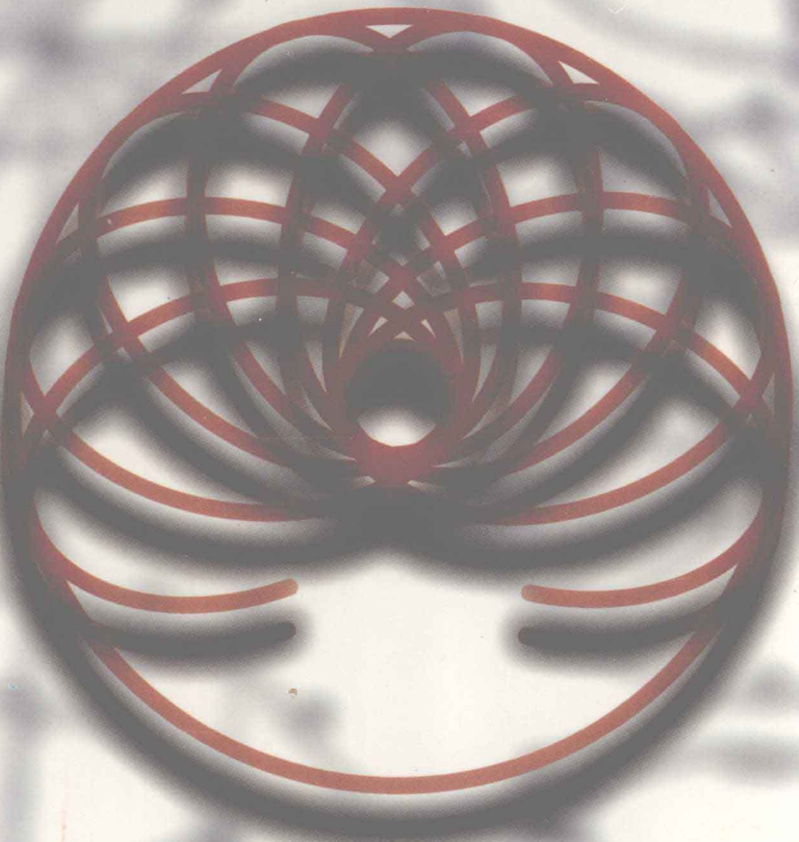
Marion Thornton

classical dynamics

of particles and systems



4 edition

CLASSICAL DYNAMICS OF PARTICLES AND SYSTEMS

FOURTH EDITION

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SAUNDERS COLLEGE PUBLISHING
Harcourt Brace College Publishers

Fort Worth Philadelphia

San Diego New York Orlando Austin San Antonio

Toronto Montreal London Sydney Tokyo

Text Typeface: Times Roman
Compositor: GTS GRAPHICS, INC.
Vice President/Publisher: John Vondeling
Developmental Editor: Jennifer Bortel
Managing Editor: Carol Field
Project Editor: Nancy Lubars
Copy Editor: Donna Regen
Manager of Art and Design: Carol Bleistine
Art Director: Robin Milicevic
Text Designer: Barbara Gibson
Cover Designer: Nicky Lindeman
Text Artwork: J.A.K. Graphics, Ltd.
Director of EDP: Tim Frelick
Production Manager: Charlene Squibb
Director of Marketing: Marjorie Waldron

Printed in the United States of America

ISBN 0-03-097302-3

Library of Congress Catalog Card Number: 94:061383

4567890123 032 10 987654321

This book was printed on acid-free recycled content paper, containing
MORE THAN
10% POSTCONSUMER WASTE

CLASSICAL DYNAMICS OF PARTICLES AND SYSTEMS

FOURTH EDITION

To

DR. KATHRYN C. THORNTON

ASTRONAUT AND WIFE

AS SHE SOARS AND WALKS THROUGH SPACE,
MAY HER LIFE BE SAFE AND FULFILLING,
AND LET OUR CHILDREN'S MINDS BE OPEN
FOR ALL THAT LIFE HAS TO OFFER.

PREFACE

In preparing the fourth edition of the late Jerry Marion's text, I have attempted to adhere to his original purpose, as I did in the third edition, of producing a modern and reasonably complete account of the classical mechanics of particles, systems of particles, and rigid bodies for physics students at the advanced undergraduate level. The purpose of this book is threefold:

1. To present a modern treatment of classical mechanical systems in such a way that the transition to the quantum theory of physics can be made with the least possible difficulty.
2. To acquaint the student with new mathematical techniques wherever possible, and to give him/her sufficient practice in solving problems so that the student may become reasonably proficient in their use.
3. To impart to the student, at the crucial period in the student's career between "introductory" and "advanced" physics, some degree of sophistication in handling both the formalism of the theory and the operational technique of problem solving.

After a firm foundation in vector methods is presented in Chapter 1, further mathematical methods are developed in the textbook as the occasion demands. It is advisable for students to continue studying advanced mathematics in a separate course. Mathematical rigor must be learned and appreciated by students of physics, but where the continuity of the physics would be disturbed by insisting on complete generality and mathematical rigor, the *physics* has been given precedence.

Changes to the Fourth Edition

The comments and suggestions of many users of *Classical Dynamics* have been incorporated into this fourth edition. Without the feedback of the many instructors who have used this text, it would not be possible to produce a textbook of significant value to the physics community. Users have requested more numerical calculations. Spread throughout the book, but especially in Chapter 2, numerical examples and end-of-chapter problems have been added. Users have also indicated that students want more examples, and I have responded.

The nonlinear oscillation material formerly in Chapter 3 has been combined with new material on chaos for a new Chapter 4 entitled **Nonlinear Oscillations and Chaos.** Chapters 12 and 13 of the third edition have been combined into a new Chapter 13, and some of the previous material has been omitted. A few more advanced or specialized topics throughout the book have also been omitted to allow new material. Particular effort was made to correct the problem solutions available in the Instructor's and Student Solutions Manuals. I thank the many users who sent comments concerning the various problem solutions. Answers to even-numbered problems have again been included at the end of the book, and the selected and general bibliography has been updated.

Course Suitability

The book is suitable for either a one-semester or two-semester upper level (junior or senior) undergraduate course in classical mechanics taken after an introductory calculus-based physics course. At the University of Virginia we teach a one-semester course based mostly on the first twelve chapters with several omissions of certain sections. Sections that can be omitted without losing continuity are denoted as optional, but the instructor can also choose to skip other sections (or entire chapters) as desired. For example, the new Chapter 4 might be skipped in its entirety for a one-semester course. Some instructors choose not to cover the calculus of variations material in Chapter 6. Other instructors may want to begin with Chapter 2, skip the mathematical introduction of Chapter 1, and introduce the mathematics as needed. This technique of dealing with the mathematics introduction is perfectly acceptable, and the community is divided on this issue with a slight preference for the method used here. The textbook is also suitable for a full academic year course with an emphasis on mathematical and numerical methods as desired by the instructor.

The textbook is appropriate for those who choose to teach in the traditional manner without computer calculations. However, it has been fun adding the numerical calculations, and I must admit that I became much more proficient doing computer calculations as a result of this revision. Practically all students, and most professors, now use computers every day, and they are a useful tool in learning physics. One difficulty is to choose among the many computer techniques available, and I decided to leave this choice to the student and instructor and have not indicated any preferred method.

Special Feature

The author has kept one popular feature of Jerry Marion's original book: the addition of historical footnotes spread throughout. Several users have indicated how valuable these historical comments have been. The history of physics has been almost eliminated from present-day curricula, and as a result, the student is frequently unaware of the background of a particular topic. These footnotes are intended to whet the appetite and to encourage the student to inquire into the history of his/her field.

Teaching Aids

Several teaching aids are available to accompany the textbook. An Instructor's Manual with solutions to all the end-of-chapter problems is available to instructors who adopt this book by contacting the local Saunders (Harcourt Brace) College Publishing sales representative. A separate Student Solutions Manual, with solutions to about 25% of the problems, is available for sale to the students if the instructor decides to order it through the local bookstore. Several instructors have requested that transparencies of the figures be provided. We have appended enlarged versions of many text figures in the Instructor's Manual to allow instructors to make their own transparencies on a copy machine if so desired.

Acknowledgments

I would like to acknowledge the many instructors who provided helpful suggestions through a questionnaire sent out before the revision began. They include

William L. Alford, *Auburn University*
Philip Baldwin, *University of Akron*
Martin Berz, *Michigan State University*
Randy A. Booker, *University of North Carolina–Asheville*
Michael E. Browne, *University of Idaho*
Richard F. Carlson, *University of Redlands*
D. Rae Carpenter, Jr., *Virginia Military Institute*
D. Casavant, *St. Michael's College*
F. Edward Cecil, *Colorado School of Mines*
Albert C. Claus, *Loyola University*
Stan Cloud, *University of Nevada–Las Vegas*
G. T. Condo, *University of Tennessee*
John. E. Crew, *Illinois State University*
Michael De Marco, *Buffalo State College*
George Dixon, *Oklahoma State University*
Warren L. Dumke, *Marshall University*
Cheng-Ming Fou, *University of Delaware*
Norman Fuchs. *Purdue University*
Elsa M. Glover, *Stillman College*
Paul M. Goldbart, *University of Illinois at Urbana-Champaign*
Chris Gould, *North Carolina State University*
Edward Hart, *University of Tennessee*
Richard Heinz, *Indiana University*
Larry D. Johnson, *Northeast Louisiana University*
Thomas Kirkman, *St. John's University*
Carl A. Kocher, *Oregon State University*
Paul L. Lee, *California State University–Northridge*
Charles Leming, *Henderson State University*
Robert R. Marchini, *Memphis State University*
Thomas R. Michalik, *Randolph-Macon Women's College*
R. D. Murphy, *University of Missouri–Kansas City*
Richard P. Olenick, *University of Dallas*
Peter Parker, *Yale University*
J. Pilcher, *University of Chicago*
Jerry Polson, *Southeastern Oklahoma State University*
Dan R. Quisenberry, *Mercer University*
Stephen P. Reynolds, *North Carolina State University*
Albert T. Rosenberger, *University of Alabama–Huntsville*

Robert Sears, Jr., *Austin Peay State University*
Sheridan Simon, *Guilford College*
Jack A. Soules, *Cleveland State University*
S. Sridhar, *Northeastern University*
Paul Stevenson, *Rice University*
N. S. Sullivan, *University of Florida*
Ronald G. Tabak, *Youngstown State University*
Larry Tankersley, *United States Naval Academy*
Philip L. Taylor, *Case Western Reserve University*
Noboru Wada, *Colorado School of Mines*
Bruce Weems, *East Central University*
Hugh D. Young, *Carnegie Mellon University*
Alma C. Zook, *Pomona College*

I would especially like to thank those individuals who either wrote me with suggestions on the text or problems or who reviewed parts of the fourth edition. Their efforts have helped considerably in producing this fourth edition. They include

William L. Alford
Auburn University

Philip Baldwin
University of Akron

Robert P. Bauman
University of Alabama, Birmingham

Michael E. Browne
University of Idaho

Melvin G. Calkin
Dalhousie University

F. Edward Cecil
Colorado School of Mines

Arnold J. Dahm
Case Western Reserve University

Dan de Vries
University of Colorado

George Dixon
Oklahoma State University

John J. Dykla
Loyola University of Chicago

Thomas A. Ferguson
Carnegie Mellon University

Shun-Fu Gao
University of Minnesota, Morris

Reinhard Graetzer
Pennsylvania State University

Thomas M. Helliwell
Harvey Mudd College

Stephen Houk
College of the Sequoias

Joseph Klarmann
Washington University, St. Louis

Kaye D. Lathrop
Stanford University

Robert R. Marchini
Memphis State University

Robert B. Muir
University of North Carolina, Greensboro

Richard P. Olenick
University of Dallas

Tao Pang
University of Nevada, Las Vegas

Peter Parker
Yale University

Peter Rolnick
Northeast Missouri State University

Albert T. Rosenberger
University of Alabama, Huntsville

William E. Slater
University of California, Los Angeles

Herschel Snodgrass
Lewis and Clark College

J. C. Sprott
University of Wisconsin, Madison

Paul Stevenson
Rice University

Joseph S. Tenn
Sonoma State University

Larry Tankersley
United States Naval Academy

In addition, I would like to acknowledge the assistance of Warren Griffith who helped considerably with the problem solutions for the fourth edition and Brian Giambattista who did a similar service for the third edition. Thanks also to Jennie Metz for typing the Instructor's Manual.

The guidance and help of the Saunders College Publishing professional staff is greatly appreciated. These persons include John Vondeling, Vice President and Publisher; Jennifer Bortel, Developmental Editor; Nancy Lubars, Project Editor; Robin Milicevic, Art Director; Marjorie Waldron, Marketing Manager; Randi Misher, Marketing Coordinator.

I would appreciate receiving suggestions or notices of errors in any of these materials. I can be contacted by electronic mail at STT@Virginia.EDU.

Stephen T. Thornton
Charlottesville, Virginia

CONTENTS

C H A P T E R

1

MATRICES, VECTORS, AND VECTOR CALCULUS

1.1 INTRODUCTION

Physical phenomena can be discussed concisely and elegantly through the use of vector methods.* In applying physical "laws" to particular situations, the results must be independent of whether we choose a rectangular or bipolar cylindrical coordinate system. The results must also be independent of the exact choice of origin for the coordinates. The use of vectors gives us this independence. A given physical law will still be correctly represented no matter which coordinate system we decide is most convenient to describe a particular problem. Also, the use of vector notation provides an extremely compact method of expressing even the most complicated results.

In elementary treatments of vectors, the discussion may start with the statement that "a vector is a quantity that can be represented as a directed line segment." To be sure, this type of development will yield correct results, and it is even beneficial to impart a certain feeling for the physical nature of a vector. We assume that the reader is familiar with this type of development, but we forego the approach here because we wish to emphasize the relationship that a vector bears to a coordinate transformation. Therefore, we introduce matrices and matrix notation to describe

*Josiah Willard Gibbs (1839–1903) deserves much of the credit for developing vector analysis around 1880–1882. Much of the present-day vector notation was originated by Oliver Heaviside (1850–1925), an English electrical engineer, and dates from about 1893.