m.g. singh

dynamical hierarchical control

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DYNAMICAL HIERARCHICAL CONTROL

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DYNAMICAL HIERARCHICAL CONTROL

To Beeba,
Christine
and my parents

A C K N O W L E D G E M E N T S

This book would not have been possible without the help of a number of individuals and organisations. I am fortunate in having had the opportunity to work with some highly imaginative scientists both at the Laboratoire d'Automatique et d'Analyse des Systèmes (LAAS) of the CNRS in Toulouse and at the Control Engineering Laboratory of the University of Cambridge. I have benefited particularly from discussions with Professor André TITLI, Mr. M. HASSAN, Dr. M. RICHETIN and Dr. J.L. CALVET of Toulouse, Professor J.F. COALES and Dr. A.T. FULLER of Cambridge and Professor H. TAMURA of the University of Osaka in Japan.

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* Currently "Directeur Scientifique" of the C.N.R.S.

≠ Currently Director of the L.A.A.S.

P R E F A C E

Although Large Systems Theory has been actively studied for over a decade with pioneering work by Mesarovic, Macko, Lasdon, Lefkowitz, Pearson, Takahara, Titli, Tamura and others, the theory at least for large interconnected dynamical systems has been largely inaccessible to the Engineers concerned with the design of hierarchies for the solution of practical problems. There are two main reasons for this, the first being the somewhat abstract nature of earlier works whilst the second is the fact that the work is described in papers which are relatively scattered in the literature. In this book an attempt is made to bring together the practically applicable techniques which can be used for synthesising hierarchical structures for the control and optimisation of large interconnected dynamical systems. The book is aimed at a wide audience ranging from practising Engineers to Graduate Students in Systems Engineering.

The bulk of the book provides a set of design techniques for synthesising hierarchical structures but before starting with the mathematical treatment, Chapter I deals with a number of fundamental questions ; what are hierarchies ? how do they arise "naturally" in certain systems and is it desirable to generate them to solve problems of control and optimisation for large dynamical systems ? The rest of the book is divided into 4 parts each consisting of one or two chapters. In Part I the techniques for hierarchical optimisation yielding open loop control are developed for both linear and non-linear systems. Some of these techniques are extended in Part II to provide closed loop control.

Whilst Part I and II deal with methods which yield optimal control, in Part III, some suboptimal methods are considered which utilise the structural peculiarities of certain systems to provide near optimal control. The ideas are illustrated on many practical problems including a detailed study of a hot steel rolling mill. Finally, in Part IV, the practical case of systems subject to stochastic disturbances is considered.

To emphasise the practical utility of the methods developed in the book, simulation studies are described of the application of the methods to practical problems taken from diverse fields ranging from Traffic Control Systems and Environmental Systems to more traditional Engineering Systems.

In the main text, it is assumed that the reader is reasonably familiar with standard single level functional optimisation techniques such as dynamic programming and Pontryagin's Maximum Principle as well as the Theory of Lagrange Duality, although in the appendices, a brief overview of these techniques is given for the sake of completeness.

Madan G. SINGH

Toulouse, December, 1976

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CHAPTER 1

INTRODUCTION

In this monograph we describe the "state of the art" of hierarchical control for large scale interconnected dynamical systems. To provide a basis for the discussion of the synthesis procedures for generating hierarchical structures which forms the bulk of this book, we examine in this chapter a number of fundamental questions : What are hierarchical structures ? Why are they so common ? Are there any specific reasons why they are more (or less) desirable than other structures ? Can such structures be generated artificially for solving problems of optimisation and control in large scale systems and is it advantageous to do so ? Although the discussion in this chapter is somewhat heuristic, it is tacitly assumed in subsequent chapters that hierarchical structures are indeed useful structures to synthesise and the book attempts to provide concrete synthesis procedures in the context of practical problems in the fields of Transportation Systems, Environmental Systems and more traditional Engineering Systems.

1. WHAT IS A HIERARCHY ?

We are all familiar with hierarchies since they pervade so much of our daily lives. But what characterises them ? The simplest possible hierarchy has two levels as shown in Fig. 1. Such a hierarchy could arise for example in an office where the first level consists of the staff members doing their own particular jobs and the supervisor (or coordinator) ensuring that the work load on each staff member is acceptable and that the overall objective of the office is fulfilled according to a suitable schedule. This schedule itself is determined through an

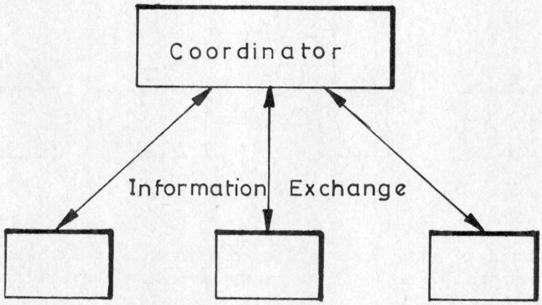

Fig. 1. : A typical two level hierarchy

iterative information exchange in which the coordinator defines a target for each staff member taking into account the interactions with the others whilst the staff member lets him know whether he can fulfil the target or not. If he cannot, the supervisor suggests an improved target taking into account those of the other workers and this process continues until an acceptable target is achieved for each of the staff members and which also fulfils the target of the office. It should be noted that whereas the higher level is interested in a longer term fulfilling of targets, the lower level is concerned with the more day to day running of the office.

Having described in a very rudimentary form a particularly simple hierarchy, let us now try to see if there are any fundamental properties which this hierarchy possesses and which it perhaps has in common with other hierarchies. The following properties might be proposed :

1. Hierarchies consist of decision making units arranged in a pyramid where at each level, a number of such units operate in parallel. Fig. 2 shows the pyramid structure.

2. Hierarchical structures exist in systems which have an overall goal and the goals of all the decision makers who constitute the hierarchy are in harmony.

3. There is an iterative information exchange between the decision making units on the various levels of the hierarchy with a precedence for the information going down which is treated as a command by the lower levels which try to obey it if they possibly can.

4. The time horizon of interest increases as one goes up the hierarchy.

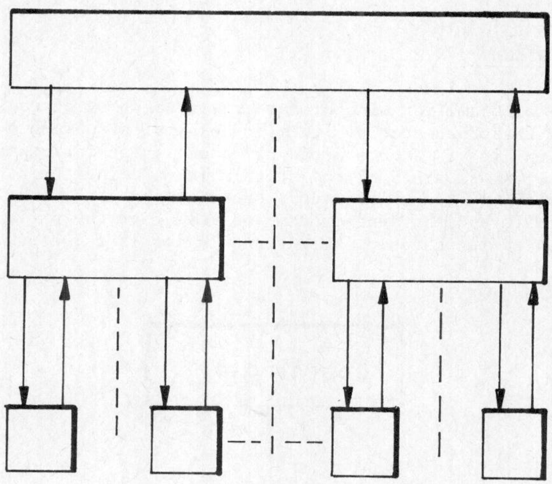

Fig. 2 : A Pyramid Structure

If we go back to our simple office hierarchy, it is clear that these properties are fundamental to it ; it has a pyramid structure (Fig. 1), an overall goal (efficiency perhaps), iterative information exchange (between staff and supervisor) and increasing time horizon of interest as one goes up the hierarchy.

Bearing these properties in mind, let us move on from our simple two level hierarchy to a more complex one : a general production process as shown in Fig. 3. Consider a company producing one industrial product. The objective of the system is perhaps the maximisation of profit or the production of a high quality product or the welfare of the workers or the welfare of the managers or some combination of these factors. Let us start at the top of the production hierarchy. This level (Top Management) decides overall policy by examining market forecasts and defining an appropriate production schedule to meet the forecasted demand.

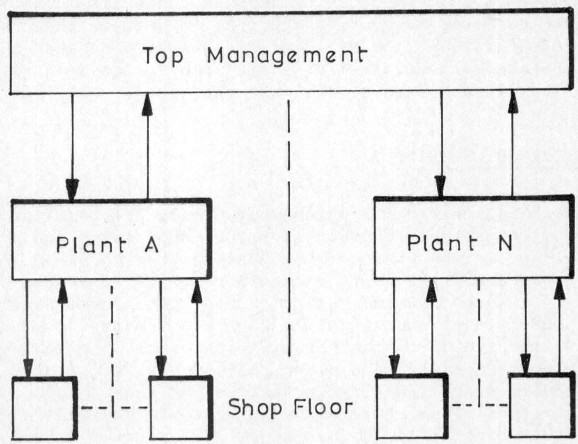

Fig. 3 : A Production Hierarchy

The top level schedule is thus a fairly coarse one since it merely fixes output targets. The next stage is to find out if, given the resources allocated by the Top Management, it is possible actually to satisfy these targets. This is clearly an iterative process where the managers of the various plants which actually produce the product (level II) receive information about their individual plants' inputs and send back estimates as to whether they can satisfy their part of the schedule. This information exchange is iterative since the individual plant managers are each concerned with their own plant only and it is the job of the Top Management to coordinate the flow between the various plants. Top Management does this by sending down orders for each plant and the plant manager indicates if the plant can carry them out or not given its own constraints. If it cannot do so, the plant manager tells Top Management the degree of imbalance between what it can achieve and what is required of it. This information is received from each plant, collated together and used to modify the orders so as to reduce successively the imbalance for each of the plant managers.* Note that although the Top Management has a long time horizon (say a year), the plant managers operate on a much more day to day basis since unknown disturbances can affect their schedules and require further iterative exchanges between this level and a level below it which is actually concerned with the fulfilling of the local managements' targets. It is possible to go down in this way to the shop floor where the actual production takes place.

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* It should be noted that we are considering somewhat idealised hierarchies where there is no real conflict between the goals of the subsystems and the overall systems goal, i.e. they operate as a team. It is true that in real human hierarchies conflicts often do arise in which case much of the analysis of this book would have to be reconsidered. It would be interesting to analyse these real life situations by using a combination of game theoretic concepts to take into account the conflict in goals and team theoretic for the overall system. However, such a study is outside the scope of the present work.

If we go back to the four properties which characterised our simple hierarchy, we can see that these are also to be found in this much more complex example. However, there is one very important difference between our office hierarchy and the production process just defined ; in our first example, it was not absolutely clear why there should be a hierarchy since the job of the supervisor could have been performed by communication amongst the staff members. As the number of staff members increases however, communication between them becomes much more difficult and less efficient,and it is when the system reaches a certain size that a coordinator becomes essential (i.e. a hierarchy arises) if efficiency is to be maintained. This leads us on to the next question :

2. WHY DO HIERARCHIES EXIST ?

The hierarchies with which most people are familiar are found primarily in socioeconomic systems. In order to try to understand the circumstances under which they arise, let us consider the case of five people shipwrecked on a desert island. Is a hierarchy likely to emerge here≠ ? If we assume that the goal of the five men is to ensure that the maximum number among them survive, then we have a common goal which is of course one of the requirements for the existence of a hierarchy. If we then go on to the organisation of the five, however, it may well be useful for them to operate in a decentralised way, in parallel with each other*, but it is not necessarily desirable that one of them should do nothing but coordinate the other four. Thus here, although we could envisage a decentralised structure, the actual control could just as well be carried out if they worked as a team with each member either using some fixed strategy or communicating with all the other members. To understand what we mean by a fixed strategy, suppose that man 1 has been given the job of fishing. The team knows that they can consume n fish per day and that fish left over at the end of the day will go bad. So his fixed strategy could be to stop when n fish have been caught. Similar fixed strategies could be generated for the other men. If a sufficiently good fixed strategy could be developed, there is no reason why the whole system should not be able to operate as a team even if the number of people became much larger. Except in very special cases, however, successful fixed strategies are very difficult to generate in real life situations and communication with other members of the group generally becomes necessary. Here we come to the nub of the matter. As the number of decision makers becomes large, it becomes progressively more inefficient to transmit information between them for coordination and it becomes necessary to have a specialist coordination function, i.e. if our number of survivors became say 100, it might be much more efficient for m of them to perform the various tasks necessary for survival in parallel and the remaining (100-m) of them to coordinate their activities as opposed to all one hundred of them communicating with each other all the time. Thus the main reason why hierarchies arise is that decentralised systems having a particular goal are much too complex for one decision maker to control and it is too inefficient for parallel decision makers to communicate their interactions to each other.

Summing up these ideas, the reason why hierarchies arise are :

(a) The system having a definite goal is too complex for one decision maker to comprehend let alone control since decision makers have limited information handling capacities

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* Since time flows sequentially, it is possible for them to do many more jobs in a given period of time if they do them in parallel.

≠ The author is grateful to Prof. Murray WONHAM of the University of Toronto for posing this problem.