Lecture Notes in Mathematics

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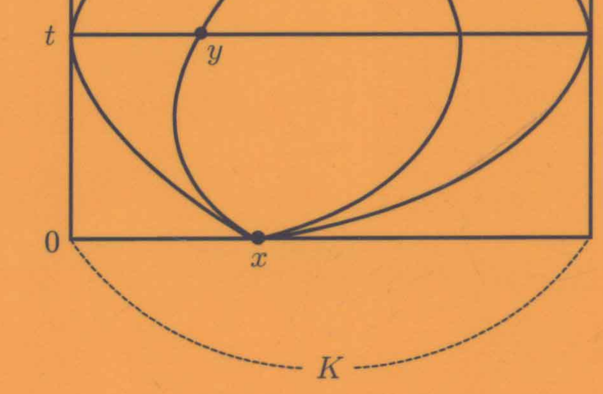
Kazuaki Taira

Boundary Value Problems and Markov Processes

2nd Edition



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Boundary Value Problems and Markov Processes

Second Edition

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To the memory of
Professor Kiyosi Itô
(1915–2008)

Preface to the Second Edition

This monograph is an expanded and revised version of a set of lecture notes for the graduate courses given by the author both at Hiroshima University (1995–1997) and at the University of Tsukuba (1998–2000) which were addressed to the advanced undergraduates and beginning-graduate students with interest in functional analysis, partial differential equations and probability.

The first edition of this monograph, which was based on the lecture notes given at the University of Tsukuba (1988–1990), was published in 1991. This edition was found useful by a number of people, but it went out of print after a few years.

This second edition has been revised to streamline some of the analysis and to give better coverage of important examples and applications. The errors in the first printing are corrected thanks to kind remarks of many friends. In order to make the monograph more up-to-date, additional references have been included in the bibliography.

This second edition may be considered as a short introduction to the more advanced book "*Semigroups, boundary value problems and Markov processes*" which was published in the Springer Monographs in Mathematics series in 2004. For graduate students working in functional analysis, partial differential equations and probability, it may serve as an effective introduction to these three interrelated fields of analysis. For graduate students about to major in the subject and mathematicians in the field looking for a coherent overview, it will provide a method for the analysis of elliptic boundary value problems in the framework of L^p spaces.

My special thanks go to the editorial staffs of Springer-Verlag for their unfailing helpfulness and cooperation during the production of this second edition.

This research was partially supported by Grant-in-Aid for General Scientific Research (No. 19540162), Ministry of Education, Culture, Sports, Science and Technology, Japan.

Last but not least, I owe a great debt of gratitude to my family who gave me moral support during the preparation of this book.

Tsukuba, Kazuaki Taira
March 2009

Preface to the First Edition

This monograph is devoted to the functional analytic approach to a class of *degenerate* boundary value problems for second-order elliptic differential operators which includes as particular cases the Dirichlet and Neumann problems. We prove that this class of boundary value problems provides a new example of *analytic semigroups* both in the L^p topology and in the topology of uniform convergence. As an application, we show that there exists a strong *Markov process* corresponding to such a diffusion phenomenon that either absorption or reflection phenomenon occurs at each point of the boundary. Furthermore, we study a class of initial-boundary value problems for *semilinear* parabolic differential equations.

This monograph is an expanded version of a set of lecture notes for the graduate courses given by the author at the University of Tsukuba between 1988 and 1990. We confined ourselves to the simple boundary condition. This makes it possible to develop our basic machinery with a minimum of bother and the principal ideas can be presented concretely and explicitly. I hope that this monograph will lead to a better insight into the study of three interrelated subjects: elliptic boundary value problems, analytic semigroups and Markov processes. For additional information on many of the topics discussed here, I would like to call attention to my previous book *Diffusion Processes and Partial Differential Equations*, Academic Press, 1988.

I would like to express my hearty thanks to many colleagues and graduate students in my courses, whose helpful criticisms of my lectures resulted in a number of improvements. The manuscript is typeset in a camera-ready form using AMS-TEX.

Tsukuba,
April 1991

Kazuaki Taira

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Edited by J.-M. Morel, F. Takens, B. Teissier, P.K. Maini

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Contents

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Introduction and Main Results

In this introductory chapter, our problems and results are stated in such a fashion that a broad spectrum of readers could understand, and also described how these problems can be solved, using the mathematics presented in Chapters 2 through 4.

In 1828, the English botanist R. Brown observed that pollen grains suspended in water move chaotically, incessantly changing their direction of motion. The physical explanation of this phenomenon is that a single grain suffers innumerable collisions with the randomly moving molecules of the surrounding water. A mathematical theory for Brownian motion was put forward by A. Einstein in 1905 ([Ei]). Einstein derived an accurate method of measuring Avogadro's number by observing particles undergoing Brownian motion. Einstein's theory was experimentally tested by J. Perrin between 1906 and 1909.

Brownian motion was put on a firm mathematical foundation for the first time by N. Wiener in 1923 ([Wi]). Wiener characterized the "starting afresh" property of Brownian motion that if a Brownian particle reaches a position, then it behaves subsequently as though that position had been its initial position.

Markov processes are an abstraction of the idea of Brownian motion. In the first works devoted to Markov processes, the most fundamental was A. N. Kolmogorov's work in 1931 ([Ko]) where the general concept of a Markov transition function was introduced for the first time and an analytic method of describing Markov transition functions was proposed. From the viewpoint of analysis, the transition function is something more convenient than the Markov process itself. In fact, it can be shown that the transition functions of Markov processes generate solutions of certain parabolic partial differential equations such as the classical diffusion equation; and, conversely, these differential equations can be used to construct and study the transition functions and the Markov processes themselves.

In the 1950s, the theory of Markov processes entered a new period of intensive development. We can associate with each transition function in a natural

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way a family of bounded linear operators acting on the space of continuous functions on the state space, and the Markov property implies that this family forms a semigroup. The Hille–Yosida theory of semigroups in functional analysis made possible further progress in the study of Markov processes. The semigroup approach to Markov processes can be traced back to the pioneering work of Feller in early 1950s ([Fe1], [Fe2]).

Now let D be a bounded domain of Euclidean space $\mathbf{R}^N$, with smooth boundary ∂D; its closure $\overline{D} = D \cup \partial D$ is an N-dimensional, compact smooth manifold with boundary. We let

$$A = \sum_{i,j=1}^{N} a^{ij}(x) \frac{\partial^2}{\partial x_i \partial x_j} + \sum_{i=1}^{N} b^i(x) \frac{\partial}{\partial x_i} + c(x)$$

be a second-order, *elliptic* differential operator with real smooth coefficients on $\overline{D}$ such that:

(1) $a^{ij}(x) = a^{ji}(x)$ for all $x \in \overline{D}$ and $1 \leq i, j \leq N$.
(2) There exists a positive constant a_0 such that

$$\sum_{i,j=1}^{N} a^{ij}(x)\xi_i\xi_j \geq a_0|\xi|^2 \quad \text{for all } (x, \xi) \in \overline{D} \times \mathbf{R}^N.$$

(3) $c(x) \leq 0$ on $\overline{D}$.

We consider the following boundary value problem with spectral parameter: Given functions $f(x)$ and $\varphi(x')$ defined in D and on ∂D, respectively, find a function $u(x)$ in D such that

$$\begin{cases} (A - \lambda)u = f & \text{in } D, \\ Lu = \mu(x')\frac{\partial u}{\partial \mathbf{n}} + \gamma(x')u = \varphi & \text{on } \partial D. \end{cases} \tag{1.1}$$

Here:

(4) λ is a *complex* parameter.
(5) $\mu(x')$ and $\gamma(x')$ are real-valued, smooth functions on the boundary ∂D.
(6) $\mathbf{n} = (n_1, n_2, \ldots, n_N)$ is the unit interior normal to the boundary ∂D (see Figure 1.1).

We remark that if $\mu(x') \neq 0$ and $\gamma(x') \equiv 0$ on ∂D (resp. $\mu(x') \equiv 0$ and $\gamma(x') \neq 0$ on ∂D), then the boundary condition L is essentially the so-called Neumann (resp. Dirichlet) condition.

It is easy to see that problem (1.1) is non-degenerate (or coercive) if and only if either $\mu(x') \neq 0$ on ∂D or $\mu(x') \equiv 0$ and $\gamma(x') \neq 0$ on ∂D. The generation theorem of analytic semigroups is well established in the non-degenerate case both in the L^p topology and in the topology of uniform convergence (cf. Friedman [Fr1], Tanabe [Tn], Masuda [Ma], Stewart [Sw]).