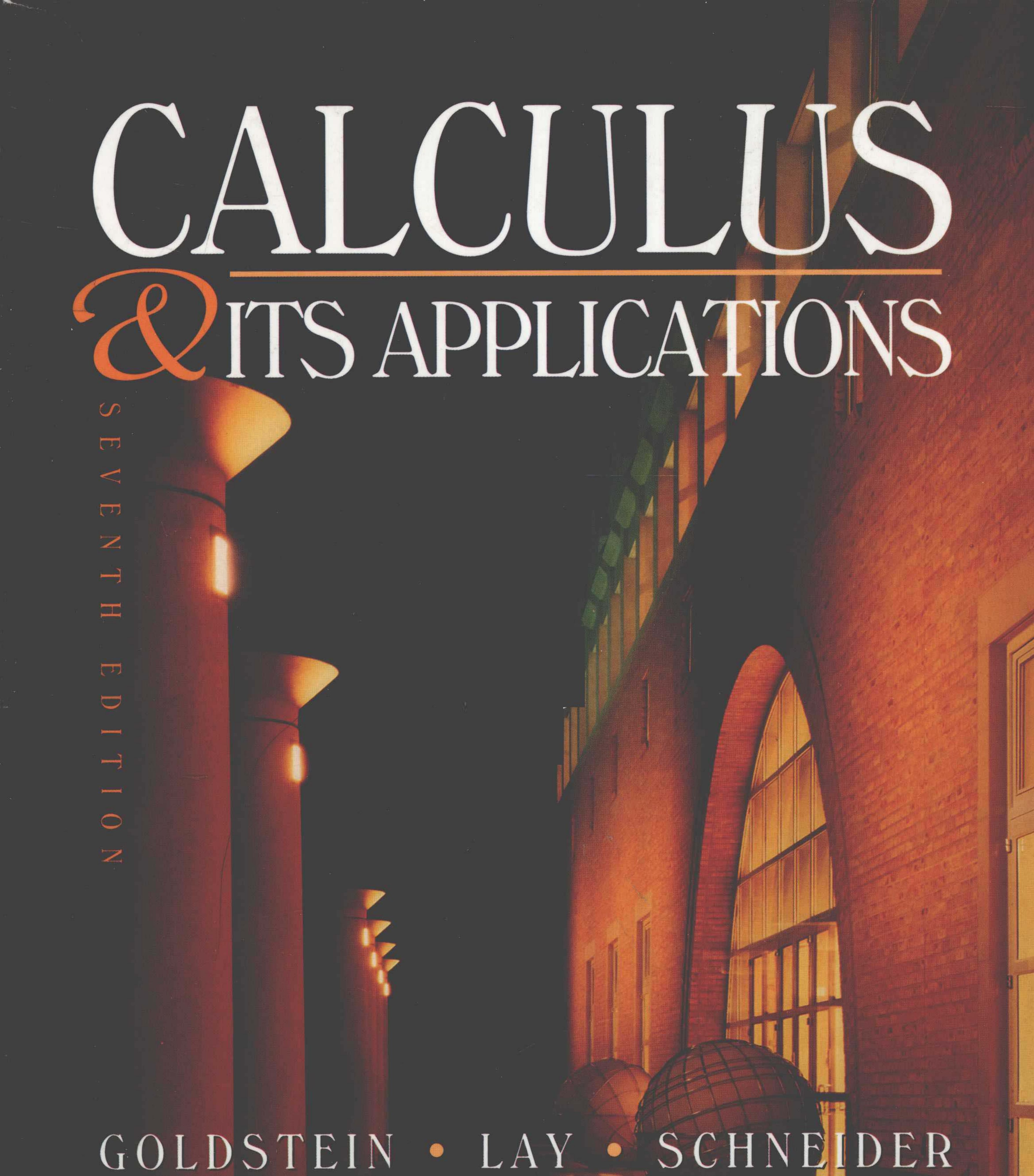
CALCULUS & ITS APPLICATIONS

SEVENTH EDITION



GOLDSTEIN • LAY • SCHNEIDER

7TH EDITION

CALCULUS AND ITS APPLICATIONS

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PREFACE

We have been very pleased with the enthusiastic response to the sixth edition of ***Calculus and Its Applications*** by teachers and students alike. The present work incorporates many of the suggestions they have put forward.

Although there are many changes, we have preserved the approach and the flavor. Our goals remain the same: to begin the calculus as soon as possible; to present calculus in an intuitive yet intellectually satisfying way; and to illustrate the many applications of calculus to the biological, social, and management sciences. We have tried to achieve these goals while paying close attention to students' real and potential problems in learning calculus. Our main concern, as always, is: Will it work for the students? Listed on the following pages are some of the features that illustrate various aspects of this student-oriented approach.

APPLICATIONS

We provide realistic applications that illustrate the uses of calculus in other disciplines. The reader may survey the variety of applications by turning to the Index of Applications on the inside cover. Wherever possible, we have attempted to use applications to motivate the mathematics.

EXAMPLES

We have included many more worked examples than is customary. Furthermore, we have included computational details to enhance readability by students whose basic skills are weak.

EXERCISES

The exercises comprise about one-quarter of the text—the most important part of the text in our opinion. The exercises at the ends of the sections are usually arranged in the order in which the text proceeds, so that the homework assignments may easily be made after only part of a section is discussed. Interesting applications and more challenging problems tend to be located near the ends of the exercise sets. Supplementary exercises at the end of each chapter expand the other exercise sets and provide cumulative exercises that require skills from earlier chapters.

PRACTICE PROBLEMS

The practice problems introduced in the second edition have proved to be a popular and useful feature and are included in the present edition. The practice problems are carefully selected exercises that are located at the end of each section, just before the exercise set. Complete solutions are given following the exercise set. The practice problems often focus on points that are potentially confusing or are likely to be overlooked. We recommend that the reader seriously attempt the practice problems and study their solutions before moving on to the exercises. In effect, the practice problems constitute a built-in workbook.

MINIMAL PREREQUISITES

In Chapter 0, we review those facts that the reader needs to study calculus. A few important topics, such as the laws of exponents, are reviewed again when they are used in a later chapter. A reader familiar with the content of Chapter 0 should begin with Chapter 1 and use Chapter 0 as a reference, whenever needed.

NUMERICAL METHODS

With the common availability of microcomputers, numerical methods assume more significance than ever. We have included many discussions of numerical methods, including the differential in one variable (Section 1.8) and several variables (Section 7.5), numerical integration (Section 9.4), Euler's method for approximating the solutions to linear differential equations (Section 10.3), Taylor polynomials (Section 12.1), the Newton-Raphson algorithm (Section 12.2), and infinite series (Sections 12.3–12.6).

NEW IN THIS EDITION

Among the many changes in this edition, the following are the most significant.

1. *Technology Discussions.* Throughout the text, we have added discussions of how calculations and mathematical ideas can be elucidated using technology, including graphing calculators and symbolic calculation packages.

2. *Technology Projects.* Each chapter includes from three to eight projects that apply various aspects of technology to the chapter material. Some of these projects are sequences of technical exercises to allow students to learn how the technology is used. Others are mathematical models too complicated or "messy" to explore without the benefit of technology. Many of these models make use of data taken from current newspapers and magazines. In all, the technology projects include over 400 separate exercises. For those not technologically oriented, we have structured the technology projects so that they may be skipped in whole or in part without any loss of continuity.
3. *Additional Examples and Exercises.* We have added new examples and exercises, many based on real-world data. Graphs and tables based on real data are indicated throughout by a shadowed border.
4. *Treatment of Compound Interest.* We have moved the introduction of compound interest to Chapter 0, so that it may be treated with the discussion of exponents. This makes for a more lucid comparison of compound interest and continuous interest in Chapter 5.
5. *Incorporation of Reviewer and User Suggestions.* Throughout, we have made use of the many generous suggestions of colleagues, users, and reviewers.

This edition contains more material than can be covered in most two-semester courses. In addition, the level of theoretical material may be adjusted to the needs of the students. For instance, only the first two pages of Section 1.4 are required in order to introduce the limit notation.

Answers to the odd-numbered exercises are included at the back of the book. Answers to all the exercises are contained in the Instructor's Edition. A Study Guide for students is available that contains detailed explanations and solutions of every sixth exercise. The Study Guide also includes helpful hints and strategies for studying that will help students improve their performance in the course.

An Instructor's Solutions Manual contains worked solutions to every exercise. A Test Item File provides nearly 1000 suggested test questions, keyed to chapter and section. These Test Items are also available on an IBM-compatible disk with Prentice Hall's Custom Test program.

The technology discussions and projects are not calculator- or program-specific. However, instructors wishing to use a particular graphing calculator or symbolic program can make use of the various *Technology Guides* which particularize the technology discussions and projects to Texas Instruments Hewlett Packard graphing calculator, and *Maple.* Computer software for IBM-compatible computers is available at a nominal cost through your book store. *Visual Calculus* by David Schneider contains over twenty routines that provide additional insight into the topics discussed in the text. Although this software has much of the computing power of standard calculus software packages, it is primarily a teaching tool that focuses on understanding mathematical concepts, rather than on computing. These routines incorporate graphics whenever possible to illustrate topics such as secant lines, tangent lines, velocity, optimization, the relationship between the graphs of f, f', and f'', and the various approaches to approximating definite integrals. All the routines in this software package are menu-driven and very easy to use. The software is accompanied by a manual with instructions and additional exercises for the student. Hardware requirements are an IBM-compatible computer with at least 384 K of memory and a graphics adapter: CGA, EGA, VGA, or Hercules.

ACKNOWLEDGMENTS

While writing this book, we have received assistance from many persons. And our heartfelt thanks goes out to them all. Especially, we should like to thank the following reviewers, who took the time and energy to share their ideas, preferences, and often their enthusiasm with us.

Reviewers of the first edition: Russell Lee, Allan Hancock College; Donald Hight, Kansas State College of Pittsburg; Ronald Rose, American River College; W. R. Wilson, Central Piedmont Community College; Bruce Swenson, Foothill College; Samuel Jasper, Ohio University; Carl David Minda, University of Cincinnati; H. Keith Stumpff, Central Missouri State University; Claude Schochet, Wayne State University; James E. Huneycutt, North Carolina University.

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Reviewers of the sixth edition: Dennis Brewer, University of Arkansas; Melvin D. Lax, California State University, Long Beach; Lawrence J. Lardy, Syracuse University; Arlene Sherburne, Montgomery College, Rockville; Gabriel Lugo, University of North Carolina, Wilmington; W. R. Hintzman, San Diego State University; George Pyrros, University of Delaware.

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The authors would like to thank the many people at Prentice Hall who have contributed to the success of our books over the years. We appreciate the tremendous efforts of the production, art, manufacturing, and marketing departments. Our sincere thanks to Judy Winthrop, who has acted as production editor for this edition.

The authors wish to extend a special thank you to Mr. George Lobell, whose ideas, encouragement, and enthusiasm have nurtured us as we prepared this revision.

Larry J. Goldstein
David C. Lay
David I. Schneider

CONTENTS

INTRODUCTION

Often it is possible to give a succinct and revealing description of a situation by drawing a graph. For example, Fig. 1 describes the amount of money in a bank account drawing 5% interest, compounded daily. The graph shows that as time passes, the amount of money in the account grows. In Fig. 2 we have drawn a graph that depicts the weekly sales of a breakfast cereal at various times after advertising has ceased. The graph shows that the longer the time since the last advertisement, the fewer the sales. Figure 3 shows the size of a bacteria culture at various times. The culture grows larger as time passes. But there is a maximum size that the culture cannot exceed. This maximum size reflects the restrictions imposed by food supply, space, and similar factors. The graph in Fig. 4 describes the decay of the radioactive isotope iodine 131. As time passes, less and less of the original radioactive iodine remains.

Each of the graphs in Figs. 1 to 4 describes a change that is taking place. The amount of money in the bank is changing as are the sales of cereal, the size of the bacteria culture, and the amount of the iodine. Calculus provides mathematical tools to study each of these changes in a quantitative way.

FIGURE 1 Growth of money in a savings account.

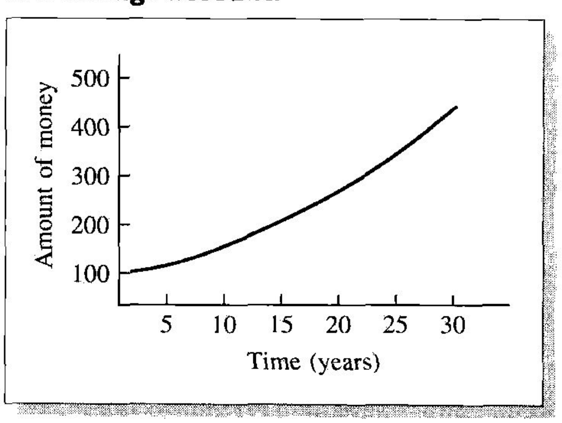
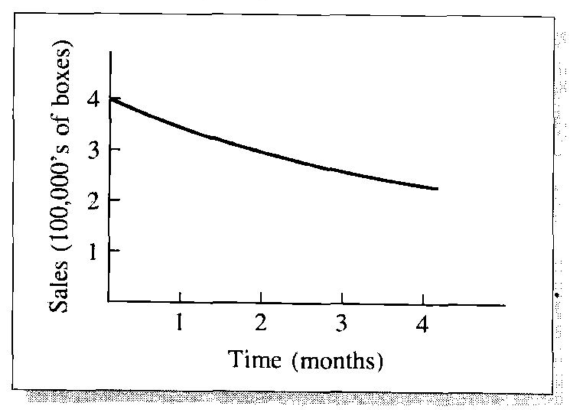


FIGURE 2 Decrease in sales of breakfast cereal.



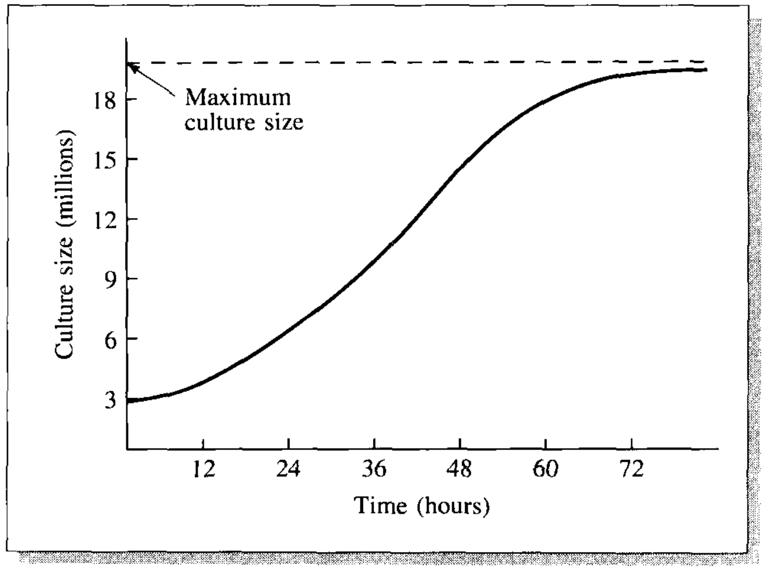


FIGURE 3 Growth of a bacteria culture.

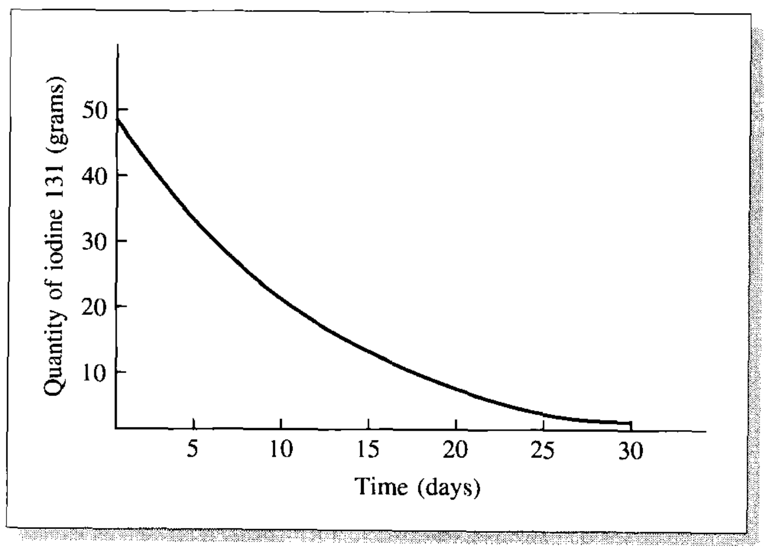


FIGURE 4 Decay of radioactive iodine.

CHAPTER 0

FUNCTIONS

Each of the graphs in Figs. 1 to 4 of the Introduction depicts a relationship between two quantities. For example, Fig. 4 illustrates the relationship between the quantity of iodine (measured in grams) and time (measured in days). The basic quantitative tool for describing such relationships is a *function*. In this preliminary chapter, we develop the concept of a function and review important algebraic operations on functions used later in the text.

0.1 FUNCTIONS AND THEIR GRAPHS

Real Numbers Most applications of mathematics use real numbers. For purposes of such applications (and the discussions in this text), it suffices to think of a real number as a decimal. A *rational* number is one that may be written as a finite or infinite repeating decimal, such as

$$-\frac{5}{2} = -2.5, \qquad 1, \qquad \frac{13}{3} = 4.333\ldots \qquad \text{(rational numbers).}$$

An *irrational* number has an infinite decimal representation whose digits form no repeating pattern, such as

$$-\sqrt{2} = -1.414214\ldots, \qquad \pi = 3.14159\ldots \qquad \text{(irrational numbers).}$$

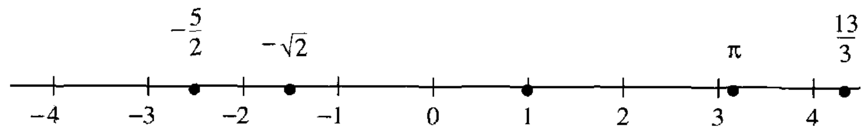


FIGURE 1 The real number line.

The real numbers are described geometrically by a *number line,* as in Fig. 1. Each number corresponds to one point on the line, and each point determines one real number.

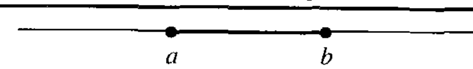
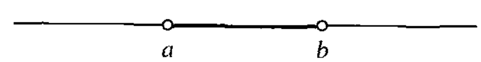
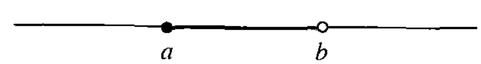
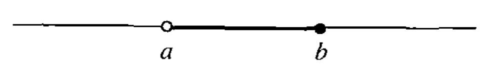
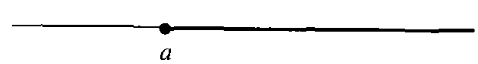
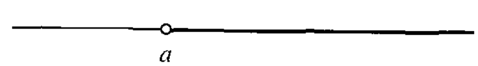
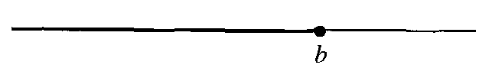
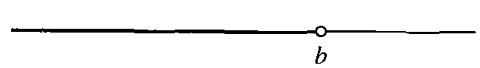
We use four types of inequalities to compare real numbers.

$x < y$	x is less than y
$x \leq y$	x is less than or equal to y
$x > y$	x is greater than y
$x \geq y$	x is greater than or equal to y

The double inequality $a < b < c$ is shorthand for the pair of inequalities $a < b$ and $b < c$. Similar meanings are assigned to other double inequalities, such as $a \leq b < c$. Three numbers in a double inequality, such as $1 < 3 < 4$ or $4 > 3 > 1$, should have the same relative positions as on the number line as in the inequality (when read left to right or right to left). Thus $3 < 4 > 1$ is never written because the numbers are "out of order."

Geometrically, the inequality $x \leq b$ means that either x equals b or x lies to the left of b on the number line. The set of real numbers x that satisfy the double inequality $a \leq x \leq b$ corresponds to the line segment between a and b, including the endpoints. This set is sometimes denoted by $[a, b]$ and is called the *closed*

TABLE 1 Intervals on the Number Line

Inequality	Geometric Description	Interval Notation
$a \leq x \leq b$		$[a, b]$
$a < x < b$		(a, b)
$a \leq x < b$		$[a, b)$
$a < x \leq b$		$(a, b]$
$a \leq x$		$[a, \infty)$
$a < x$		(a, ∞)
$x \leq b$		$(-\infty, b]$
$x < b$		$(-\infty, b)$

interval from a to b. If a and b are removed from the set, the set is written as (a, b) and is called the *open interval* from a to b. The notation for various line segments is listed in Table 1.

The symbols ∞ ("infinity") and $-\infty$ ("minus infinity") do not represent actual real numbers. Rather, they indicate that the corresponding line segment extends infinitely far to the right or left. An inequality that describes such an infinite interval may be written in two ways. For instance, $a \leq x$ is equivalent to $x \geq a$.

EXAMPLE 1 Describe each of the following intervals.

(a) $(-1, 2)$ (b) $[-2, \pi]$ (c) $(2, \infty)$ (d) $(-\infty, \sqrt{2}]$

Solution The line segments corresponding to the intervals are shown in Fig. 2(a)–(d).

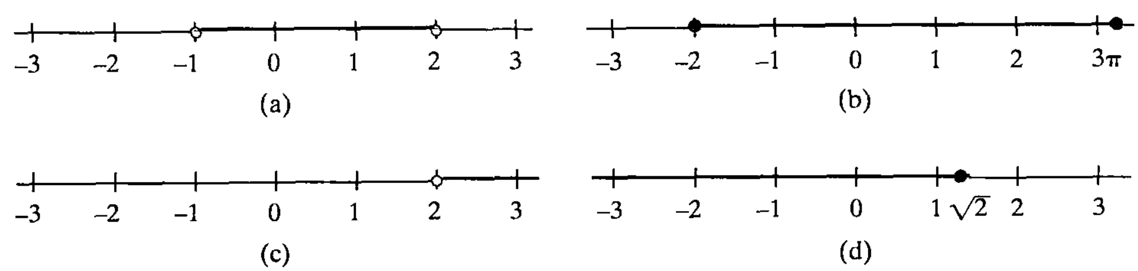


FIGURE 2 Line segments. ●

EXAMPLE 2 The variable x describes the profit that a company is anticipated to earn in the current fiscal year. Ths business plan calls for a profit of at least 5 million dollars. Describe this aspect of the business plan in the language of intervals.

Solution The business plan requires that $x \geq 5$ (where the units are millions of dollars). This is equivalent to saying that x lies in the infinite interval $[5, \infty)$. ●

Functions A *function* of a variable x is a *rule* f that assigns to each value of x a unique number $f(x)$, called *the value of the function at* x. [We read "$f(x)$" as "f of x."] The variable x is called the *independent variable*. The set of values that the independent variable is allowed to assume is called the *domain* of the function. The domain of a function may be explicitly specified as part of the definition of a function or it may be understood from context. (See the following discussion.) The *range of a function* is the set of values that the function assumes.

The functions we shall meet in this book will usually be defined by algebraic formulas. For example, the domain of the function

$$f(x) = 3x - 1$$

consists of all real numbers x. This function is the rule that takes a number, multiplies it by 3, and then subtracts 1. If we specify a value of x, say $x = 2$, then we find the value of the function at 2 by substituting 2 for x in the formula:

$$f(2) = 3(2) - 1 = 5.$$