THEORETICAL PHYSICS 2

Text and Exercise Books

W. Greiner B. Müller

Quantum Mechanics

Symmetries

Foreword by D. A. Bromley

Springer-Verlag

Walter Greiner Berndt Müller

QUANTUM MECHANICS

Symmetries

With a Foreword by D. A. Bromley

With 81 Figures

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Professor Dr. Walter Greiner
Professor Dr. Berndt Müller

Institut für Theoretische Physik, Universität Frankfurt, Robert-Mayer-Str. 8–10
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Foreword

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics – with mathematics, the most fundamental of sciences – using Walter Greiner's textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicists remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck and by Landau and Lifshitz. From the students' viewpoint, there are a great many obvious advantages to be gained through use of consistent notation, logical ordering of topics and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

The present five volume set, *Theoretical Physics*, is in fact only that part of the complete set of textbooks developed by Greiner and his students that presents the quantum theory. I have long urged him to make the remaining volumes on classical mechanics and dynamics, on electromagnetism, on nuclear and particle physics, and on special topics available to an English-speaking audience as well, and we can hope for these companion volumes covering all of theoretical physics some time in the future.

What makes Greiner's volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common "it follows that..." which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools under study are applied to actual problems of interest to a working physicist. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1798-1857) in his *Positive Philosophy* who noted, "To understand a science it is necessary to know its history". This is all too often forgotten in modern physics teaching and the bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner's lectures, which underlie these volumes, are internationally noted for their clarity, their completeness and for the effort that he has devoted to making physics an integral whole; his enthusiasm for his science is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again the reader will find that, after dipping into a particular volume to review a specific topic, he will end up browsing, caught up by often fascinating new insights and developments with which he had not previously been familiar.

Having used a number of Greiner's volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

D. Allan Bromley
Henry Ford II Professor of Physics
Yale University
New Haven, CT USA

Preface

Theoretical physics has become a many-faceted science. For the young student it is difficult enough to cope with the overwhelming amount of new scientific material that has to be learned, let alone obtain an overview of the entire field, which ranges from mechanics through electrodynamics, quantum mechanics, field theory, nuclear and heavy-ion science, statistical mechanics, thermodynamics, and solid-state theory to elementary-particle physics. And this knowledge should be acquired in just 8–10 semesters, during which, in addition, a Diploma or Master's thesis has to be worked on or examinations prepared for. All this can be achieved only if the university teachers help to introduce the student to the new disciplines as early on as possible, in order to create interest and excitement that in turn set free essential, new energy. Naturally, all inessential material must simply be eliminated.

At the Johann Wolfgang Goethe University in Frankfurt we therefore confront the student with theoretical physics immediately, in the first semester. Theoretical Mechanics I and II, Electrodynamics, and Quantum Mechanics I – An Introduction are the basic courses during the first two years. These lectures are supplemented with many mathematical explanations and much support material. After the fourth semester of studies, graduate work begins, and Quantum Mechanics II – Symmetries, Statistical Mechanics and Thermodynamics, Relativistic Quantum Mechanics, Quantum Electrodynamics, the Gauge Theory of Weak Interactions, and Quantum Chromodynamics are obligatory. Apart from these, a number of supplementary courses on special topics are offered, such as Hydrodynamics, Classical Field Theory, Special and General Relativity, Many-Body Theories, Nuclear Models, Models of Elementary Particles, and Solid-State Theory. Some of them, for example the two-semester courses Theoretical Nuclear Physics and Theoretical Solid-State Physics, are also obligatory.

The form of the lectures that comprise *Quantum Mechanics – Symmetries* follows that of all the others: together with a broad presentation of the necessary mathematical tools, many examples and exercises are worked through. We try to offer science in as interesting a way as possible. With symmetries in quantum mechanics we are dealing with a particularly beautiful theme. The selected material is perhaps unconventional, but corresponds, in our opinion, to the importance of this field in modern physics.

After a short reminder of some symmetries in classical mechanics, the great importance of symmetries in quantum mechanics is outlined. In particular, the consequences of rotational symmetry are described in detail, and we are soon led to the general theory of Lie groups. The isospin group, hypercharge, and SU(3) symmetry and its application in modern elementary-particle physics are broadly outlined. Essential mathematical theorems are first quoted without proof and heuristically illustrated to show their importance and meaning. The proof can then be found in detailed examples and worked-out exercises.

A mathematical supplement on root vectors and classical Lie algebras deepens the material, the Young-tableaux technique is broadly outlined, and, by way of a chapter on group characters and another on charm, we lead up to very modern questions of physics. Chapters on special discrete symmetries and dynamical symmetries round off these lectures. These are all themes which fascinate young physicists, because they show them that as early as the fifth semester they can properly address and discuss questions of frontier research.

Many students and collaborators have helped during the years to work out examples and exercises. For this first English edition we enjoyed the help of Maria Berenguer, Snježana Butorac, Christian Derreth, Dr. Klaus Geiger, Dr. Matthias Grabiak, Carsten Greiner, Christoph Hartnack, Dr. Richard Herrmann, Raffael Matiello, Dieter Neubauer, Jochen Rau, Wolfgang Renner, Dirk Rischke, Thomas Schönfeld, and Dr. Stefan Schramm. Miss Astrid Steidl drew the graphs and prepared the figures. To all of them we express our sincere thanks. We are also grateful to Dr. K. Langanke and Mr. R. Könning of the Physics Department of the University in Münster for their valuable comments on the German edition.

We would especially like to thank Mr. Béla Waldhauser, Dipl.-Phys., for his overall assistance. His organizational talent and his advice in technical matters are very much appreciated.

Finally, we wish to thank Springer-Verlag; in particular, Dr. H.-U. Daniel, for his encouragement and patience, and Mr. Michael Edmeades, for expertly copy-editing the English edition.

Frankfurt am Main
July 1989

Walter Greiner
Berndt Müller

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1. Symmetries in Quantum Mechanics

1.1 Symmetries in Classical Physics

Symmetries play a fundamental role in physics, and knowledge of their presence in certain problems often simplifies the solution considerably. We illustrate this with the help of three important examples.

a) Homogeneity of Space. We assume space to be *homogeneous*, i.e. of equal structure at all positions $\boldsymbol{r}$. This is synonymous with the assumption that space is invariant under translations, because in this case the area surrounding any point can be mapped exactly by a translation from a similar area surrounding an arbitrary point (Fig. 1.1). This "translation invariance" implies the conservation of momentum for an isolated system. Here we define homogeneity of space to mean that the Lagrange function $L(\boldsymbol{r}_i, \dot{\boldsymbol{r}}_i, t)$ of a system of particles remains invariant if $\boldsymbol{r}_i$ is replaced by $\boldsymbol{r}_i + \boldsymbol{a}$, where $\boldsymbol{a}$ is an arbitrary constant vector. (A more general concept of "homogeneity of space" would require only the invariance of the equations of motion under spatial translations. In this case a conserved quantity can also be shown to exist, but it is not necessarily the canonical momentum. See Exercises 1.3 and 1.5 for a detailed discussion of this aspect.) Thus

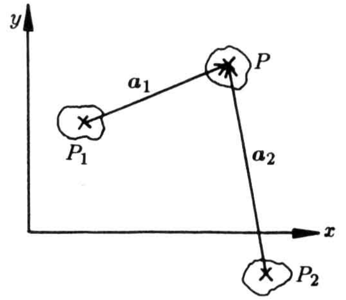

Fig. 1.1. Homogeneity or translational invariance of space means that the area around P follows from that of any other arbitrary point (e.g. $P_1, P_2, \ldots$) by translations ($\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots$)

$$\delta L = \sum_i \frac{\partial L}{\partial \boldsymbol{r}_i} \cdot \delta \boldsymbol{r}_i = \boldsymbol{a} \cdot \sum_i \frac{\partial L}{\partial \boldsymbol{r}_i} = 0 \tag{1.1}$$

must be valid. Since $\boldsymbol{a}$ is arbitrary this implies

$$\sum_i \frac{\partial L}{\partial \boldsymbol{r}_i} = 0 = \left\{ \sum_i \frac{\partial L}{\partial x_i}, \ \sum_i \frac{\partial L}{\partial y_i}, \ \sum_i \frac{\partial L}{\partial z_i} \right\} \ . \tag{1.2}$$

Here we have abbreviated

$$\frac{\partial L}{\partial \boldsymbol{r}_i} = \left\{ \frac{\partial L}{\partial x_i}, \frac{\partial L}{\partial y_i}, \frac{\partial L}{\partial z_i} \right\} \ ,$$

the gradient of L with respect to $\boldsymbol{r}_i$. From the Euler-Lagrange equations

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} = 0 \ , \quad \text{etc.}$$

it follows immediately with (1.2) that

$$\frac{d}{dt} \sum_i \frac{\partial L}{\partial \dot{x}_i} = \frac{d}{dt} P_x = 0 \quad , \quad \text{thus} \quad P_x = \text{const.} \quad ,$$

where P_x is the x component of the total momentum

$$P = \left\{ \sum_i P_{x_i}, \ \sum_i P_{y_i}, \ \sum_i P_{z_i} \right\} = \sum_i \boldsymbol{P}_i \quad . \tag{1.3}$$

This is the *law of momentum conservation in classical mechanics*. In nonrelativistic physics, it allows for the definition of a centre of mass. This is due to the fact that this law holds in all inertial systems, because in all of these space is homogeneous. Let $\boldsymbol{P} = \sum_i m_i \boldsymbol{v}_i$ denote the total momentum in the system K. Then in the system, K', which moves with the velocity $\boldsymbol{v}$ with respect to K, it is given by

$$\boldsymbol{P}' = \sum_i m_i \boldsymbol{v}_i' = \sum_i m_i(\boldsymbol{v}_i - \boldsymbol{v}) = \boldsymbol{P} - \boldsymbol{v}\sum_i m_i \quad ,$$

because nonrelativistically $\boldsymbol{v}_i' = \boldsymbol{v}_i - \boldsymbol{v}$. The centre-of-mass system is defined by the condition that the total momentum $\boldsymbol{P}'$ vanishes. In K it moves with the velocity

$$\begin{aligned} \boldsymbol{v}_\mathrm{s} &= \frac{\boldsymbol{P}}{\sum_i m_i} = \left(\sum_i m_i \boldsymbol{v}_i\right) \Big/ \left(\sum_i m_i\right) \\ &= \left(\sum_i m_i \frac{d\boldsymbol{r}_i}{dt}\right) \Big/ \left(\sum_i m_i\right) = \frac{d}{dt}\left[\left(\sum_i m_i \boldsymbol{r}_i'\right) \Big/ \left(\sum_i m_i\right)\right] \equiv \frac{d\boldsymbol{R}}{dt} \quad , \end{aligned} \tag{1.4}$$

where

$$\boldsymbol{R} = \left(\sum_i m_i \boldsymbol{r}_i'\right) \Big/ \left(\sum_i m_i\right) \tag{1.5}$$

is the coordinate of the nonrelativistic centre of mass.

b) Homogeneity of Time. The homogeneity of time has no less importance than the homogeneity of space. It stands for the invariance of the laws of nature in isolated systems with respect to translations in time, i.e. at time $t+t_0$ they have the same form as at time t. This is expressed mathematically by the fact that the Lagrange function does not depend on time explicitly, i.e.

$$L = L(q_i, \dot{q}_i) \quad . \tag{1.6}$$

Then it follows that

$$\frac{dL}{dt} = \sum_i \frac{\partial L}{\partial q_i}\dot{q}_i + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i \quad . \tag{1.7}$$

[Note that if L depends explicitly on time, the term $\partial L/\partial t$ has to be added on the right-hand side (rhs) of (1.7).]

Making use of the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

one finds

$$\frac{dL}{dt} = \sum_i \dot{q}_i \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} + \sum_i \frac{\partial L}{\partial \dot{q}_i}\ddot{q}_i = \sum_i \frac{d}{dt}\left(\dot{q}_i \frac{\partial L}{\partial \dot{q}_i}\right) \quad ,$$

or