Wavelet Applications in Signal & Image Processing VIII

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Akram Aldroubi
Andrew F. Laine
Michael A. Unser
Chairs/Editors

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Invited Paper

Tomographic reconstruction with non-linear diagonal estimators

Jérôme Kalifa[a], Andrew Laine[a] and Peter D. Esser[b]

[a]Department of Biomedical Engineering, Columbia University, New York, NY

[b]Department of Radiology, Columbia-Presbyterian Medical Center, New York, NY

ABSTRACT

In tomographic reconstruction, the inversion of the Radon transform in the presence of noise is numerically unstable. Reconstruction estimators are studied where the regularization is performed by a thresholding in a wavelet or wavelet packet decomposition. These estimators are efficient and their optimality (minimax sense for bounded variation images) can be established when the decomposition provides a near-diagonalization of the inverse Radon transform operator and a compact representation of the object to be recovered. Several new estimators are investigated in different decompositions. First numerical results already exhibit a strong metrical and perceptual improvement over current reconstruction methods. These estimators are implemented with fast non-iterative algorithms, and are expected to outperform Filtered Back-Projection and iterative procedures for PET, SPECT and X-ray CT devices.

Keywords: Global tomographic reconstruction, wavelets and wavelet packets, bounded variation images, minimax optimality.

1. INTRODUCTION

1.1. Tomographic reconstruction

We are interested in the recovery of an image f from its tomographic projections Y, also called sinograms, and defined as:

$$Y = \mathcal{R}f + W \tag{1}$$

where $f[n_1, n_2] \in \mathbb{C}^{N^2}$ is the observed image, W is an additive noise, usually modeled as Gaussian or Poisson, and $\mathcal{R}$ is the discrete Radon transform. The discrete Radon transform is derived from its continuous version $\mathcal{R}_c$, which is equivalent to the X-ray transform in two dimensions and is defined as[1]

$$(\mathcal{R}_c f_c)(t, \alpha) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_c(x_1, x_2)\delta(x_1 \cos\alpha + x_2 \sin\alpha - t)dx_1 dx_2. \tag{2}$$

where $f_c(x_1, x_2) \in \mathbf{L}^2(\mathbb{R}^2)$, δ is the Dirac mass, $\alpha \in [0, 2\pi)$, and $t \in \mathbb{R}$. In the discrete Radon transform, a line integral along $x_1 \cos\alpha + x_2 \sin\alpha = t$ can be approximated by a summation of the pixel values inside the strip $t - 1/2 \leq n_1 \cos\alpha + n_2 \sin\alpha < t + 1/2$.

Tomographic reconstruction is ubiquitous in medical imaging. Imaging devices such as X-ray CTs, Positron Emission Tomography (PET), Single Positron Emission Computerized Tomography (SPECT) measure the density or the metabolic activity of a section of the patient's body (ie, roughly speaking, produce sinograms Y), and an estimation of the image f representing the observed section is derived by a tomographic reconstruction procedure from the sinograms.

A direct inversion of the Radon transform is computed with an amplification of high frequency components of Y in the direction of t, followed by a spatial interpolation. A fundamental difficulty of tomographic reconstruction comes from the fact that the Radon transform is a smoothing transform, and inverting the Radon transform in presence of additive noise is an *ill-posed* inverse problem,[2] because $\mathcal{R}_c^{-1}$ is not a bounded linear operator; numerically speaking, a direct computation of $\mathcal{R}^{-1}f$ would be contaminated by a huge additive noise $Z = \mathcal{R}^{-1}W$.

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1.2. Regularization Procedures

Filtered Back-Projection (FBP) and its derivatives are the most popular regularization methods for tomographic reconstruction. These are linear filtering techniques in the Fourier space. In these procedures, the amplification of high frequencies to compute the inverse Radon transform is attenuated to damp out high frequency components in which the amplitude of the noise $Z = \mathcal{R}^{-1}W$ would be too high. However, FBP suffers from serious performance limitations, due to the fact that the vectors of the Fourier basis provide a good representation (diagonalization) of the Radon operator, but are not adapted to represent spatially inhomogeneous data such as medical images. In other words, the Fourier basis is suboptimal for the denoising and restoration of piece-wise regular signals because it does not provide a compact representation of this type of objects.

To improve the performance of tomographic reconstruction procedures, researchers have studied iterative statistical model-based techniques, designed to implement Expectation-Maximization (EM) and Maximum A Posteriori (MAP) estimators. [3,4] In some cases, these approaches can provide a significant improvement over FBP. However, these estimators suffer from the following drawbacks:

- Computation time. Almost all the corresponding algorithms are too computer-intensive and not usable yet for clinical applications, with the exception of OS-EM[5] (an accelerated implementation of an EM estimator). In MAP methods, useful priors usually give local maxima, and the computational cost of relaxation methods is prohibitive.
- Theoretical understanding and justification. EM estimators lack theoretical foundations to understand and characterize the estimation error. Some MAP estimators are in some cases better understood, yet no optimality for a realistic model has been established.
- Convergence. EM estimators are ill-conditioned, in the sense that the corresponding iterative algorithms have to be stopped after a limited number of iterations. Beyond this critical number, the noise is magnified, and EM and OS-EM converge to a non-ML (Maximum Likelihood) solution.

This study aims at building a family of non-linear estimators, for which the estimation error is understood and the optimality is established for a realistic prior model of the data, which can be implemented with fast non-iterative algorithms, and which provide better numerical results, both metrically and perceptually. Such algorithms should also be flexible enough to be compatible with other specific needs, such as local tomographic reconstruction, limited angle tomography, and post-processing algorithms.

1.3. Non-linear Diagonal Estimators

A general strategy was recently advocated by Kalifa and Mallat[6] to solve linear inverse problems:

1. "Diagonalization" of the problem. Design a decomposition in which the Gram inverse operator -here, $\mathcal{R}^{-1*}\mathcal{R}^{-1}$- of the operator to be inverted is nearly diagonal, and in which the object to be estimated is compactly represented.
2. Design diagonal operators (typically thresholding rules) in this representation to estimate the decomposition coefficients of the signal to be recovered.

The Wavelet-Vaguelette Decomposition of Donoho,[7] which has been studied numerically by Kolaczyk[8] for tomographic reconstruction and refined by Lee and Lucier,[9] and the curvelet-based estimator of Candès and Donoho,[10] are particular examples. Section 2 briefly recalls the underlying formalism and the justifications of this approach. The asymptotic optimality is characterized on a set modeling the prior information on the signals, which often happens, especially for natural images, to be a more realistic and simpler model than probabilistic priors. Section 3 implements this strategy to build simple minimax optimal estimators for bounded variation images in 1-D and 2-D orthogonal wavelet bases. Section 4 investigates the benefits of wavelet packets, as well as directional selectivity on images using better 2-D wavelet transforms. The corresponding estimators have similar minimax optimality properties than the estimators in orthogonal wavelet bases of section 3, but can provide significant numerical enhancements. Section 5 shows a numerical application on a simulation of PET acquisition of the Shepp-Logan phantom. Preliminary results already show a strong metrical and perceptual improvement over Filtered Back-Projection.

Notation: if β_1 and β_2 depend upon the parameters of the problem, such as the signal size N or any other parameter, we write $\beta_1 \sim \beta_2$ and say that they are equivalent, if there exists two constant $A, B > 0$ such that for all values of these parameters $A\beta_1 \leq \beta_2 \leq B\beta_1$.

2. DIAGONAL ESTIMATORS FOR LINEAR INVERSE PROBLEMS

The results summarized in this section are detailed in.[6] We consider the estimation problem in (1), which is also equivalent to the denoising problem

$$X = f + Z \tag{3}$$

where $X = \mathcal{R}^{-1}Y$ and $Z = \mathcal{R}^{-1}W$. The noise W is considered Gaussian of variance σ^2 to simplify the explanations. Z is still Gaussian because the inverse Radon transform is a linear operator. Its covariance operator is $K = \sigma^2 \mathcal{R}^{-1*}\mathcal{R}^{-1}$.

2.1. Thresholding Estimators

A thresholding estimator $\tilde{F}$ of f in a basis $\mathcal{B} = \{g_m\}_m$ is computed with

$$\tilde{F} = \sum_m \rho_{T_m}\left(\langle X, g_m\rangle\right) g_m \ , \tag{4}$$

when the denoising is performed after the application of the inverse operator $\mathcal{R}^{-1}$, or with

$$\tilde{F} = \mathcal{R}^{-1}\left(\sum_m \rho_{T_m}\left(\langle Y, g_m\rangle\right) g_m\right), \tag{5}$$

when the denoising is performed before the application of $\mathcal{R}^{-1}$. ρ_{T_m} is a thresholding operator. T_m are the threshold values, and are different in (4) and (5). Typical thresholding rules include hard thresholding

$$\rho_{T_m}(x) = \begin{cases} x & \text{if } |x| > T_m \\ 0 & \text{if } |x| \leq T_m \end{cases} , \tag{6}$$

and soft thresholding

$$\rho_{T_m}(x) = \begin{cases} x - T_m & \text{if } x \geq T_m \\ x + T_m & \text{if } x \leq -T_m \\ 0 & \text{if } |x| \leq T_m \end{cases} . \tag{7}$$

If the noise Z in (3) was white, the choice of the basis $\mathcal{B}$ would only depend on the prior information on the object f. Indeed, Donoho and Johnstone[11] proved that, with a proper choice of threshold values T_m, thresholding estimators in an orthonormal basis are nearly optimal for white noise removal if the basis provides a sparse signal representation, which means that the basis concentrates the energy of the signal on a few coefficients. In our situation, the choice of $\mathcal{B}$ also depends on the noise Z.

2.2. Near-Diagonalization of the Estimation Problem

The assumption underlying diagonal estimators in a basis is that each coefficient in this decomposition can be estimated independently. As a consequence, such estimators are efficient if the coefficients of the noise and of the object to be recovered are indeed nearly independent in the basis $\mathcal{B}$. This means that $\mathcal{B}$ must provide a near-diagonalization of the covariance operator K of the noise Z and of the prior information on the object f.

The near-diagonalization of the covariance operator K of Z is measured by preconditioning K with its diagonal. Let K_d be the diagonal operator in the basis $\mathcal{B}$, whose diagonal values are equal to the diagonal values of K, noted σ_m^2:

$$\sigma_m^2 = \langle K g_m, g_m\rangle .$$

The square root $K_d^{1/2}$ is the diagonal matrix whose coefficients are σ_m. The diagonal preconditioning factor of K^{-1} in the basis $\mathcal{B}$ is defined as

$$\lambda_{\mathcal{B}} = \|K_d^{1/2}\, K^{-1}\, K_d^{1/2}\|_s \ . \tag{8}$$

It satisfies $\lambda_{\mathcal{B}} \geq 1$. We have $\lambda_{\mathcal{B}} = 1$ if and only if $K = K_d$, which means that K is diagonal in $\mathcal{B}$. The closer $\lambda_{\mathcal{B}}$ is to 1 the more diagonal K.

The best method to measure the near-diagonalization of the prior information on f depends on its nature. In a classical Bayes estimation framework, f is modeled as a realization of a random process F, whose probability distribution is known a priori. However, complex signals such as natural images are highly non-Gaussian, and there is no probabilistic model that incorporates their spatial inhomogeneity. The prior information often defines a set Θ to which signals are guaranteed to belong, without specifying their probability distribution. For example, many images have some form of piecewise regularity, which can be characterized by bounded Besov norms [12] or a bounded total variation, which specify a prior set Θ. With this type of prior model, the near-diagonalization of the prior information can be measured with orthosymmetric sets. Any signal f can be decomposed as $f = \sum_m f_{\mathcal{B}}[m] g_m$. A set Θ is *orthosymmetric* in $\mathcal{B}$ if for any $f \in \Theta$ and for any $a[m]$ with $|a[m]| \leq 1$ then

$$\sum_m a[m]\, f_{\mathcal{B}}[m]\, g_m \in \Theta .$$

This means that the set Θ is elongated along the directions of the vectors g_m of $\mathcal{B}$.

2.3. Minimax estimation

When the prior model on the signals is a set Θ, the estimation error for an estimator $\tilde{F} = DX$ is characterized by the maximum risk over Θ:

$$r(D, \Theta) = \sup_{f \in \Theta} \mathsf{E}\{\|DX - f\|^2\}.$$

The *minimax risk* is the lower bound computed over all operators D:

$$r^n(\Theta) = \inf_{D \in \mathcal{O}_n} r(D, \Theta).$$

In practice, one must find a decision operator D that is simple to implement and such that $r(D, \Theta)$ is close to the minimax risk $r^n(\Theta)$.

Minimax estimation aims at studying the robustness of estimators; In a Bayesian framework, one has to be careful to design a Bayes prior which is not too optimistic, otherwise its performance will be overestimated for some natural signals. This task is difficult for complex signals such as natural images, and yields Bayes priors which are usually close to the "worst" signals of the set Θ used as a model for minimax estimation. This means that, for complex signals, a robust Bayesian prior will be roughly similar to a minimax prior, while more complex to design and with a less convenient formalism.

2.4. Minimax optimality

Let $\mathcal{B}$ be a decomposition in which the covariance K of the noise Z is nearly diagonal with a preconditioning factor $\lambda_{\mathcal{B}}$ defined in (8), and in which the set Θ is orthosymmetric. Let D be a thresholding estimator in $\mathcal{B}$, as defined in section 2.1. The corresponding maximum thresholding risk is $r^t(\Theta) = r(D, \Theta) = \sup_{f \in \Theta} \mathsf{E}\{\|DX - f\|^2\}$, and we clearly have $r^t(\Theta) \geq r^n(\Theta)$. Let us choose $T_m = \sigma_m \sqrt{2 \log_e N}$ for the threshold values in (6) and (7) when the estimator is given by equation (4), and let $T_m = \sigma \sqrt{2 \log_e N}$ when the estimator is given by equation (5). One can show [6] that

$$r^t(\Theta) \sim r^n(\Theta) \lambda_{\mathcal{B}} \log_e N. \qquad (9)$$

This shows that the thresholding risk $r^t(\Theta)$ remains of the same order as the minimax risk $r^n(\Theta)$ up to a $\lambda_{\mathcal{B}} \log_e N$ factor. In some cases, the factor $\log_e N$ can even be reduced to a constant independent of N. $\lambda_{\mathcal{B}}$ is independent of N is $\mathcal{R}^{-1*}\mathcal{R}^{-1}$ is nearly diagonal in $\mathcal{B}$.

We now apply these results to the tomographic reconstruction of images.

3. APPLICATION TO THE INVERSION OF THE RADON TRANSFORM

This section shows that orthogonal 1-D and 2-D wavelet transforms are simple examples of decompositions which satisfy the conditions of section 2 for the inversion of the Radon transform. The prior set Θ of images is defined with a bounded variation norm.

The thresholding estimation for tomographic reconstruction can be performed in the spatial domain, on the signal $X[n_1, n_2]$ of equation (3), which is obtained after the application of the inverse Radon transform. It can also be performed in the Radon domain on the sinograms Y of equation (1), before applying the inverse Radon transform.

3.1. Bounded Variation Images

The set Θ used as a prior model on the object f is defined with a bounded variation norm. Bounded variation images may include sharp transitions such as discontinuities. Large classes of images including medical images, in fact images with no fractal textures, have a bounded total variation. For square discrete image of N^2 pixels, the total variation is defined as

$$\|f\|_V = \frac{1}{N} \sum_{n_1,n_2=0}^{N-1} \left(\left| f[n_1,n_2] - f[n_1-1,n_2] \right|^2 + \left| f[n_1,n_2] - f[n_1,n_2-1] \right|^2 \right)^{1/2} .$$

We say that an image has a bounded variation if $\|f\|_V$ is bounded by a constant independent of the resolution N. Let Θ be the set of images that have a total variation bounded by C

$$\Theta = \left\{ f \ : \ \|f\|_V \leq C \right\} .$$

The co-area formula[13] proves that the total variation has a simple interpretation as the average length of level sets in the image.

3.2. Wavelet bases

1-D Wavelet Basis A discrete orthonormal wavelet basis of $\mathbb{C}^N$ is constructed with a dilated discrete wavelet $\psi_j[n]$ at each scale $1 < 2^j \leq N$. The wavelet ψ_j is translated: $\psi_{j,m}[n] = \psi_j\left[\frac{n-2^j m}{2^j}\right]$, and at the largest scale $\psi_{N,0}[n] = 1$. One can construct such discrete wavelets so that the resulting family

$$\mathcal{B}_1 = \{\psi_{j,m}\}_{1<j\leq \log_2(N),\, 0\leq m<2^j} \tag{10}$$

is an orthonormal basis of $\mathbb{C}^N$. Moreover, the N wavelet coefficients in this basis can be calculated with $O(N)$ operations, with a fast filter bank algorithm.[14]

2-D Wavelet Basis A separable wavelet basis of $\mathbb{C}^{N^2}$ is constructed with the one-dimensional wavelets ψ_j defined in (10). At each scale 2^j there are three wavelets $\psi_j^\alpha[n_1,n_2]$ for $\alpha = 1,2,3$. These wavelets are uniformly translated to define $\psi_{j,m_1,m_2}^\alpha[n_1,n_2] = \psi_j^\alpha\left[\frac{n_1-2^j m_1}{2^j}, \frac{n_2-2^j m_2}{2^j}\right]$, and $\psi_N^0[n_1,n_2] = 1/N$. The separable wavelet family

$$\mathcal{B}_2 = \left\{\psi_N^0,\ \psi_{j,m_1,m_2}^1,\ \psi_{j,m_1,m_2}^2,\ \psi_{j,m_1,m_2}^3\right\}_{1<j\leq\log_2(N),\, 0\leq m_1,m_2<2^j}$$

is an orthonormal basis of $\mathbb{C}^{N^2}$. Fast wavelet decomposition and reconstruction algorithms are implemented with $O(N^2)$ operations, with a separable filter bank algorithm.[14]

Orthosymmetry in Wavelet Bases The set Θ of bounded variation images is not orthosymmetric in the 2-D wavelet basis. However, it can be embedded in two close orthosymmetric sets Θ_1 and Θ_2 such that $\Theta_1 \subset \Theta \subset \Theta_2$. For a given estimator $\tilde{F} = DX$, the estimation risks satisfy $r(D,\Theta_1) \leq r(D,\Theta) \leq r(D,\Theta_2)$. If one can show that $r(D,\Theta_1) \sim r(D,\Theta_2)$, it follows immediately that $r(D,\Theta)$ is of the same order. The set Θ of bounded variation images can be embedded in two sets defined with discrete Besov norms[12] in which the risk equivalence is satisfied for the estimators under study.

The representation of bounded variation images using the 1-D wavelet basis is slightly more difficult to characterize, but follows a similar idea. Images are segmented in different directions, and each direction is studied independently. Once again, the set Θ is embedded in two close sets in which the orthosymmetry can be established for the different directions.

3.3. Near-Diagonalization of the Operator

For a given α, let $\mathcal{R}_\alpha f(t) = \mathcal{R}f(t,\alpha)$, and let $Y_\alpha = \mathcal{R}_\alpha f + W_\alpha$ be the noisy tomographic projection of f at the angle α. The 1-D orthogonal wavelet decomposition in $\mathcal{B}_1$ is used to estimate each projections Y_α; here, the operator to be inverted is $\mathcal{R}_\alpha$, and the operation is repeated for each value of α.

The 2-D wavelet basis $\mathcal{B}_2$ is used to perform the whole estimation on the sinograms Y or on the back-projected data $\mathcal{R}^{-1}Y$ in a single pass, without separating the estimation in each direction α. In that case, $\mathcal{R}$ is the operator to be inverted.

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