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José Luiz Fiadeiro  
Neil Harman  
Markus Roggenbach  
Jan Rutten (Eds.)

# Algebra and Coalgebra in Computer Science

First International Conference, CALCO 2005  
Swansea, UK, September 2005  
Proceedings



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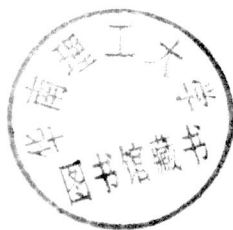
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## Preface

In April 2004, after one year of intense debate, CMCS, the International Workshop on Coalgebraic Methods in Computer Science, and WADT, the Workshop on Algebraic Development Techniques, decided to join their forces and reputations into a new high-level biennial conference. CALCO, the Conference on Algebra and Coalgebra in Computer Science, was created to bring together researchers and practitioners to exchange new results related to foundational aspects, and both traditional and emerging uses of algebras and coalgebras in computer science. A steering committee was put together by merging those of CMCS and WADT: Jiri Adamek, Ataru Nakagawa, Michel Bidoit, José Fiadeiro (co-chair), Hans-Peter Gumm, Bart Jacobs, Hans-Jörg Kreowski, Ugo Montanari, Larry Moss, Peter Mosses, Fernando Orejas, Francesco Parisi-Presicce, John Power, Horst Reichel, Markus Roggenbach, Jan Rutten (co-chair), and Andrzej Tarlecki.

CALCO 2005 was the first instance of this new conference. The interest that it generated in the scientific community suggests that it will not be the last. Indeed, it attracted as many as 62 submissions covering a wide range of topics roughly divided into two areas:

**Algebras and Coalgebras as Mathematical Objects:** Automata and languages; categorical semantics; hybrid, probabilistic, and timed systems; inductive and coinductive methods; modal logics; relational systems and term rewriting.

**Algebras and Coalgebras in Computer Science:** Abstract data types; algebraic and coalgebraic specification; calculi and models of concurrent, distributed, mobile, and context-aware computing; formal testing and quality assurance; general systems theory and computational models (chemical, biological, etc); generative programming and model-driven development; models, correctness and (re)configuration of hardware/middleware/architectures; re-engineering techniques (program transformation); semantics of conceptual modelling methods and techniques; semantics of programming languages; validation and verification.

Every submission received three or four reviews, which were generally of excellent quality. We want to thank all reviewers, the list of which is at the end of this preface, for carrying out their task with competence and precision, but also with the enthusiasm that comes from contributing to the birth of a new conference. Decisions were made during two weeks of animated e-mail discussion. In the end, a total of 25 papers were selected, the revised versions of which can be found in this volume. We were also lucky to have invited talks by three expert researchers: Samson Abramsky, Gordon Plotkin and Vladimiro Sassone. We are very grateful to the three of them.

The technical programme of the conference was preceded by a Young Researchers Workshop, CALCO-jnr, dedicated to presentations by PhD students and by those who had completed their doctoral studies in recent years, thus following on one of the traditional features of WADT. A technical report collects contributions selected from

the presentations. CALCO-jnr was organized by Peter Mosses, John Power and Monika Seisenberger. A meeting of the IFIP WG1.3 – Foundations of System Specification – took place immediately after the conference and was hosted by Peter Mosses.

The project of hosting this first edition of CALCO was seized with both hands by a young and enthusiastic team led by Neal Harman and Markus Roggenbach from the University of Wales Swansea. The organizers would like to thank John V. Tucker and Monika Seisenberger for their invaluable advice and support, and IT Wales and the support staff, especially Sue Phillips, for making the event possible. CALCO 2005 received generous contributions from the Welsh Development Agency (WDA), IFIP, BCS-FACS, Digita and IT Wales.

Alfred Hofmann and his team at Springer lent us their support from Day –1 by agreeing to publish this volume. The work of the PC was supported by the Conference Online Service; Tiziana Margaria, Bernhard Steffen and their team deserve all our applause; it was very reassuring to feel that Martin Karusseit was available 24 hours a day, 7 days a week, but the truth is that we only needed him to sort out our own silly mistakes ...

We would like to reserve our final words of thanks to all the authors who have contributed such good quality papers to CALCO 2005.

June 2005

José Luiz Fiadeiro and Jan Rutten  
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# Abstract Scalars, Loops, and Free Traced and Strongly Compact Closed Categories

Samson Abramsky

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**Abstract.** We study structures which have arisen in recent work by the present author and Bob Coecke on a categorical axiomatics for Quantum Mechanics; in particular, the notion of *strongly compact closed category*. We explain how these structures support a notion of *scalar* which allows quantitative aspects of physical theory to be expressed, and how the notion of strong compact closure emerges as a significant refinement of the more classical notion of compact closed category.

We then proceed to an extended discussion of free constructions for a sequence of progressively more complex kinds of structured category, culminating in the strongly compact closed case. The simple geometric and combinatorial ideas underlying these constructions are emphasized. We also discuss variations where a prescribed monoid of scalars can be ‘glued in’ to the free construction.

## 1 Introduction

In this preliminary section, we will discuss the background and motivation for the technical results in the main body of the paper, in a fairly wide-ranging fashion. The technical material itself should be essentially self-contained, from the level of a basic familiarity with monoidal categories (for which see e.g. [20]).

### 1.1 Background

In recent work [4,5], the present author and Bob Coecke have developed a categorical axiomatics for Quantum Mechanics, as a foundation for high-level approaches to quantum informatics: type systems, logics, and languages for quantum programming and quantum protocol specification. The central notion in our axiomatic framework is that of *strongly compact closed category*. It turns out that this rather simple and elegant structure suffices to capture most of the key notions for quantum informatics: compound systems, unitary operations, projectors, preparations of entangled states, Dirac bra-ket notation, traces, scalars, the Born rule. This axiomatic framework admits a range of models, including of course the Hilbert space formulation of quantum mechanics.

Additional evidence for the scope of the framework is provided by recent work of Selinger [25]. He shows that the framework of completely positive maps acting

on generalized states represented by density operators, used in his previous work on the semantics of quantum programming languages [24], fits perfectly into the framework of strongly compact closed categories.<sup>1</sup> He also showed that a simple construction (independently found and studied in some depth by Coecke [9]), which can be carried out completely generally at the level of strongly compact closed categories, corresponds to passing to the category of completely positive maps (and specializes exactly to this in the case of Hilbert spaces).

## 1.2 Multiplicatives and Additives

We briefly mention a wider context for these ideas. To capture the branching structure of measurements, and the flow of (classical) information from the result of a measurement to the future evolution of the quantum system, an additional *additive* level of structure is required, based on a functor  $\oplus$ , as well as the *multiplicative* level of the compact closed structure based around the tensor product (monoidal structure)  $\otimes$ . This delineation of additive and multiplicative levels of Quantum Mechanics is one of the conceptually interesting outcomes of our categorical axiomatics. (The terminology is based on that of Linear Logic [12] — of which our structures can be seen as ‘collapsed models’). In terms of ordinary algebra, the multiplicative level corresponds to the multilinear-algebraic aspect of Quantum Mechanics, and the additive level to the linear-algebraic. But this distinction is usually lost in the sea of matrices; in particular, it is a real surprise how much can be done purely with the multiplicative structure.

It should be mentioned that we fully expect an *exponential* level to become important, in the passage to the multi-particle, infinite dimensional, relativistic, and eventually field-theoretic levels of quantum theory.

We shall not discuss the additive level further in this paper. For most purposes, the additive structure can be regarded as freely generated, subject to arithmetic requirements on the scalars (see [4]).

## 1.3 Explicit Constructions of Free Structured Categories

Our main aim in the present paper is to give explicit characterizations of free constructions for various kinds of categories-with-structure, most notably, for traced symmetric monoidal and strongly compact closed categories. We aim to give a synthetic account, including some basic cases which are well known from the existing literature [20,19]. We will progressively build up structure through the following levels:

- (1) Monoidal Categories
- (2) Symmetric Monoidal Categories
- (3) Traced Symmetric Monoidal Categories

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<sup>1</sup> Selinger prefers to use the term ‘dagger compact closed category’, since the notion of adjoint which is formalized by the dagger operation  $()^\dagger$  is a separate structure which is meaningful in a more general setting.

- (4) Compact Closed Categories
- (5) Strongly Compact Closed Categories
- (6) Strongly Compact Closed Categories with prescribed scalars

Of these, those cases which have not, to the best of our knowledge, appeared previously are (3), (5) and (6). But in any event, we hope that our account will serve as a clear, accessible and useful reference.

Our constructions also serve to *factor* the Kelly-Laplaza construction [19] of the free compact closed category through the  $\mathcal{G}$  or  $\text{Int}$  construction [14,2] of the compact closed category freely generated by a traced symmetric monoidal category, which is a central part of (the mathematically civilised version of) the so-called ‘Geometry of Interaction’ [12,3].

It should be emphasized that constructions (1)–(4) are free over *categories*, (5) over categories with involutions, and (6) over a comma category of categories with involution with a specified evaluation of scalars. We note that Dusko Pavlovic has given a free construction of traced categories over *monoidal categories* [21]. His construction is elegant, but abstract and less combinatorial/geometric than ours: perhaps necessarily so, since in our situation the monoidal structure, which itself has some spatial content, is added freely. Another reference is by Katis, Sabadini and Walters [16]. They construct a free ‘feedback category’, which is a trace minus the Yanking axiom — which is very important for the dynamics of the trace — over a monoidal category, and then formally quotient it to get a traced category. A treatment in the same style as the present paper of free traced, compact closed and strongly compact closed categories over a monoidal category remains a topic for future investigation.

Furthermore, we will work entirely with the *strict* versions of the categories-with-structure we will study. Since in each case, every such category is monoidally equivalent to a strict one, this does not really lose any generality; while by greatly simplifying the description of the free constructions, it makes their essential content, especially the geometry that begins to emerge as we add traces and compact closure (paths and loops), much more apparent.

## 1.4 Diagrammatics

Our free constructions have immediate diagrammatic interpretations, which make their geometric content quite clear and vivid. Diagrammatic notation for tensor categories has been extensively developed with a view to applications in categorical formulations of topological invariants, quantum groups, and topological quantum field theories [15]. Within the purely categorical literature, a forerunner of these developments is the early work of Kelly on coherence [17,18]; while there are also several precursors in the non-categorical literature, notably Penrose’s diagrammatic notation for abstract tensors [22].

Diagrammatic notation has played an important role in our own work with Coecke on applying our categorical axiomatics to quantum informatics, e.g. to quantum protocols [4]. For example, the essence of the verification of the teleportation protocol is the diagrammatic equality shown in Figure 1. For details, see [4,5].

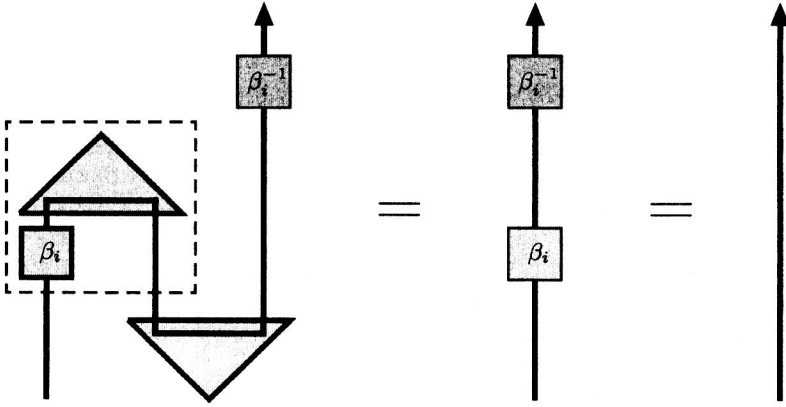


Fig. 1. Diagrammatic proof of teleportation

### 1.5 Categorical Quantum Logic

The diagrammatics of our constructions leads in turn to the idea of a *logical* formulation, in which the diagrammatic representation of a morphism in the free category is thought of as a *proof-net*, in the same general sense as in Linear Logic [11].

More precisely, morphisms in the free category will correspond to proof nets in normal form, and the definition of composition in the category gives a direct construction for normalizing a cut between two such proof nets. One advantage of the logical formulation is that we get an explicit syntactic description of these objects, and we can decompose the normalization process into cut-reduction steps, so that the computation of the normal form can be captured by a rewriting system. This provides an explicit computational basis for deciding equality of proofs, which corresponds in the categorical context to verifying the *commutativity of a diagram*.

In the categorical approach to quantum informatics [4], verifying the correctness of various quantum protocols is formulated as showing the commutativity of certain diagrams; so a computational theory of the above kind is directly applicable to such verifications.

In a joint paper with Ross Duncan [6], we have developed a system of Categorical Quantum Logic along these lines, incorporating additive as well as multiplicative features. This kind of logic, and its connection with Quantum Mechanics, is very different to the traditional notion of ‘Quantum Logic’ [8]. Duncan is continuing to develop this approach in his forthcoming thesis.

### 1.6 Overview

The further structure of the paper is as follows. In Section 2 we explore the abstract notion of scalar which exists in any monoidal category. As we will see, scalars play an important role in determining the structure of free traced and



strongly compact closed categories, as they correspond to the values of *loops*. In Section 3, we review the notions of compact closed and strongly compact closed categories. The need for the notion of strong compact closure, to capture the structure of the complex spaces arising in Quantum Mechanics, is explained. In Section 4, we turn to the free constructions themselves.

*Notation.* We set up some notation which will be useful. We define  $[n] := \{1, \dots, n\}$  for  $n \in \mathbb{N}$ . We write  $S(n)$  for the symmetric group on  $[n]$ . If  $\pi \in S(n)$  and  $\sigma \in S(m)$ , we define  $\pi \otimes \sigma \in S(n+m)$  by

$$\pi \otimes \sigma(i) = \begin{cases} \pi(i), & 1 \leq i \leq n \\ \sigma(i-n) + n, & n+1 \leq i \leq n+m. \end{cases}$$

Given  $\lambda : [n] \rightarrow X$ ,  $\mu : [m] \rightarrow X$ , we define  $[\lambda, \mu] : [n+m] \rightarrow X$  by

$$[\lambda, \mu](i) = \begin{cases} \lambda(i), & 1 \leq i \leq n \\ \mu(i-n), & n+1 \leq i \leq n+m. \end{cases}$$

We write  $\mathcal{M}(X)$  for the free commutative monoid generated by a set  $X$ . Concretely, these are the finite multisets over  $X$ , with the addition given by multiset union, which we write as  $S \uplus T$ .

## 2 Scalars in Monoidal Categories

The concept of a *scalar* as a basis for quantitative measurements is fundamental in Physics. In particular, in Quantum Mechanics complex numbers  $\alpha$  play the role of *probability amplitudes*, with corresponding probabilities  $\alpha\bar{\alpha} = |\alpha|^2$ .

A key step in the development of the categorical axiomatics for Quantum Mechanics in [4] was the recognition that the notion of scalar is meaningful in great generality — in fact, in any monoidal (not necessarily symmetric) category.<sup>2</sup>

Let  $(\mathcal{C}, \otimes, I)$  be a strict monoidal category. We define a *scalar* in  $\mathcal{C}$  to be a morphism  $s : I \rightarrow I$ , *i.e.* an endomorphism of the tensor unit.

*Example 1.* In  $\mathbf{FdVec}_{\mathbb{K}}$ , the category of finite-dimensional vector spaces over a field  $\mathbb{K}$ , linear maps  $\mathbb{K} \rightarrow \mathbb{K}$  are uniquely determined by the image of 1, and hence correspond biuniquely to elements of  $\mathbb{K}$ ; composition corresponds to multiplication of scalars. In  $\mathbf{Rel}$ , there are just two scalars, corresponding to the Boolean values 0, 1.

<sup>2</sup> Subsequently, I became aware through Martin Hyland of the mathematical literature on Tannakian categories [23, 10], stemming ultimately from Grothendieck. Tannakian categories embody much stronger assumptions than ours, in particular that the categories are abelian as well as compact closed, although the idea of strong compact closure is absent. But they certainly exhibit a consonant development of a large part of multilinear algebra in an abstract setting.