Proceedings of the

THIRD INTERNATIONAL OFFSHORE MECHANICS AND ARCTIC ENGINEERING SYMPOSIUM

VOLUME II

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OFFSHORE PLATFORM CONTROL
OFFSHORE LOADING SYSTEMS
SENSORS/MEASUREMENTS
OTEC
WAVE & CURRENT ENERGY
OCEAN CABLE SYSTEMS
UNDERWATER POWER SYSTEMS
MARINE STRUCTURES

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FOREWORD

The International Symposium on Offshore Mechanics and Arctic Engineering (OMAE) has rapidly matured since the first conference in 1982. This has been possible only through the active participation of our colleagues and their unselfish, dedicated team efforts, and through the support of authors and reviewers from industry, R & D Laboratories, and academia around the world. The door to the Symposium has been kept wide open to encourage, welcome, and invite everyone with high quality papers to join our forum. This forum belongs to you and us. This year, NIF (Norway), LCMT (England), SNAJ (Japan), and other ASME Divisions joined the ASME Petroleum Division in sponsoring the Symposium. It has been, is, and will be the purpose of this forum to promote new developments in offshore mechanics and arctic engineering, to establish a forum for meaningful transfer of technology, and to promote cooperation among us all at an international level.

This Symposium was initiated and organized in 1982 by The Editorial Board members in Offshore Mechanics and Arctic Engineering of The ASME Transactions Journal of Energy Resources Technology with the help of the ASME Petroleum Division. After that offshore mechanics group formed Offshore Mechanics Committee and the arctic engineering group returned to the Arctic Committee and the Offshore Mechanics Committee, of the Petroleum Division with the positive cooperation with the Transactions Journal. However, the major Symposium coordinating and organizing activities lie in the OMAE Technical Program Committee which has been the central organization of inter-society, individual, and the ASME organizers. That is, this Symposium belongs not only to the Societies, but to individuals.

At the inception of this Symposium in 1981, it was foreseen that interdisciplinary offshore mechanics and arctic engineering would be the backbone of offshore technology, and that the fields of offshore mechanics and arctic engineering would be merging into arctic offshore technology. This year, the merging of these interdisciplinary fields is clearly shown at this Symposium. For these interdisciplinary fields, as we have encouraged, more realistic integrated solution approaches in the industry of hydrodynamics, structural mechanics, dynamics and control are appearing at this Symposium with advanced numerical methods and applications of microprocessors.

The Proceedings contain only peer-reviewed papers presented at 40 paper sessions and the keynote paper. The paper reviews have been conducted according to ASME Review Criteria, requiring two or more reviews by experts in individual topic areas. In most cases, the reviewers have provided objective, competent, constructive opinions and comments that helped the authors prepare the final revised manuscripts.

This year, emphasis is placed on deepwater technology and arctic technology such as recent advances in theories, computational methods, model- and full-scale experiments, and on the application of these results to sound design, engineering analyses, and operational problems.

Volume I contains papers on deepwater platforms, marine risers, submarine pipelines, responses of the offshore and arctic structures to earthquakes, vortex-shedding-induced vibrations together with a large body of papers on fluid-solid interactions, fatigue and fracture, and buckling and collapse loads.

Dr. Wenk, Jr., the keynote speaker presents his views and advice to fellow engineers on socio-technical, legal, and risks of offshore mechanics, ocean and arctic engineering on the basis of his long, outstanding career in both technical and public policy on ocean and arctic engineering affairs. On Tension Leg Platforms, the platform is treated as a deformable body, results being supported by carefully conducted experimental data. Linear and nonlinear analyses and experimental results of detailed TLP and guyed tower systems are presented.

Fluid-structure-(soil) interactions are extensively treated with a broad range of problems. Starting with the influence of surface waves on seabottom topography, papers present linear and nonlinear interactions of waves with platforms and structural members and revisit the Morison-type equations. Platform motions, stresses, and multibody interactions are investigated with comparisons with experimental measurements. Three-dimensional effects on hydrodynamic coefficients and comparisons of FEM results with experimental data and experimental and analytical investigation of floating platform members are presented together with full-scale measurements and analyses of a jack-up platform.

On the topics of vortex-induced vibrations, recent investigations on the effect of current and waves on pipeline vibrations are presented along with experimental investigation of the effects of steady and wave-induced currents. For the first time for this Symposium, papers discuss broad aspects of the responses of offshore and arctic offshore structures to earthquakes. These range from theoretical to experimental to design and safety analyses of structure, and to soil for gravity structures and SPM system.

Papers on fatigue treat welded joints and offshore structures. Noteworthy are fatigue crack growth, corrosion fatigue and low temperature effects for welded joints, ARMA simulation for fatigue tests, and aerodynamic load effects. The less-often investigated, fatigue in submersible design is also discussed.

Marine risers, drill pipes, and submarine pipelines are extensibly tested, bringing out new approaches and results. New modeling and case study of submarine pipeline buckling and collapse loads are presented with plastic and elasto-plastic analyses. Design philosophy, waves-pipeline-soil interactions, theoretical model for the seabottom effect, hydroelastic fracture model, and design practice are brought out to readers. Papers on marine risers present new facets in approaches to solutions of single risers together with experimental data on riser arrays, that include hydrodynamic coefficients. Also new are papers on recent applications of structural mechanics and finite element methods to the designs of pipe threads and casings.

Volume II contains papers on offshore platform controls, offshore loading systems and dynamics, offshore sensors and measurements, and NDE for the maintenance of offshore structural systems. Also contained are papers on ocean energy systems from many countries with R & D activities (OTEC, wave, and current energy), and ocean engineering papers on ocean cable systems and ship fenders.

Papers on controls are refreshing ones. Offshore production systems with articulated structures or multibody systems have been investigated, some including their interactions with tanker motions or mooring systems. Investigation results are technically sound, and practical in applications. Complementing the analytical design methods, full-scale measurements and discussions of analyzed data with predictions are well documented for a free standing conductor pipe, that would be of interest to the designers and operators. New findings on gyroscope and gyrocompass are revealed. Underwater inspection and maintenance of offshore platforms and underwater systems have been of great importance: four papers present various automated NDE inspection techniques and systems.

New in a large scale to this year's Symposium are ocean energy systems. Through the participation of Northern Ireland, Japan, England, Ireland, and U.S.A., papers present OTEC (ocean thermal energy conversion) systems and systems of energy from ocean current and salinity difference. OTEC papers deal with engineering aspects of both closed- and open cycle systems with biofouling and corrosion of heat exchanger materials. CWP cold water pipe) systems developments, construction, and at-sea deployment and tests, including an 8-ft vertically suspended fiberglass pipe are reviewed. Many CWP systems are of pipeline concept with non-conventional pipeline materials for deployment along steep slopes. Papers on wave energy conversion present different concepts and devices, and their status are reviewed. Also, some full- and model- scale test results are presented with carefully conducted hydrodynamic studies. Other ocean energy systems utilize current and salinity differences.

In the vein of traditional ocean engineering, papers on ocean cables present bending and friction characteristics and operations for armored cables and for underwater power transmission. Also presented are studies on ship-fender dynamics and the Coast Guard's ocean ice problems.

Volume III contains papers on arctic engineering that merge arctic offshore with conventional offshore mechanics.

The success of this Third Symposium should be credited to our organized teamwork members of the OMAE Symposium Technical Program Committee who have helped solicit papers, handle technical reviews, or organized sessions. Furthermore, we would like to thank the individual reviewers, who have helped us maintain technical accuracy and quality of the papers within the short review time being given. As a part of our staff team, Mr. Paul Drummond, ASME Petroleum Division Executive Secretary and Mrs. Catherine Natal, Staff Coordinator have excellently assisted in the administrative matters of the Final Program and given the final touch on the OMAE Symposium Proceedings preparation.

Jin S. Chung
Symposium Chairman and
Chairman, Offshore Mechanics
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The success of this Symposium should be credited to our organized teamwork members who have helped either solicit the papers or handle the technical reviews.

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OFFSHORE MECHANICS II

CONTROL OF MARINE STRUCTURES

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ABSTRACT

Active control of the motion of marine structures may produce structural and operational savings over conventional designs.

It is shown how equations of motion of typical marine structures, including those with frequency dependent coefficients, may be rendered into a form compatible with modern controller design methods. Linear quadratic Gaussian control theory is summarised and applied to a simple example. The limitations of the theory are discussed and the effects of a force limited actuator are quantified.

NOMENCLATURE

A Dynamic state matrix
B Input matrix
C Output matrix
D Differential operator
$\underline{e}$ Error vector
G Observer gain
h Wave elevation
I Roll inertia
J Cost function
K Feedback gain
k Spring constant
R Cost matrix
$S(\omega)$ Spectrum
T Exciting torque
t Time
$\underline{u}$ Input
W White noise power
$\underline{x}$ State vector
$\underline{y}$ Output
λ Damping
θ Angular displacement
ω Frequency

Subscripts
s System
d Disturbance

INTRODUCTION

Conventional marine structures are designed with 'passive' control of their dynamic response. If predicted displacements caused by environmental or operational loads are too large, the usual remedies are:

a) To reduce the coupling between the force and the structure by changing the geometry.

b) To increase the stiffness of the structure to decrease deflections and raise natural frequencies above the excitation frequency.

c) Less commonly, mass may be added to decrease natural frequencies below the exciting range, or passive dissipative components can increase the damping ratio.

Extra inertia, spring or damping components carry out the three functions of a feedback controller; they sense displacement, 'compute' an appropriate response and generate the desired force.

The limited repertoire of simple objects which possess only stiffness, inertia and damping may be inadequate in some applications. If the three functions of measurement, computation and force generation are separated, structures with sophisticated adaptive characteristics may be designed. In the aerospace industry where constraints on weight are tight and the value of increased performance is high, the benefits of active control are widely recognised. The Lockheed Airbus, for instance, uses wing tip accelerometers to control the ailerons to minimise wing root bending stresses and hence to extend the structures fatigue life with negligible weight penalty. (ref. 1)

Active control offers some substantial advantages over conventional passive designs. Some difficult analytic problems may be made irrelevant by feedback control; dynamic ship positioning is effective even if the relationship between the second order drift forces and the first order waves is not well known. (ref. 2)

Otherwise attractive designs with poor dynamic response might be improved by a controller which need only be active during severe conditions. Passive systems are designed to withstand the complete envelope of operating conditions while actively controlled systems can adapt to optimise their response to prevailing conditions and objectives.

In the onshore civil engineering field, there is considerable interest in the control of bridges and buildings (ref. 3). Most of this work is directly applicable to the control of structural modes of marine systems; in this paper, we survey possible applications of active control in ocean engineering. Subsequently, we summarise the mathematical models required and outline the structure and design of an optimal controller. The technique is illustrated by an application to the suppression of the roll of a barge.

APPLICATIONS OF CONTROL IN OCEAN ENGINEERING

The most difficult and novel aspect of control in marine systems is the design of the actuator which applies the control forces. Fortunately, recent work on wave energy has created much interest in the design of systems which can extract energy from the motion of marine structures; these power conversion devices are, of course, ideal motion suppression actuators.

A wide variety of pneumatic, hydraulic and mechanical systems have been suggested. Air pressure over the free surface can force the heave, pitch and roll modes of semi-submersibles, tension leg platforms, barges and ships. Roll and pitch can also be suppressed by Salter's novel gyroscope power canister (ref. 4). Tube pumps (ref. 5) may be incorporated in mooring systems as active or passive dissipative elements to control longitudinal motions. Slosh tanks (ref. 6) or servo driven inertias (ref. 7) may be used to modify the sway dynamics of jackets, guyed tower or jackups.

More conventionally, a mooring system incorporating actively controlled winches (ref. 8) can decrease peak mooring forces and suppress complicated non-linear oscillations in single buoy mooring configurations, amongst others. Thrusters are commonly used to control slow drift, while ballast tanks can be used to trim or suppress heave, roll and pitch. Active fin stabilisers are only useful if a substantial ahead speed can be maintained; roll of slowly moving or stationary vessels is best controlled by gyros or pneumatic systems.

The same mathematical techniques can be applied to design controllers for all the systems listed above; passive systems may only represent an approximation to the calculated optimum but may be more economical and reliable in some applications. Broadly speaking, active systems are likely to be acceptable in the lower frequency ranges while (perhaps actively tuned) passive systems will be applied to suppress higher frequency vibration.

MATHEMATICAL MODELS

Most structures of interest can be represented by approximately linear equations if the motions are small which will be the case if the controller works well. These equations can be modified to the form needed by Linear Quadratic Gaussian control theory (ref. 9), a powerful controller design method for linear systems disturbed by random signals. Small non-linearities are usually unimportant since the closed loop system dynamics are robust with respect to small variations in the system model (ref. 9).

A more important problem is the description of those components usually characterised by a function of frequency (ref. 10) such as added mass and damping, exciting force operators and, indeed, wave spectra. A variety of numerical 'identification' (ref. 11) techniques may be used to approximate these functions of frequency by a small set of differential equations whose frequency response is the one desired. A particularly powereful technique, due to Maciejowski (ref. 12) can generate guaranteed stable, minimum phase approximate differential equation models from either spectra or complex amplitude responses; it has been used in the example described below.

The theory which follows is illustrated by application to a simple but important marine problem, the rolling of a barge. Frequency dependent variations of the added mass and damping have been neglected since they are usually small with respect to the frequency independent inertia and viscous damping (ref. 13); in other applications where they materially affect the system dynamics, these coefficients may be represented by a small number of linear differential equations (ref. 14) whose coefficients can be calculated by Maciejowski's technique.

Roll inertia is denoted by I, damping by λ and spring by k. An exciting force $T(t)$ and a control force, $u(t)$ generated by the actuator, affect the roll displacement $\theta(t)$ according to the second order differential equation

$$(ID^2 + \lambda D + k)\ \theta(t) = T(t) + u(t) \qquad (1)$$

We define $x_1(t) = \theta(t)$, $x_2(t) = \dot{\theta}(t)$ to transform this second order equation into two first order equations.

$$\frac{d}{dt}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K/I & -\lambda/I \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I \end{bmatrix} (T(t)+u(t)) \qquad (2)$$

or, more concisely

$$\dot{\underline{x}}_s(t) = A_s \underline{x}_s(t) + B_s(T(t)+u(t))$$

A differential equation model which transforms white noise into a signal with the spectrum of $T(t)$, the exciting torque, is used in the controller design. This set of differential equations must have a transfer function which approximates the product of the wave spectrum, $S(\omega)$ with the exciting force operator, $R(\omega)$. The spectrum may be a parametric spectrum such as Pierson-Moskwitz or a measured spectrum, specified at a number of discrete frequency points. In any case, the exciting force operator is likely to be specified at a set of discrete frequency points.

In Maciejowski's algorithm, the product spectrum $R(\omega) \times S(\omega)$ is inverse Fourier transformed to yield an autocorrelation function; this is used to generate a Hankel matrix which is subequently processed by a combination of algorithms proposed by Kung and Faurre (refs. 15, 16) to give a discrete time approximate realisation. The model is guaranteed to be stable and minimum phase.

For a continuous time controller design, the discrete time model must be transformed into continuous time; this is easily achieved by inversion of the standard continuous - discrete relations given in ref. 17. The final result is the set of matrices A_d, B_d, C_d, in a a linear differential equation (3) which transforms white noise, $w(t)$, into an exciting torque signal, $T(t)$, with the desired spectrum, $R(\omega)\times S(\omega)$.

$$\dot{\underline{x}}_d(t) = A_d \underline{x}_d(t) + B_d w(t) \qquad T(t) = C_d \underline{x}(t) \tag{3}$$

The states $\underline{x}_d(t)$ do not correspond to physically obvious variables; they define as much of the future of the signal as can be inferred from its past behaviour.

Figure 1 illustrates the result of applying the procedure to a Pierson - Moskwitz spectrum; the agreement is excellent except at very low frequencies where the barge does not respond particularly strongly; a higher order model could remove this discrepency. The model has 5 states; the A_d, B_d and C_d matrices which define the transfer function are shown below.

$$A_d = \begin{bmatrix} .00254 & -.449 & -.00474 & .0149 & .0117 \\ .449 & -.216 & -.0808 & .164 & .0996 \\ .00475 & -.808 & -.0638 & .445 & .165 \\ .0149 & -.16 & -.445 & -.317 & -.330 \\ .0117 & -.0996 & -.165 & -.330 & -.617 \end{bmatrix} \quad B_d = \begin{bmatrix} .229 \\ -.489 \\ -.496 \\ -.441 \\ -.741 \end{bmatrix} \tag{4}$$

$$C_d = [-.103 \quad -.333 \quad -.0877 \quad .0656 \quad .0343]$$

The body and noise dynamics can be combined

$$\frac{d}{dt}\begin{bmatrix} \underline{x}_s(t) \\ \underline{x}_d(t) \end{bmatrix} = \begin{bmatrix} A_s & B_s C_d \\ 0 & A_d \end{bmatrix} \begin{bmatrix} \underline{x}_s(t) \\ \underline{x}_d(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_d \end{bmatrix} w(t) + \begin{bmatrix} B_s \\ 0 \end{bmatrix} \underline{u}(t) \tag{5}$$

$$\underline{y}(t) = [C_s \quad 0] \begin{bmatrix} \underline{x}_s(t) \\ \underline{x}_d(t) \end{bmatrix}$$

This is the formulation of the system dynamics needed for the design of a controller which will generate the 'best' values for the actuator force, $\underline{u}(t)$.

Controller design

Optimal linear regulator theory (Linear, Quadratic Gaussian, LQG theory) (ref. 9), is a controller design method for linear systems, disturbed by white noise. Only the bare essentials of the technique are described here.

The controller is designed to minimise a weighted sum of the mean square values of the system inputs, $u(t)$ and outputs, $y(t)$.

$$J = \int_0^\infty (\underline{u}^T(t) \; R_1 \underline{u}(t) \; + \underline{y}^T(t) \; R_2 \underline{y}(t))dt \tag{6}$$

The outputs are a linear combination of the system states and do not include any of the disturbance states, $\underline{x}_d$, which cannot be controlled or measured since they do not correspond to any obvious physical quantities. In the barge example, the output y might correspond to displacement, θ, in which case $C_s = [1.0]$.

The matrices R_1 and R_2 are weightings which fix the cost of actuator power relative to the 'cost' of output mean square. Their choice is a matter of engineering judgement; if R_1 is too small, excessive actuator power will be demanded while if R_2 is too small, the actuators will be under used and motions will be larger than necessary. Different values of R_1 and R_2 will be appropriate to different conditions and operating objectives. No 'hard' constraints on input or output value can be specified in LQG design; the controller weightings must be chosen so that the input value demanded exceeds that deliverable by the actuator 'acceptably' infrequently. If the actuators are non-linear and the costs of input and output are not strictly commensurable, a number of simulations may need to be carried out with different cost functions before a solution which meets more complicated realistic constraints and objectives is found.

The regulator generates the desired input signal as a linear combination of the states.

$$\underline{u}(t) = [K_s \; K_d] \begin{bmatrix} \underline{x}_s(t) \\ \underline{x}_d(t) \end{bmatrix} \tag{7}$$

The gain K_s can be calculated from the body dynamics and the cost weightings R_1 and R_2 while K_d depends additionally on the nature of the noise dynamic. Not all the states are measurable, so they must be reconstructed from the measured outputs (which may be contaminated by noise) and the measurable control input. The state reconstructor, usually called an observer or Kalman - Bucy Filter should be an exact model of the system and noise dynamics. It is driven by the measurable system inputs and some linear function of the difference between its own output and that of the system. Figure 2 shows a block diagram of the physical system - observer combination; the equations of the observer are, with $\hat{\underline{x}}(t)$ denoting the reconstructed state, and A_T, B_T the combined noise and system matrices

$$\dot{\hat{x}}(t) = A_T \hat{\underline{x}}(t) + B_T u(t) + G(\underline{y}(t) - \hat{\underline{y}}(t) - w_2(t)) \tag{8}$$

The behaviour of the error, $\underline{e}(t)$, between the exact and estimated states is easily found.

$$\dot{\underline{e}}(t) = \dot{\underline{x}}(t) - \dot{\hat{\underline{x}}}(t) = (A_T - GC)\underline{e}(t) + w_1(t) - G\, w_2(t) \tag{9}$$

Once again, standard techniques can calculate a G which will minimise the mean square value of the error, given values for the matrices A and C and covariances of the driving and measurement white noise signals $w_1(t)$ and $w_2(t)$. The gains, K_s, K_d are the same, whether or not the states are reconstructed by an observer. The gain matrix K_s is independent of the nature of the disturbance. Control costs are increased by the error between the estimated and exact (but inaccessable) states.

The performance of the composite system - observer - controller can be evaluated from their state equations.

$$\frac{d}{dt}\begin{bmatrix} \underline{x}_s \\ \underline{x}_d \\ \hat{\underline{x}}_s \\ \hat{\underline{x}}_d \end{bmatrix} = \begin{bmatrix} A_s & B_s C_d & B_s K_s & B_s K_d \\ 0 & A_d & 0 & 0 \\ G_1 C_s & 0 & (A_s - G_1 C_s + B_s K_s) & C_d + B_s K_d \\ G_2 C_s & 0 & -G_2 C_s & A_d \end{bmatrix} \begin{bmatrix} \underline{x}_s \\ \underline{x}_d \\ \hat{\underline{x}}_s \\ \hat{\underline{x}}_d \end{bmatrix} + \begin{bmatrix} 0 \\ B_d w_1 \\ G_1 w_2 \\ G_2 w_2 \end{bmatrix} \tag{10}$$

or, for brevity

$$\dot{X} = Z\underline{X} + W \tag{11}$$

The mean square response, $Y = E(XX^T)$ is (ref. 9) the solution to the Lyapunov equation, which may be found by standard methods.

$$ZY + YZ^T + E(WW^T) = 0 \tag{12}$$

Barge roll suppression

For illustrative purposes, we have taken the roll dynamics of a barge and assumed it to be excited by the rate of change of wave height of the fifth order Pierson Moskwitz spectrum discussed earlier.

$$I\ddot{\theta} + \lambda\dot{\theta} + k\theta = k\dot{h}(t) \tag{13}$$

With $x_1 = \theta$, $x_2 = \int \theta dt$

$$\dot{x}_1 = -\lambda\, x_2/I - kx_1/I + kh(t)/I \tag{14}$$
$$\dot{x}_2 = x_1$$

Note that the states have been chosen to eliminate the differential of the wave elevation signal since the model of the PM spectrum generates wave height, not its rate of change.

The parameters k and I were chosen so that the roll resonance of the barge was 6.86s, relatively close to the peak of the forcing spectrum (10s) and λ was set to give a realistic damping constant of 0.1. Since the entire problem is linear, the process noise w_1 was chosen to yield a significant wave height of one metre. The measurement noise w_2 was (arbitrarily) set to be about 1% of the uncontrolled response signal.

The response was calculated for a number of different cases:

1) Control with exact state feedback.
2) Control with states reconstructed by an exact observer.
3) Control with states reconstructed by an observer incorporating an approximate disturbance model.
4) Control with states reconstructed by an observer incorporating no disturbance model.
5) No control.

As the problem is scalar, only the ratio, r, of output cost is significant; case 2 was run for a range of values of this ratio. Roll position was the only available output.

Figure 3 presents the degree of motion suppression achieved, and the input root mean square for a range of cost weightings, r. The rms roll displacement is halved for an r value of about 4.25 and the input root mean square required to achieve this is some 55% of the rms exciting force. At high r values, the benefits in reduced motions are necessarily finally overwhelmed by increases in control cost. All the subsequent calculations involved controllers designed with an r value of 4.25; the cost function was therefore

$$J = \int_0^\infty (4.25x_1{}^2 + u^2)dt \tag{15}$$

The cost achieved for the five cases listed above are listed below; the cost given is the expected average cost.

Cost(kNm)		Configuration
0.1002	:	state feedback
0.1188	:	feedback with states reconstructed by an exact observer.
0.1369	:	feedback from an observer with an approximate disturbance model.
0.1719	:	feedback from an observer with no disturbance model.
0.3013	:	no control.

The approximate disturbance model was a second order system with roughly the same peak frequency and bandwidth as the PM spectrum. This test was run to evaluate the sensitivity of the control to the quality of the disturbance model.

Costs for the approximate model rise by 15% while removal of the disturbance model increases them by 45%. In contrast, switching the controller off completely increases costs by 153%; it is clear that most of the reduction in roll amplitude is due to feedback from the system states and not to feedforward from the disturbance model states. This is because the exciting force spectrum is fairly broad so the disturbance signal is fairly unpredictable. Had it been multiplied by a narrow exciting force operator, it would have been narrower, hence more predictable and the noise states would have had a greater effect on the control. In some applications where the exciting force spectrum is wide with respect to the dynamic response of the system, no disturbance model is needed for good control.

Where the disturbance prediction materially improves the control, a practical system must contain an adaptive subsystem (ref. 2) to continuously update the model and controller parameters to adapt to changes in the exciting force spectrum. Alternatively, a self tuning regulator might provide acceptable performance (ref. 18).

Any practical actuator will be limited to some maximum force and may well be subject to more complex dynamic constraints. Salter's gyro system, which might be used in this application, can provide any restoring torque, up to some maximum value determined by the system parameters; additionally, there will be a limit to the speed of response which should lie outside the frequency range of interest.

Here, we have run a simulation of the system and noise dynamics to investigate the change in controller performance as the force limit is changed. Figure 4 displays the motion suppression, mean square controller force and proportion of the time spent with the control force on the limit as a function of the force limit. It is clear that the limit of the controller must be at least twice the rms force required if a substantial part of the theoretically achievable motion suppression is to be realised in practice. Note that the man square actuator force is above its unconstrained values if the force limit is high; the controller does extra work to minimise motions and applies force at non-optimum times to make up for being unable to generate enough force at the right time.

THEORETICAL PROBLEMS

Two of the most obvious problems, the modelling of the exciting spectrum and limits on controller force have been investigated in a cursory way above. It has been shown that even an approximate disturbance model improves control quality and that the force limit of the actuator does not degrade control quality unacceptably so long as the limit is more than 2.5 times the rms force required.

An adaptive subsystem (ref. 2) or a self tuning controller should be able to handle the effects of slowly varying forcing spectra. It seems much more difficult to derive an algorithm which takes some account of actuator non-linearity. In marine control, controller power or force constraints are likely to be far more serious than computational speed limitations since the time constants are an order of magnitude slower than those common in aerospace applications.