Major American Universities Ph. D. Qualifying Questions and Solutions

Problems and Solutions on Optics

Compiled by:
The Physics Coaching Class
University of Science and Technology of China

Refereed by:
Bai Gui-ru, Guo Guang-can

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PREFACE

This series of physics problems and solutions, which consists of seven parts — Mechanics, Electromagnetism, Optics, Atomic, Nuclear and Particle Physics, Thermodynamics and Statistical Physics, Quantum Mechanics, Solid State Physics — contains a selection of 2550 problems from the graduate school entrance and qualifying examination papers of seven U. S. universities — California University Berkeley Campus, Columbia University, Chicago University, Massachusetts Institute of Technology, New York State University Buffalo Campus, Princeton University, Wisconsin University — as well as the CUSPEA and C. C. Ting's papers for selection of Chinese students for further studies in U.S.A. and their solutions which represent the effort of more than 70 Chinese physicists.

The series is remarkable for its comprehensive coverage. In each area the problems span a wide spectrum of topics while many problems overlap several areas. The problems themselves are remarkable for their versatility in applying the physical laws and principles, their uptodate realistic situations, and their scanty demand on mathematical skills. Many of the problems involve order of magnitude calculations which one often requires in an experimental situation for estimating a quantity from a simple model. In short, the exercises blend together the objectives of enhancement of one's understanding of the physical principles and practical applicability.

The solutions as presented generally just provide a guidance to solving the probelms rather than step by step manipulation and leave much to the student to work out for him/herself, of whom much is demanded of the basic knowledge in physics. Thus the series would provide an invaluable complement to the textbooks.

In editing no attempt has been made to unify the physical terms and symbols. Rather, they are left to the setters' and solvers' own preference so as to reflect the realistic situation of the usage today.

The present volume for Optics consists of three parts: Geometrical Optics, Physical Optics, Quantum Optics, and comprises 160 problems.

Lim Yung-kuo
Editor

INTRODUCTION

Solving problems in school work is the exercise of mental faculties, and examination problems are usually the pick of the problems in school work. Working out problems is a necessary and important aspect of the learning of Physics.

The *American University Ph. D. Qualifying Questions and Solutions* is a series of seven volumes. The subjects of the volumes and their respective referees (in parentheses) are as follows:

1. Mechanics (Qiang Yuan-qi, Gu En-pu, Cheng Jia-fu, Li Ze-hua, Yang De-tian)
2. Electromagnetism (Zhao Shu-ping, You Jun-han, Zhu Jun-jie)
3. Optics (Bai Gui-ru, Guo Guang-can)
4. Atomic, Nuclear and Particle Physics (Jin Huai-cheng, Yang Bao-zhong, Fan Yang-mei)
5. Thermodynamics and Statistical Physics (Zheng Jiu-ren)
6. Quantum Mechanics (Zhang Yong-de, Zhu Dong-pei, Fan Hong-yi)
7. Solid State Physics and Comprehensive Topics (Zhang Jia-lü, Zhou You-yuan, Zhang Shi-ling)

These books cover almost all aspects of university physics and contain 2550 problems, most of which are solved in detail.

These problems have been carefully chosen from a collection of 3100 problems some of which came from the China-U.S.A. Physics Examination and Application Programme and Ph.D. Qualifying Examination on Experimental High Energy Physics sponsored by Chao Chong Ting, while the others from the graduate preliminary or qualifying examination questions of the following seven top American universities during the last decade: Columbia University, University of California at Berkeley, Massachusetts Institute of Technology, University of Wisconsin, University of Chicago, Princeton University, State University of New York at Buffalo.

In general, examination problems on physics in American universities do not involve too much mathematics. Rather, they can be categorized into the following three types. Many of the problems that involve the various frontier subjects and overlapping domains of science have been selected by the professors directly from their own research and show a "modern style". Some of the problems involve a wide field and require a quick mind to analyse, while the others are often simple to solve but are practical and require a full "touch of physics." We think it reasonable to take these problems as a reflection, to some extent, of the characteristics of American science and culture, as well as the tenet of American education.

This being so, we believe it worthwhile to collect and solve these problems and then introduce them to the students and teachers, even though the effort involved is formidable. Nearly a hundred teachers and graduate students took part in this time-consuming task.

There are 160 problems in this volume, which is divided into three parts: part I consists of 41 problems in geometric optics, part II consists of 89 problems in wave optics, part III consists of 30 problems in quantum optics.

The depth of knowledge involved in solving these problems is not beyond the contents of common textbooks on optics used in colleges and universities in China, although the scope of the knowledge and techniques needed in solving some of the problems go beyond what we are usually familiar with. Furthermore, some new scientific research results (e.g. some newly developed lasers) are introduced in the problems. This will not only enhance the understanding of the established theories and knowledge, but also encourage the interaction between teaching and research which cannot but enliven academic thoughts and excite the mind.

The physicists who contributed to solving the problems in this volume are Shi De-xiu, Yao Kun, Lu Hong-jun, Chen Xiang-li, Gu Chun, Han Wen-hai and Wu Zhi-qiang. The initial translation from Chinese into English was carried out by Xuan Zhi-hua. Some revisions have been made in this English edition by the compilers, the translator and the editor.

CONTENTS

PART 1 GEOMETRICAL OPTICS

1001

A rainbow is produced by:

(a) refraction of sunlight by water droplets in the atmosphere.

(b) reflection of sunlight by clouds.

(c) refraction of sunlight in the human eye.

(*CCT*)

Solution:

The answer is (a).

1002

A horizontal ray of light passes through a prism of index 1.50 and apex angle 4° and then strikes a vertical mirror, as shown in the figure. Through what angle must the mirror be rotated if after reflection the ray is to be horizontal?

(*Wisconsin*)

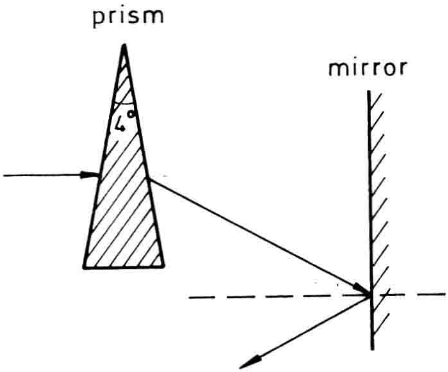

Fig. 1.1

Solution:

As the apex angle is very small ($\alpha = 4°$), the angle of deviation δ can be obtained approximately:

$$\delta = (n-1)\alpha = (1.5-1) \times 4° = 2° \ .$$

From Fig. 1.2 we see that if the reflected ray is to be horizontal, the mirror must be rotated clockwise through an angle γ given by

$$\gamma = \frac{\delta}{2} = 1° \ .$$

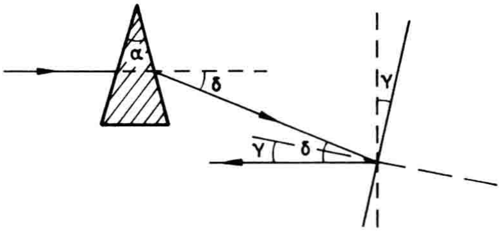

Fig. 1.2

1003

A narrow beam of light is incident on a $30^\circ - 60^\circ - 90^\circ$ prism as shown (Fig. 1.3). The index of refraction of the prism is $n = 2.1$. Show that the entire beam emerges either from the right-hand face, or back along the incident path.

(*Wisconsin*)

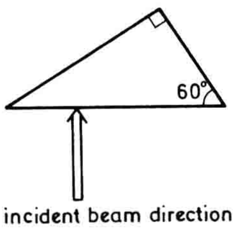

Fig. 1.3

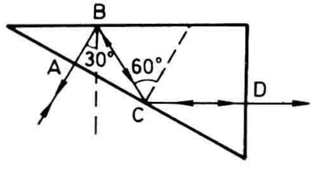

Fig. 1.4

Solution:

As seen from Fig. 1.4, for normal incidence at the bottom face the angle of incidence at B is 30°, and that at C is 60°, both of which are larger than the critical angle of the prism,

$$\theta_c = \sin^{-1}\frac{1}{n} = 28^\circ 26' .$$

Hence the ray is totally reflected at B and C. Also, the ray is partially reflected back at the bottom and the right-hand faces. Therefore, the entire beam emerges either from the right-hand face, or back along the incident path.

1004

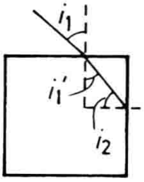

Fig. 1.5

A glass cube has a refractive index of 1.5. A light beam enters the top face obliquely and then strikes the side of the cube. Does light emerge from this side? Explain your answer.

(*Wisconsin*)

Solution:

Assume that the angles of incidence and refraction at the top face are i_1 and i_1' respectively. According to Snell's law of refraction,

$$\sin i_1 = n \sin i_1' \ .$$

From the geometry (see Fig. 1.5) $i_1' + i_2 = 90°$, where i_2 is the angle of incidence at the right side. Thus, we have

$$\sin i_1 = n \cos i_2 \ ,$$

or

$$i_2 = \cos^{-1}\left(\frac{\sin i_1}{n}\right) \ .$$

When $i_1 = 90°$, i_2 has the minimum value

$$i_2 = \cos^{-1}\frac{1}{1.5} = 48°10' > i_c = 42° \ \text{(critical angle)} \ .$$

Hence no light emerges from this side.

1005

A glass rod of rectangular cross-section is bent into the shape shown in the Fig. 1.6. A parallel beam of light falls perpendicularly on the flat surface A. Determine the minimum value of the ratio R/d for which all light entering the glass through surface A will emerge from the glass through surface B. The index of refraction of the glass is 1.5.

(*Wisconsin*)

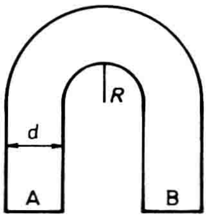

Fig. 1.6

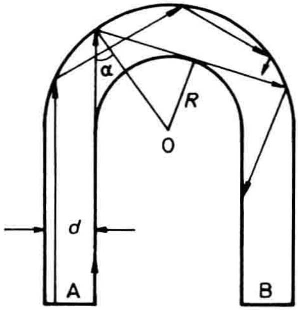

Fig. 1.7

Solution:

Consider the representative rays shown in Fig. 1.7. A ray entering the glass through surface A and passing along the inner side of the rod will be reflected by the outer side with the smallest angle α, at which the reflected ray is tangent to the inner side. We have to consider the conditions under which the ray will undergo total internal reflection before reaching B.

If $\alpha > \theta_c$, the critical angle, at which total internal reflection occurs, all the incident beam will emerge through the surface B. Hence we require

$$\sin\alpha > \frac{1}{n} \, .$$

The geometry gives

$$\sin\alpha = \frac{R}{(R+d)} \, .$$

Therefore

$$\frac{R}{R+d} \geq \frac{1}{n} \, ,$$

or

$$\left(\frac{R}{d}\right)_{\min} = \frac{1}{n-1} = \frac{1}{1.5-1} = 2 \, .$$

1006

A small fish, four feet below the surface of Lake Mendota is viewed through a simple thin converging lens with focal length 30 feet. If the lens is 2 feet above the water surface (Fig. 1.8), where is the image of the fish seen by the observer? Assume the fish lies on the optical axis of the lens and that $n_{\text{air}} = 1.0, n_{\text{water}} = 1.33$.

(*Wisconsin*)

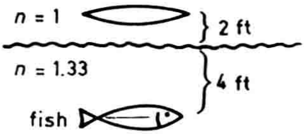

Fig. 1.8

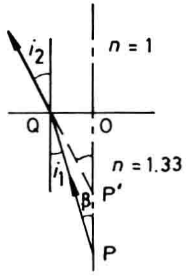

Fig. 1.9

Solution:

An object at P in water appears to be at P′ as seen by an observer in air, as Fig. 1.9 shows. The paraxial light emitted by P is refracted at the water surface, for which

$$1.33 \sin i_1 = \sin i_2 \ .$$

As i_1, i_2 are very small, we have the approximation $1.33 i_1 = i_2$. Also,

$$i_2 = \alpha \approx \frac{\overline{\text{OQ}}}{\overline{\text{OP}'}} , \quad i_1 = \beta \approx \frac{\overline{\text{OQ}}}{\overline{\text{QP}}} \ .$$

Hence, we have

$$\overline{\text{OP}'} = \frac{1}{1.33} \cdot \overline{\text{OP}} = 3 \text{ ft} \ .$$

Let the distance between the apparent location of the fish and the center of the lens be u, then

$$u = 2 + \overline{\text{OP}'} = 5 \text{ ft} \ .$$

From $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, we have

$$v = -6 \text{ ft} \ .$$

Therefore, the image of the fish is still where the fish is, four feet below the water surface.

1007

The index of refraction of glass can be increased by diffusing in impurities. It is then possible to make a lens of constant thickness. Given a disk of radius a and thickness d, find the radial variation of the index of refraction $n(r)$ which will produce a lens with focal length F. You may assume a thin lens ($d \ll a$).

(*Chicago*)

Solution:

Let the refractive index of the material of the disk be n and the radial distribution of the refractive index of the impurity-diffused disk be represented by $n(r)$, with $n(0) = n_0$. Incident plane waves entering the lens refract and converge at the focus F as shown in Fig. 1.10. We have

$$[n(r) - n_0]d = -\sqrt{F^2 + r^2} + F \ ,$$

i.e.,

$$n(r) = n_0 - \frac{\sqrt{F^2 + r^2} - F}{d} \ .$$

For $F \gg r$, we obtain

$$n(r) = n_0 - \frac{r^2}{2dF} \ .$$

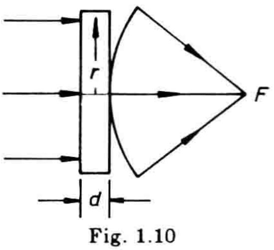

Fig. 1.10

1008

The index of refraction of air at 300 K and 1 atmosphere pressure is 1.0003 in the middle of the visible spectrum. Assuming an isothermal atmosphere at 300 K, calculate by what factor the earth's atmosphere would have to be more dense to cause light to bend around the earth with the earth's curvature at sea level. (In cloudless skies we could then watch sunset all night, in principle, but with an image of the sun drastically compressed vertically.) You may assume that the index of refraction n has the property that $n-1$ is proportional to the density. (Hint: Think of Fermat's Principle.) The $1/e$ height of this isothermal atmosphere is 8700 metres.

(UC, Berkeley)

Solution:

We are given that

$$n(r)-1=\rho e^{-\frac{r-R}{8700}} ,$$

where $R=6400\times 10^3$ m is the earth's radius and ρ is the density coefficient of air. Then

$$n(r)=1+\rho e^{-\frac{r-R}{8700}} , \tag{1}$$

$$\frac{dn(r)}{dr}=n'(r)=-\frac{1}{8700}\rho e^{-\frac{r-R}{8700}} . \tag{2}$$

It is also given that air is so dense that it makes light bend around the earth with the earth's curvature at sea level, as shown in Fig. 1.11.

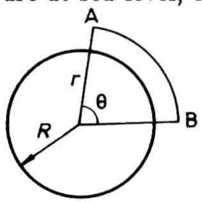

Fig. 1.11

The optical path length from A to B is

$$l=n(r)r\theta .$$

According to Fermat's Principle, the optical path length from A to B should be an extremum. Therefore,

$$\frac{dl}{dr}=[n'(r)r+n(r)]\theta=0 ,$$

i.e.,

$$n'(r)=\frac{-n(r)}{r} . \tag{3}$$

Substituting (3) into (2) yields

$$\frac{1}{8700}\rho e^{-\frac{r-R}{8700}}=\frac{n(r)}{r} . \tag{4}$$